Decimals

Teacher Lessons

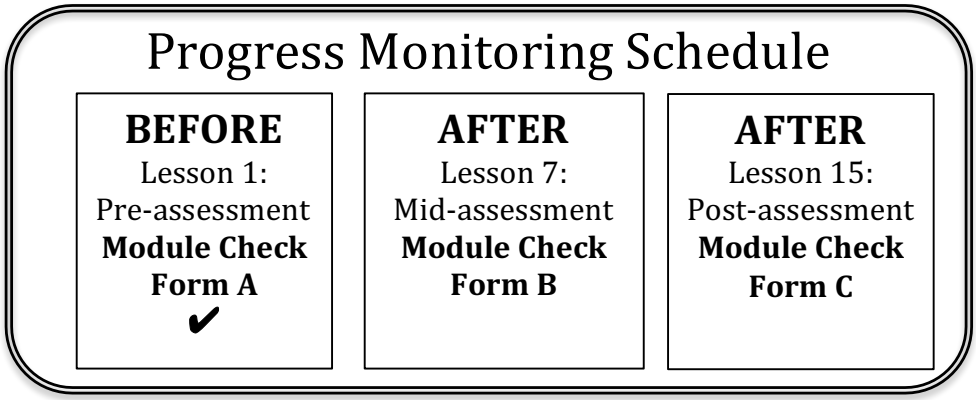
The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Decimals
Lesson 1

Lesson 1: Introduction to Decimals

Lesson Objectives	Students express fractions with denominators of 10 and 100 as decimals. Students express decimals as decimal fractions. Students use area models to represent decimals and decimal fractions. Students reason quantitatively with decimals and fractions. (SMP 2) Students construct arguments and critique the reasoning of others. (SMP 3)	
Vocabulary	**Deci-:** prefix meaning 1 tenth **Decimal fraction:** a fraction that has a denominator of a power of 10 (10, 100, 1,000, and so on) **Equivalent decimals:** 2 or more decimal numbers that name the same quantity or amount **Equivalent fractions:** 2 or more fractions that name the same quantity or amount **Generalization:** formulating and producing statements about patterns and relationships and evaluating their reasonableness	
Requisite Vocabulary	Decimal number, numerator, denominator	
Misconceptions	Students think that the numbers that follow the decimal point are whole numbers.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • 10-by-10 grids • Whiteboard • Dry erase marker

	• Chart paper or Poster Board • Decimal and Fraction Cards (see page 126 of Teacher Masters)	• Decimal and Fraction Cards (see page 126 of Teacher Masters) • Red colored pencil

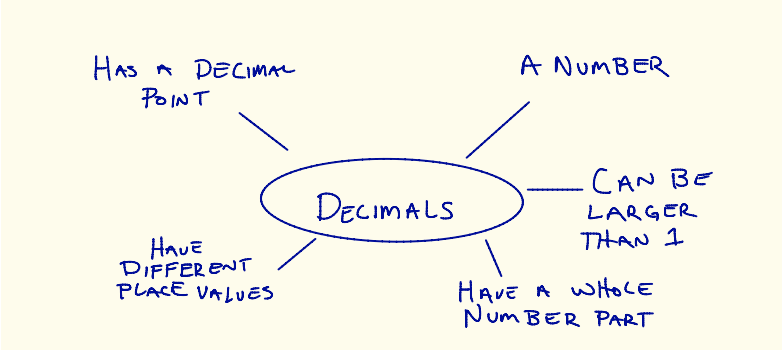


Warming Up

Have students turn to the Warming Up sheet in their Student Booklets. Draw a circle on the chart paper or poster board with "Decimals" in the middle (similar to the one below). Display it for students to see.

Today, we will start by thinking about what you know about decimals. Write everything you know about decimals in your Student Booklet.

Ask for student responses. Ask for students to clarify or provide examples to aid in the brainstorming of ideas. Add ideas to the concept map.



What are some important ideas? What is a decimal? What are examples of your ideas?

Display the concept map for the duration of this module. The concept map will be used again at the end of this module. Displaying during the course of the module will allow both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

Teacher Notes

Be sure to read 444.4 as "444 **and** 4 tenths." Refrain from reading it as "444 **point** 4." Reading it with "**and**" helps to emphasize the whole number and parts of a whole. At times in the lesson(s), you will be directed to rad, for example, zero point 4, but these are fairly rare and obvious in the lesson.

When students are asked to write decimal fractions, they may not make the connection that the denominator has to be a power of 10. For students who do not write a correct decimal fraction, write an example, ask what they notice about the denominator, and then show how the denominator of the decimal fraction shows us how to write a decimal.

The grids that students use in this lesson model decimals with area. If students do not make this connection, begin with a simple rectangle and a fraction like $\frac{3}{4}$. Have students partition

and show the fraction. Then show how the grids can be used for the fractions in this lesson. Each square can be thought of one tenth of the area, rather than, for example, 30 squares out of 100. This focuses on a continuous model, rather than a discrete model.

When students write decimals, model writing those less than 1 as "0.2," not ".2." Writing this way will help students understand that the digits to the right of the decimal point represent a part of a whole. The 0 acts as a placeholder for the whole number portion. Writing decimals this way links with the way in which we read decimals.

1. Students will model decimal fractions.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   On the Learning to Solve sheet in your Student Booklet are 4 10-by-10 grids. What do you notice about the grids? *(answers will vary, such as they have 10 rows and 10 columns or there are 100 squares in the grid)*

   By shading, use the first 2 10-by-10 grids to represent $\frac{6}{10}$ in 2 different ways.

   Again, by shading, use the second 2 10-by-10 grids to represent $\frac{60}{100}$ in 2 different ways.

   Allow students time to work. As you monitor their work, look for students who use different representations by shading the grids in different ways. Ask 4 to 6 students who have used different shadings to share their work. Project their work or provide another way so the rest of the class can view their representations.

   What do all of these grids have in common? *(they all have 60 area units or squares shaded)*

What does it mean that all of the grids have the same amount of area shaded? *(they all represent the same quantity)*

When fractions represent the same amount or quantity, we can say they are equivalent fractions. Below the fractions $\frac{6}{10}$ and $\frac{60}{100}$, write "equivalent fractions."

Now turn to the Notes section of your Student Booklet. Write the fraction $\frac{7}{10}$ in your Notes section. What is an equivalent fraction that has 100 has the denominator? *($\frac{70}{100}$)* **Write $\frac{70}{100}$ next to $\frac{7}{10}$; you now have 2 examples of equivalent fractions.**

What would the numerator be of a fraction that is equivalent to $\frac{7}{10}$ and that has 1,000 as its denominator? *(700)* **Write $\frac{700}{1000}$ next to $\frac{7}{10}$ and $\frac{70}{100}$; you now have 3 examples of equivalent fractions.**

2. Students will identify and write decimal fractions.

   Give each student a whiteboard.

   There is a special name for fractions that have denominators such as 10, 100, 1,000, 10,000, and so on. We call them decimal fractions. We use the term "decimal" because "deci-" is related to the number 10. "Decimal" means that the numbers are based on powers of 10.

   On your whiteboard, write 2 decimal fractions.

   Ask students to share their fractions on the classroom whiteboard. Watch for fractions that do not have powers of 10 as the denominators.

   After the fractions have been written on the classroom whiteboard, have students make the generalization. Note that some students may say multiples of 10. This is partially correct because, for example, 70 is a multiple of 10, but using it as a denominator will not create a decimal fraction. Model writing the generalization on the board and have students copy it into their notes.

What do you notice about all of the fractions that are listed? *(they all have powers of 10 as a denominator)*

When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed that decimal fractions all have powers of 10 as a denominator. Powers of 10 means 10 multiplied by itself a given number of times.

Display the Generalization sheet in the Notes section in the Teacher Masters (p. 140). Have students turn to the Generalization sheet in the Notes section on page 140 in their Student Booklets.

On the Generalization sheet, write this generalization in the first column: "Decimal fractions all have powers of 10 as a denominator." In the next column, draw a picture if possible to represent the generalization. In the last column, write the example of the decimal fraction that you represented in the picture.

3. Students will connect decimal fractions to traditional decimal notation and the appropriate place value.

Decimal fractions can be written in a way that is not a fraction. Instead of writing fractions, we can represent them by using a decimal point.

Point to problem 1 on the Learning to Solve sheet.

This number is 444 and 4 tenths. This number has been written in a table, so we can show the value of each place.

Point to the hundreds place.

This 4 is in what place value? *(hundreds)* **Write the word "Hundreds" in the first column of the first row. Now, write 100 below the word "Hundreds."**

What is the value of four one-hundreds? *(400)* **Write 400 below the 4 in the table.**

Point to the tens place.

> **This 4 is in what place value?** *(tens)* **Write the word "Tens" in the first row, and 10 below it.**
>
> **What is the value of four tens?** *(40)* **Write 40 below the 4 in the table.**

Point to the ones place.

> **This 4 is in what place value?** *(ones)* **Write the word "Ones" in the first row, and 1 below it.**
>
> **What is its value?** *(4)* **Write a 4 below the 4 in the table.**
>
> **What is the relationship between 4 and 40?** *(answers may vary, such as you can multiply 4 by 10 or divide 40 by 10; the 10s place value position is 10 times larger than the 1s place value position, the 10s place value position is one-tenth of the 100s place value position)*

Write "multiply by 10 or divide by 10" on the board. Write the examples as the script progresses.

> **What is the relationship between the tens place value position and the ones place value position?** *(you can multiply 4 by 10 or divide 40 by 10; the 10s place value position is 10 times larger than the 1s place value position, the 10s place value position is one-tenth of the 100s place value position)*

Point to the blank box at the top row of the 4 tenths.

> **If the pattern is to multiply or divide by 10 as you move to a new place value, what is the place value for this 4?** *(tenths)*
>
> **Write "Tenths" above the last 4. The 4 is in the tenths place because when we divide the ones place by 10, we get $\frac{1}{10}$, or tenths. Write zero point one below Tenths.**

Point to the tenths place again.

What is a way to write 4 tenths without using a decimal point? ($\frac{4}{10}$)

Write "$\frac{4}{10}$" and "0.4" on the board.

4 tenths can be written as $\frac{4}{10}$ or zero point four. Write zero point four below four tenths in the table.

From this example, we can see 2 ways to write a number that represents parts of a whole. We can use a decimal number or a fraction.

Write "$\frac{3}{10}$" on the whiteboard.

What is the name of this fraction? *(3 tenths)*

Just as the fraction $\frac{3}{10}$ refers to 3 one-tenths of the total area, the decimal notation of zero point three also refers to three-tenths of the total area.

What is another fraction that is equivalent to $\frac{3}{10}$? *(answers may vary, such as $\frac{30}{100}$, which is a likely response)*

Using the 10 by 10 grids on the Learning to Solve sheet, work with your neighbor to model $\frac{3}{10}$ and $\frac{30}{100}$ by shading. Be prepared to share with the class how you know that they are equivalent.

Go over the student graphs and call upon one student to explain how $\frac{3}{10}$ and $\frac{30}{100}$ are equivalent.

Display the 10 by 10 grid representation of 0.23 on the Learning to Solve sheet from the Teacher Masters. Do not write "23 hundredths" on the board—just refer to the quantity verbally. Show the grid with 2 columns and 3 squares shaded.

How much of the area of this grid is shaded? *(23 hundredths)*

Based on this picture, how many tenths are in 23 hundredths? *(2 tenths)* **How do you know?** *(because the equivalent of 2 columns has been shaded and each column is 1 tenth of the total area)*

How much area is shaded in addition to the 2 tenths? *(3 hundredths)*

Based on that, what is another way to describe 23 hundredths? *(2 tenths and 3 hundredths)*

On your whiteboard, write a decimal and a fraction to represent the portion of the area of this grid that is shaded. *(answers may vary—for example, 0.23 or 0.230, $\frac{23}{100}$ or $\frac{230}{1{,}000}$)*

Ask students to share their decimal or fraction on the board. Be sure to ask for any different ways to write this quantity.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

For problems 1 through 8, write each of the decimal fractions as a decimal number and each of the decimal numbers as a decimal fraction. Work independently. When you are finished, compare your answers with your partner.

Ask multiple students to share their answers for each part of the sheet. Write, or have students write, their numbers on the board.

Did anyone write a different decimal number? If so, how do you know that your number is equivalent?

Is there another decimal number that is equivalent?

If no one shares an alternate form of the decimal number, ask students to think of another decimal number that is equivalent by looking at the 10 by 10 grids they completed. For example, 0.3 could also be written as 0.30. Write equivalent numbers on the board under the original representation. As students justify their answers, they may point out that

because 3 tenths and 30 hundredths cover the same area, they are equivalent decimals.

> **Pair with a partner and solve problem 9 together. Pay careful attention to the four response choices and choose the one that provides the best answer. When you are done, compare your answer to that of another student pair.**

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 1A to provide extended practice before proceeding to Lesson 2.

Wrapping It Up

Cut and distribute the Decimal and Fraction Cards to each student (see page 126 of Teacher Masters). If less than 12 students, ensure that the distributed cards all have matches. If an odd number of students, keep one for yourself as a participant.

> **Each of you** (or **us**) **has a card with either a fraction or a decimal number. Find the person whose number represents an equivalent amount.**

Give students time to find their match. Have them share the two equivalent amounts. Watch for students who partner with the wrong person.

Decimals
Lesson 2

Lesson 2: Comparing Decimals and Fractions

Lesson Objectives	Students compare decimals with the same number of digits (up to thousandths) to identify the greater (or lesser) decimals, using models (length and area). Students compare decimals and fractions. Students model decimals in multiple ways. (SMP 4) Students reason quantitatively with decimals. (SMP 2) Students construct arguments and critique the reasoning of others. (SMP 3)	
Vocabulary	**None**	
Requisite Vocabulary	Decimal fractions, decimals	
Misconceptions	Students may think that the number of digits determines the size of the decimal—for example, 2.034 is greater than 2.3.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Decimal and Fraction Cards from Lesson 1: Mixed for Lesson 2 (see page 126 of Teacher Masters) • Red colored pencil

Warming Up

Students will review writing decimals from decimal fractions and decimal fractions from decimals.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **What is a decimal fraction?** *(A fraction with a power of 10 as a denominator)*
>
> **Look at the decimal numbers and decimal fraction in your Student Book. We used them in the first lesson. Match them to show equivalent quantities or amounts. Use letters to show the matches; so place an A in both boxes of your first match, B on both boxes of your second match, and so on.**

Provide time for students to work. Have students share their matches.

Then, show a decimal number from one of the cards.

> **How do you read this decimal number?**

Select a student to read. Listen for students to read the decimal correctly using "and" rather than saying "point." As the student reads, model how to write the decimal. Then, ask the student to write the decimal fraction that is equivalent. Then reverse the process; identify a decimal fraction and call upon a student to write and say the equivalent decimal number.

Model if students have difficulty in writing and saying decimal numbers from decimal fractions.

Learning to Solve

Teacher Notes

Students will use number lines in this lesson to order decimals. This practice will help students compare magnitudes of decimals and other equivalent forms, such as fractions.

> Make sure that students read decimals appropriately. Model how to read the decimals as needed. For example, if students read decimals with "point," model reading the decimals with "and." For example, 3.25 should be read as "3 and 25 hundredths."

1. Students will model decimals on a number line.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   In Lesson 1, you modeled decimals with an area model, the 10 by 10 grids. Today, we will use a linear model, a number line, to model decimals.

   A decimal number is similar to a fraction; it can represent part of a whole. For this number line, the whole is 0 to 1. How is the number line partitioned? *(into hundredths)*

   Work with your partner to decide where you would find the decimals on the number line.

   Have students share their number lines and solutions.

   How did you decide where the decimals belonged? *(answers will vary, such as for 16 hundredths we counted from 0, until we counted 16 one one hundredths and saw that it matched with B)*

   When students read the decimal numbers or provide their explanations, they may refer to place value ideas as part of how they decided where to place the decimals. As students share their answers, label the points on the number line with the matching decimal.

   Point A did not have a match. How could you name point A with a decimal? *(0.21)*

   What do you notice about the decimals on the number line? *(answers will vary, such as the tenths are in order, ordering decimals between 0 and 1 is comparable to ordering whole numbers)*

2. Students will compare decimals.

> **Look at the next 2 decimals on the Learning to Solve sheet. Read each decimal aloud.** *(37 hundredths, 42 hundredths)*
>
> **We can compare decimals by comparing their place values. In this case, 0.42 has the larger value in the tenths place. What does the 4 in 0.42 represent?** *($\frac{4}{10}$)* **What does the 3 in 0.37 represent?** *($\frac{3}{10}$)*
>
> **Is 4 tenths always greater than 3 tenths plus 7 hundredths, or 37 hundredths?** *(yes)* **Why?** *(answers will vary, such as 4 tenths will always be farther to the right from 0 on the number line)*
>
> **When we compare numbers between 0 and 1, the number with the larger tenths value is always larger.**
>
> **What compare symbol, greater than or less than, should be placed between 0.37 and 0.42?** *(less than)* **Write it.**

Direct students to the second compare problem: 0.137 ____ 0.132.

> **Look at the next two decimals. How would you read these decimals?** *(137 thousandths and 132 thousandths)*
>
> **With your partner, identify which decimal represents the lesser amount in this pair. Be prepared to explain your reasoning.**

Provide time for students to work.

> **What compare symbol, greater than or less than, should be placed between 0.137 and 0.132?** *(greater than)* **How do you know?** *(accept reasonable answers, such as the tenths and hundredths are equal, 7 thousandths is greater than 2 thousandths)* **Write it, and then do the last comparison.**

Direct students to the last compare problem:
0.53 ___ $\frac{1}{2}$.

> **How could we compare this decimal and fraction?** *(accept reasonable answers, such as 0.53 is greater than 0.50 which is $\frac{1}{2}$,*

changing the decimal to a fraction, changing the fraction to a decimal, or modeling both numbers with a linear or area model)

If any of these methods are not mentioned, suggest them to the class.

How are decimals and fractions similar? *(accept reasonable answers such as they can both represent parts of a whole)*

How could I write the fraction $\frac{1}{2}$ as a decimal? *(0.5)* **How do you know?** *(one-half is equivalent to five-tenths, which is written as a decimal 0.5)*

Select a student to share his or her symbol (greater than).

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner, small groups, or as a class to solve the problems.

   Solve the Practicing Together problems. Be ready to share how you decided on your answers.

2. When students have finished, ask them to share their answers. Discuss how they decided on their answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes sheet in their Student Booklets.

> **In the Notes section of your Student Booklet, describe how you would decide which decimal is larger: seven hundred and three thousandths or seven hundredths and thirty thousandths.** *(0.703 < 0.730)*

When students have finished, ask them to share their answers.

Lesson 3: Order Decimals and Fractions

Lesson Objectives	Students compare and order decimals (up to thousandths) and fractions. Students reason quantitatively with decimals. (SMP 2) Students construct arguments and critique the reasoning of others. (SMP 3)	
Vocabulary	**Descending:** to decrease, order from greatest to least **Ascending:** to increase, order from least to greatest	
Requisite Vocabulary	**None**	
Misconceptions	Students may think that the number of digits determines the size of the decimal.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Decimal and Fraction Cards (I set per pair, see page xxx of Teacher Masters)	• Student Booklet • Whiteboard • Dry erase marker • Decimal and Fraction Cards (1 per pair, see page 127 of Teacher Masters) • Red colored pencil

Warming Up

Students will write a decimal number with specific conditions. By restricting the range to $0.9 < x < 1.0$, students will create decimals with multiple digits, allowing you to explore the magnitude of decimals with a diverse set of digits. This list will be used in Learning to Solve.

Give each student a whiteboard.

> **On your whiteboard, write a decimal number that is greater than 0.9 but less than 1.0. Everyone will have to share at least 1 decimal number, so if your decimal number is already given by another student, be prepared to say another decimal number.**

Make a list on the board of students' decimal numbers. Be sure not to write the numbers in ascending or descending order.

> **Look at the list. Which decimal numbers are greater than 0.95? Which decimal numbers are less than 0.95?**
>
> **As we look at the order of this list, what would make it more useful?** *(answers may vary but should include that an ordered list can be more useful than a list that is not in order)*
>
> **When would an ordered list be helpful?** *(answers will vary, such as when looking for the greatest or least in a list, finding the range of a list; encourage students to be creative)*

Learning to Solve

Teacher Notes

As students order decimals, watch for students who use the number of digits as a way of determining magnitude. This misconception stems from overgeneralizing the process used with whole numbers. Monitoring these students is important during the Practicing Together section. Students may need to refer to a place value chart or to write the equivalent fractions to better understand that place value determines magnitude.

1. Students will order decimals from greatest to least.

   Point to the list created from the Warming Up section.

   Let's start today by putting these decimal numbers in order from greatest to least, or descending order. Descending order means we start with the greatest number and end with the least number.

   We can start by finding the greatest number—the number that is larger than each of the other numbers.

   Point to the list. As students state the decimal numbers, create a new list.

   Which of these decimal numbers is the greatest?

   What do you notice about all of the decimals? *(they all have 9 in the tenths place value position)*

   The tenths is the largest place value position of all of the decimals we have in the list. Because the digit in the tenths place value position is the same for all of the numbers, what should we do next? *(look at the next place value position, hundredths)*

Have students check the hundredths place value position. Follow a similar process until students decide which is the greatest.

> **So we don't forget a number from our original list, we could cross off each number as we rewrite it in our ordered list.**
>
> **How will we know which number is the next largest number?** *(accept reasonable answers, encouraging students to look for the number that is larger than all the remaining numbers by using place valuer; there may be numbers that are the same or have to move to thousandths or beyond to order)*
>
> **What is the next largest number?**

Continue this pattern until only 1 number is left.

> **How do we know that this is the least number in the list?** *(answers may vary. For example, all of the other numbers were larger than it because we compared the place values)*

2. Students will order rational numbers from least to greatest.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Let's put this list in order from least to greatest, or in ascending order. What is different about this list from the last list we looked at?** *(it has fractions and decimals)*
>
> **How can we compare fractions and decimals?** *(answers may vary, such as changing all of the numbers to decimals, changing all of the numbers to fractions, modeling the numbers on 10 by 10 grids, or modeling the numbers on a number line)*
>
> **What does ascending mean?** *(to increase, order from least to greatest)*
>
> **Put this list in ascending order by either changing all of the decimals to fractions or all of the fractions to decimals.**

Have students share their list.

> **Does everyone agree with this order?**

Who changed the fractions to decimals? Why? *(answers will vary, such as there were fewer decimals to change or the student prefers decimals instead of fractions)*

Who used this method for a different reason?

Who changed the decimals to fractions? Why? *(answers may vary, such as the student prefers fractions)*

Who used this method for a different reason?

It is important to check the instructions after we finish a problem to make sure that we answer the question correctly. This question asked us to put these numbers in ascending order. We should check to be sure the numbers increase in size.

Select one or two students to share their order and discuss their answers as needed. If the selected students do not provide the correct order, share their answers, write the list on the board as 0.908, $\frac{93}{100}$, 0.98, 1.27, $\frac{23}{10}$, 2.54.

Practicing Together

Have the students work in pairs. Distribute a stack of the Decimal and Fraction Cards (see page 127 of Teacher Masters), 1 set to each pair of students. Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets. As the student work, circulate and provide error feedback as needed.

Each pair of students has a stack of Decimal and Fraction Cards. When I say "Go," take turns flipping over the top 2 cards in your stack. Work together to identify the greater number. When you finish the stack, shuffle all of the cards and repeat. The second time you do this, in your Student Booklet, write which cards were drawn on each turn and write the number you decided was greater.

Go.

Allow students time to work.

> **Stop. Shuffle the deck.**
>
> **We will change the game. When I say "Go," take turns flipping over the top 3 cards in your stack. Work together to put the cards in ascending order. When you finish the stack, shuffle all of the cards and repeat. The second time you do this, write in your Student Booklet the numbers pulled and then write the numbers in ascending order.**
>
> **Go.**

Allow students time to work.

> **Stop. Shuffle the deck.**
>
> **We will change the game one last time. When I say "Go," take turns flipping over the top 4 cards in your stack. Work together to put the cards in descending order.**
>
> **Notice that we changed the order. Be careful that you put the cards in the right order. When you finish the stack, shuffle all of the cards and repeat. The second time you do this, write in your Student Booklet the numbers pulled and then write the numbers in descending order.**
>
> **Go.**

Allow students time to work; walk around and check for understanding as the students work in their pairs.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 3A to provide extended practice before proceeding to Lesson 4.

Wrapping It Up

Have students answer the following questions by writing in the Notes section of their Student Booklets.

Write the answers to these questions in the Notes section of your Student Booklets. What is one place where you have seen an ordered list? Was it an ascending or descending list? Why was the order important for that list?

As time permits, have students share their answers.

Decimals
Lesson 4

Lesson 4: Using Benchmark Decimals to Compare Decimals

Lesson Objectives	Students compare decimals by using benchmark decimals (0, 0.25, 0.5, 0.75, and 1). Students reason quantitatively. (SMP 2) Students construct arguments and critique the reasoning of others. (SMP 3)	
Vocabulary	**Benchmark decimals**: decimals that can be used to judge the magnitude or size of other decimals (0, 0.25, 0.5, 0.75, and 1)	
Requisite Vocabulary	Expanded form, estimate, equivalent decimals	
Misconceptions	Students may think that there is 1, and only 1, benchmark decimal to which another decimal can be close. However, some decimals have multiple benchmark decimals that would be appropriate such as 0.35.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 10 x 10 grids (see page 128 of Teacher Masters)	• Student Booklet • Whiteboard • Dry erase marker • Red colored pencil

Warming Up

Students will review writing decimals in a restricted range.

Give each student a whiteboard.

> **Write 3 decimals that are less than five-tenths but greater than 0.**

Allow time for students to write the decimals. Have students share their decimals by either writing them on the class whiteboard or saying the decimals as you write them.

Display a 10 by 10 grid (see page xxx of Teacher Masters).

> **Look at our list of decimals. Are all of the decimals listed less than 0.5 but greater than 0? Let's think how we could determine whether we agree with all of them.**
>
> **How could we determine whether a decimal is less than five-tenths using 10 by 10 grids?** *(create a grid for 0.5, which will have 50 squares shaded, and for the other decimal; the grid that covers less area is the smaller decimal)*
>
> **How could we determine whether a decimal is greater than 0.5 by looking at the digits?** *(if the digit in the tenths place is greater than 5, the decimal is greater than 0.5 and if there are any digits other than zero to the left of the decimal point, the decimal is greater than 0.5)*
>
> **What are some other ways we could determine whether a decimal is less than 0.5?** *(allow a few students to explain their ideas and help them refine the idea if they are not correct)*
>
> **Based on our ideas of how to determine whether a decimal is less than 0.5 but greater than 0, are there any decimals that are listed that are not within this range?**

Discuss any decimals that students identify, using a method described above.

As time permits, have students erase their whiteboards. Select 2 decimals from the list they created. Have them write a comparison statement on their whiteboard, such as 0.47 > 0.351. Have them share their statements by holding up their whiteboards. Discuss as appropriate.

Learning to Solve

Teacher Notes

If you find that students try to be too precise in associating a decimal to a benchmark decimal, you may want to use only 0, 0.5, and 1.

Benchmark decimals are closely related to the fraction benchmarks (0, $\frac{1}{2}$, and 1). Students may be familiar with benchmark fractions. If so, as you go through the lesson, you may want to refer to benchmark fractions to provide the link between decimals and fractions.

1. Students will model benchmark decimals.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Select a student to read the problem.

   Follow along as [student] reads.

   Pause while the student reads the problem.

   Complete the problem now.

   Allow students time to work.

   How did you solve? Why did you choose the number line or 10 by 10 grids to support your answer?

   Discuss students' answers.

How does the 10 by 10 grid representation show that eight-tenths is closer to seventy-five hundredths than five tenths? *(answers may vary, such as because 80 squares are shaded in the 0.8 model, 75 squares shaded in the 0.75 model, and 50 squares shaded in the 0.5 model, the area shaded in the 0.8 model is closest to the 0.75 model)*

How does the number line representation show that eight-tenths is closer to seventy-five hundredths than five tenths? *(answers may vary, such as because there is less distance between 0.8 and 0.75 than between 0.8 and 0.5, 0.8 is closer to 0.75 than to 0.5)*

Write the benchmark decimals 0.25, 0.5, 0.75, and 1 on the board.

The decimals 0.25, 0.5, 0.75, and 1 are called benchmark decimals. These decimals are common decimals that you are familiar with. Why do you think you are familiar with these decimals? *(accept reasonable answers, such as they are used with money representing a quarter, 2 quarters, 3 quarters, and a dollar)*

Display the Learning to Solve sheet in the Teacher Masters (10 x10 Grid; see page 128 of Teacher Masters). Have students turn to the four 10 x 10 Grids in Learning to Solve in their Student Booklets.

Model the decimals 0.25, 0.5, 0.75, and 1 on the 10 by 10 grids on the next page of the Learning to Solve.

After students complete their models, have students share their models and discuss any differences that exist among the models.

Even though some of you may have shaded differently, what do you notice about all of your models for the same benchmark decimal? *(all have the same number of squares shaded or the same area shown)*

The shaded area might be configured differently, but the same number of shaded squares means that each model for the same decimal represents the same area.

Now, model each benchmark decimal on the number lines on the next page of the Learning to Solve. For each benchmark

decimal, write an equivalent decimal, meaning a different decimal number that names the same quantity or amount.

Allow time for students to work.

What did you write?

As students share their equivalent decimals, make a list.

What do you notice about the equivalent decimals? *(equivalent decimals have 0s in the place values to the right)*

2. Students will use benchmark decimals (0, 0.25, 0.5, 0.75, and 1) to compare and order decimals.

We can use these benchmark decimals to compare and order other decimals. How do you think we could do that? *(determine which benchmark decimal each decimal is closer to and then compare the decimal numbers)*

When might it be helpful to use benchmark decimals to compare and order decimals? *(responses will vary but should include when the decimals are not common decimals or as a way to check our answers)*

Discuss how to determine which benchmark decimal is the closest to another decimal.

How could we determine which benchmark decimal is closest to seven-tenths without using a model? *(accept reasonable answers, such as 0.7 is only .05 less than 0.75 but 0.2 more than 0.5)*

Have students turn to the Notes section in their Student Booklets. Discuss how to use benchmark decimals to order decimals.

Write the following decimals in the Notes section of your Student Booklet: 0.7, 0.87, and 0.59.

I can use benchmark decimals to order in ascending order, or smallest to largest. For example, we could say that seven-tenths is just a little smaller than seventy-five hundredths, eighty-seven hundredths is larger than seventy-five

hundredths, and fifty-nine hundredths is just a little larger than fifty hundredths. Notice that fifty hundredths and five tenths are equivalent decimals.

Write the comparisons: 0.7 < 0.75, 0.87 > 0.75, and 0.59 > 0.50.

Order the non-benchmark decimals in ascending order.

What is the order? *(0.59, 0.7, 0.87)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

In pairs, have students determine which benchmark decimal each decimal number is closest to.

With your partner, decide which benchmark decimal is closest to each decimal. Circle your choice. Refer back to the models of the benchmark decimals from earlier in the lesson if you need to. I will ask some of you to share your answers and how you decided on your answers.

After students have completed the task, discuss students' answers as a class, using the language from the lesson.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **We have talked about the benchmark decimals 0, twenty-five hundredths, five tenths, seventy-five hundredths, and 1. What decimals might we use as benchmark decimals that are larger than 1? Write your answer on your sheet.**

As time permits, have students share their answers.

Lesson 5: Adding Decimal Numbers

Lesson Objectives	Students estimate the sum of decimal numbers. Students add decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)
Vocabulary	**Expanded form:** rewriting a number to show it as the sum of its place values **Expanded form algorithm:** an addition algorithm that uses the expanded forms for the addends to add each place value **Front-end estimation: a method for estimating where the digits to the right of the leading digit are truncated** **Compatible numbers:** addends are changed to be "easy" numbers to work with but are close to the original addends **Traditional algorithm:** the conventional algorithm that begins by adding the furthest right place value and progresses to the left, regrouping as needed
Requisite Vocabulary	Place value, estimate, benchmark, partial sums
Misconceptions	Students may not align the appropriate place-value digits. Students may instead align from the right-most digits or the left-most digits, resulting in an incorrect sum.

Instructional Materials	Teacher	Student
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry erase marker • Red colored pencil

Warming Up

Students will review the expanded form of whole numbers.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Write the number 627 on the board.

> **How would you read this number?** *(627)*
>
> **Write 627 in expanded form at the bottom of your sheet.** *(600 + 20 + 7 = 627)*
>
> **Use your understanding of place value of decimals and the process we used with 627 to write each number in expanded form.**

Have students display their answers. Watch for students who incorrectly write out the decimals with a 0 to the right of the decimal point. For example, some students may incorrectly write that 6.01 = 6 + 0.1.

Learning to Solve

TEACHER NOTES

Students will use benchmark decimals to estimate sums. Estimating may not always match due to the benchmark decimal used. You may want to discuss this idea if it becomes an issue and students want one correct answer. Other estimation strategies may include front-end (the numbers to the right of the leading digit are truncated) or compatible numbers (addends are changed to be "easy" numbers to work with but are close to the original addends).

Students will have the opportunity to select an algorithm that matches their skill level. The traditional algorithm and expanded form method will be introduced. Students may feel more comfortable using the expanded form until they transition to the traditional algorithm. Another effective algorithm is partial sums. Students add the digits in each place value. Students can add from left-to-right or right-to-left. This method allows students who struggle with directionality of the more traditional algorithm to preserve place value and be more accurate in their adding.

1. Students will determine how to estimate the sum of 2 decimal numbers.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   > **I was adding 2 decimal numbers last night. I added 0.79 to 0.03. I said the sum was 1.09. Without using pencil, paper, whiteboards, or calculators, work with a partner and decide whether you agree with my calculation.**

   Allow students time to work.

   > **Is this answer reasonable?** *(no)*

How did you determine that the sum was not reasonable? How can we use benchmark decimals? *(accept reasonable answers, such as using benchmark decimals to see that 0.79 is close to 0.75 and 0.03 is close to 0; 0 + 0.75 = 0.75, so the sum should be close to 0.75)*

Another way to check the reasonableness of the sum is to use front-end estimation. Many of you may have used this method but did not know the formal name.

In front-end estimation, you "chop off," or truncate, the digits to the right of the leading digit. So, 0.79 is thought of as 0.7 and 0.03 is thought of as 0.0. What is 0.7 + 0.0? *(0.7)*

Both of these methods, benchmark decimals and front-end estimation, are helpful when we want to estimate. Why would it be important to estimate when we are adding? *(allows you to check reasonableness of an answer and can help you predict the size of your answer before you start to work)*

Have students turn to the Notes section in their Student Booklets. Write "0.07 + 1.21" on the board.

Turn to the Notes section of your Student Booklet.

Let's describe the 2 ways we just used to estimate. Use 0.07 + 1.21 as your example problem. Write it.

When estimating the sum with benchmark decimals, what would we do first? *(decide which benchmark decimal each addend is closer to)*

What benchmark decimal is 0.07 and 1.21 closer to? *(0.0 and 1.25)*

Write "0.0 + 1.25" under the original expression. Also write "Decide which benchmark decimal each addend is closer to."

By using benchmark decimals, what should the sum be close to? *(1.25)*

Using the same example, let's try front-end estimation. What would we do first? *(rewrite each addend, using only the first digits)*

What digits are "chopped off," or truncated? *(7 and 1)*

Write 0.0 and 1.2 under the original expression.

By using front-end estimation, what should our sum be close to? *(1.2)* **Will it be greater than 1 or less than 1?** *(greater than 1)* **How do you know?** *(by chopping off, you reduce the sum, so the sum would be greater than one)*

Note that with front-end estimation, you can also truncate or "chop off" all of the numbers following the decimal point. Thus, another solution is 0.0 and 1.0, which sum to 1.

2. Students will add decimal numbers.

Once again, display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Call attention to the problem 1.47 + 0.91.

Write an estimate of the sum in your Student Booklet, using either benchmark decimals or front-end estimation. Write which method you used to make your estimate.

What was your estimate? *(if using front-end estimation, the estimate is 1 or 2.3; if using benchmark decimals, the estimate is 2.5)*

What method did you use? *(front-end estimation or benchmark decimals)*

Did anyone get a different estimate?

If students have different estimates, write them on the board

Are any of these estimates unreasonable? How do you know?

Let's add the decimals to get an exact sum. Keeping the place value positions lined up is important when adding decimals. It

may help to rewrite any problems written in a horizontal form and have the place-value positions line up.

Let's rewrite 1.47 + 0.91 in the vertical format and find the sum.

$$\begin{array}{r} 1.47 \\ +\ 0.91 \\ \hline \end{array}$$

What is the sum? *(2.38; check for students who incorrectly respond two point three eight)* **How did you solve? Does anyone disagree with the sum? Did anyone solve it differently?**

Let's try solving it another way to check our sum. This time we will add, using expanded form. First, write both numbers in expanded form.

What is 1.47 in expanded form? *(1 + 0.4 + 0.07)* **What is 0.91 in expanded form?** *(0.9 + 0.01)*

Now we can add the numbers in the same place value. Start in the hundredths. What do we add? *(0.07 + 0.01)* **What is seven hundredths + one one-hundredth?** *(0.08)* **Now add the tenths and the ones places. What is the sum?** *(2.38; if students respond 1 + 1.3 + 0.08, encourage students to further evaluate the expression)*

Complete the rest of the problems on the Learning to Solve sheet. First, estimate by using benchmark decimals, front-end estimation or other strategies you may know. Then, solve by lining up the decimal points or using expanded form.

Allow students time to work.

What is your estimate? What method did you use? Did anyone have a different estimate?

What is the sum? How did you solve? Does anyone have a different sum?

As time allows, provide additional problems to practice the algorithms.

Practicing Together

To allow more time for Learning to Solve, this lesson has no Practicing Together section.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students use their whiteboards and markers. Give them 2 decimals to add. Rather than finding the exact sum, ask them to write their estimate.

> **I'm going to give you an addition problem. Do not find the exact sum. Use one of your strategies to write an estimate of the sum**.

Consider using problems such as: 1.25 + 0.4; 0.15 + 2.03; 1.01 + 1.9; and so on. After you give the problem, allow students time to write an estimate. Since this is only an estimate, you should keep the time relatively short, such as 20 seconds. Have students hold up their whiteboards so that you and others can see. Compare their estimates as time allows.

Decimals
Lesson 6

Lesson 6: Solving Contextual Problems by Adding Decimal Numbers

Lesson Objectives	Students solve contextual problems by adding decimal numbers. Students estimate the sum of decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Estimate, front-end estimation, benchmark, partial sums, expanded form	
Misconceptions	Students often search for key words to help them decide on the operation to use with contextual problems. These key words include "altogether" and "total." Students may also look at the magnitude of numbers to decide which operation to use. Students should be encouraged to think about the actions or relationships in problems to determine which operation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Notecard, 1 per student • Calculator (optional) • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Look at the computation. It was done incorrectly. First determine how you could use estimation to explain why the answer isn't reasonable. Then, identify the error and, finally, do the computation correctly.**

Have students complete the sheet individually or, if necessary, as a whole class, to review how to estimate and add decimals.

> **How did you estimate?** *(answers may vary, such as front-end estimation or benchmark decimals)* **Did anyone use a different method?**
>
> **How did you identify the error? What was done wrong?** *(Incorrect alignment of place value positions)*
>
> **How did you compute?** *(answers may vary, such as used partial sums, traditional, or expanded form)* **Does anyone disagree with the sum? Did anyone solve it differently?**

Learning to Solve

Teacher Notes

In this lesson, students will apply the skill of adding decimal numbers to solve contextual problems. The focus is on the summing of parts to determine the whole. The contexts in the problems are focused on the joining aspect of addition.

Focus student attention, if they are using the traditional algorithm, on aligning place-value positions rather than lining up the decimal point. This will emphasize the conceptual understanding rather than procedural.

1. Students will solve contextual problems involving addition of decimal numbers by first estimating and then completing the computation.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as the lesson progresses.

   Select a student to read the first problem.

   Follow along as [student] reads the first problem.

   Pause while the student reads.

   How much cheese did Sandra need for the recipe? *(2 pounds)* **So, could these 2 packages of cheese be used to make her dinner?** *(no)* **Why?** *(less than 2 pounds)*

   If Sandra were at the store, could she have used her estimation skills or would she have had to complete the actual computation? *(estimation)*

   How could estimation have helped her decide that it was not enough cheese? *(each package of cheddar cheese is a little less than 1 pound, so both packages will be just less than 2 pounds)*

   Now, let's add to find the exact total.

   When we add decimal numbers, what is 1 important thing we need to remember? *(digits in the same place value should be added)*

   Add the numbers to find the amount of cheese she has.

   Allow students time to write.

   How many pounds of cheddar cheese? *(1.79 pounds)*

   Ask a student to explain the method used to add the decimal numbers. Have the students turn to the Notes section of their Student Booklets.

   How did you add the 2 decimal numbers?

   Did anyone use a different method to add?

In the Notes section of your Student Booklets, write the method and steps that you used in words to find the sum of the 2 decimal numbers.

Give students time to record their process. As they work, monitor their writing. If students are not precise, help them individually or as a group to use the appropriate vocabulary and include significant details.

Display the Learning to Solve sheet in the Teacher Masters. Have students return to the Learning to Solve sheet in their Student Booklets.

Students may use calculators to complete the magic square. You may also write the decimal numbers on the board as a type of "number bank."

The next problem is a magic square. A magic square is an array of numbers in which the sum of all rows, columns, and diagonals are the same. For this magic square, the sum of every row, column, and diagonal is 15.3.

We have to find 5 different numbers to complete the square. How can we start? *(allow reasonable answers, such as work across the first row or first column)*

There are a couple of spots to begin, but let's start with the first row. What decimal numbers are in the first row? *(6.12 and 1.02)* **We know that when we add these 2 numbers and a third number, the sum will be 15.3. What do we do first?** *(add 6.12 and 1.02)*

What is the sum of 6.12 and 1.02? *(7.14)* **How can we find the third number?** *(accept reasonable answers, such as fact family knowledge, subtract 7.14 from 15.3, count up from 7.14 to 15.3)*

There a few ways to solve, but I will count up from 7.14 to 15.3. Let's estimate. How many do we need to add to 7 for a sum of 15? *(8)* **Based on this estimate, the missing number will be close to 8.**

Right now, I have 14 hundredths, but I need to have only tenths to sum to 15.3. What number added to 4 would result in 0? *(6)* **Write a 6 in the hundredths place.**

If we were adding 4 + 6 = 10, we would regroup and now we have 1 tenth plus 1 tenth. What is 1 tenth plus 1 tenth? *(2 tenths)* **Remember, we are trying to fund the sum 15.3. What digit do we need in the tenths place?** *(3)*

What number added to 2 tenths equals 3 tenths? *(1 tenth)* **Write a 1 in the tenths place. Now we need to identify the digit in the ones place. 7 plus what number equals 15?** *(8)* **Write an 8 in the ones place. What number did we form?** *(8.16)*

Check your work. What is 6.12 + 1.02 + 8.16? *(15.3)*

Working with a partner, find the remaining missing numbers.

Allow students to work. Review the missing numbers.

How did you solve? Did anyone use a different method?

Have students work in pairs to complete the last item. Select one or two groups to provide responses to A and B.

Practicing Together

To allow more time for Learning to Solve, this lesson has no Practicing Together section.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Give each student a notecard. Using the decimals 3.35, 2.15, and 5.5, they will each write a word problem. Their word problem should include all 3 decimals in some way. One of the decimals may be the answer to the problem or the situation may include all 3 decimals with a different answer.

> **Each of you has a notecard. You are going to write a word problem that uses the decimals 3.35, 2.15, and 5.5. One of the decimals may be the answer to your problem or the situation may include all 3 decimals with a different answer.**

Allow students time to write their problem. Then, have them switch cards with someone in the class. That person will solve the problem and return the card back to the original student to check it. Discuss as needed.

Lesson 7: Estimating the Difference of Decimal Numbers

Lesson Objectives	Students estimate the difference of decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Estimate, difference, front-end estimation, rounding, benchmark decimals	
Misconceptions	Students often think subtraction can solve only "take-away" situations. Other contexts, such as comparison and missing addends, can also be solved with subtraction.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Select a student to read the problem.

Look at the problem and follow along as [student] reads.

Pause while the student reads the problem.

What do we need to find? *(how much Samson weighs)*

What is an estimate of Samson's weight? *(about 350 pounds; accept a range of 330 to 350 pounds)*

How did you determine your estimate? *(accept reasonable answers, such as 302.78 is close to 300 pounds, 38.49 pounds is close to 50 pounds, and 300 + 50 = 350)*

Have students solve the problem. Select a student to share the solution.

How much does Samson weigh? (*341.27 pounds)* **Is the answer reasonable?** *(yes, because it is close to the estimate)*

Did anyone solve the problem in a different way?

Discuss the solution as needed.

Learning to Solve

Teacher Notes

Students will continue to develop estimation skills and apply these skills to subtraction of decimal numbers. The same methods of front-end estimation, benchmark decimals, or others can be used.

Monitor students to verify that they are estimating instead of computing and rounding the difference. That procedure defeats the purpose of estimation to check reasonableness of the answer. Encourage students to use multiple methods to estimate.

1. Students will estimate differences of decimal numbers.

   What ways have we used to estimate sums when we add decimals? *(front-end estimation, benchmark decimals, or others such as rounding)*

   Another method we can use to estimate involves rounding, which means to make a number less exact but easier to use when calculating. When we round the whole number 8, for example, 8 is close to 10, so we round to 10. For decimal numbers, 0.2 is closer to 0 than 1, so we round 0.2 to 0. In many ways, rounding estimation is similar to benchmark estimation.

   The same methods can be used with subtraction. We can use estimation to determine about how large the difference is. What does "difference" mean in relation to subtraction? *(the answer to a subtraction problem)*

   Display the table on the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Look at the table. We will not compute the answers to these problems; instead, we will estimate. Remember, the difference is the answer to a subtraction problem. How are estimates different from answers from actually subtracting? *(an estimate is not an exact answer)* **Let's do the first problem together.**

   Look at the first number. What is that decimal close to? *(accept a reasonable range, depending on method)*

Look at the second number. What is that decimal close to? *(accept a reasonable range, depending on method)*

What is the difference? *(accept a range of 26 to 31)*

Is it okay if we have different estimates? *(yes)* **Why?** *(different methods may result in different estimates of the difference)*

With your partner, estimate the difference of each problem in this table.

Write the estimates in the table. Remember, don't compute the exact answer; instead, just estimate. Be sure you are able to explain how you arrived at your estimate.

Monitor students. If you notice that students have difficulty estimating, you may want to complete the activity sheet as a whole class.

Once students have completed the table, ask pairs of students to give their estimates and explain how they found those estimates. Write the different estimates on the board to demonstrate that it is possible to have a range of estimates. Without a context for the problem, you cannot know which estimate is most appropriate.

Are these estimates reasonable? How do you know? *(answers may vary, depending on the method and how accurately students applied their methods)*

Why might we estimate before we compute? *(to determine whether our answer is reasonable)*

When might an estimate be sufficient and no computation be necessary? *(answers will vary, such as when accuracy is not important, if the original problem was an estimate to begin with, or when the problem asks for an answer that is "about" or approximated)*

Will everyone's estimate be the same? *(not necessarily)* **Why or why not?** *(different people may use different estimating methods)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the problems on the Practicing Together sheet. This is a time for students to practice writing and discussing.

   Think about and estimate the answer to each problem. Write your estimate, but also explain in a few sentences how you found your estimate. You may be asked to explain your process, so be sure you understand how you determined the estimate.

2. Monitor the solutions of the groups. Ask 2 pairs of students who have different answers to the same problem to write their solutions on the board. Have the students talk about how they decided on their answers. Each of these problems has multiple solutions.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Write a problem that fits the following clues:

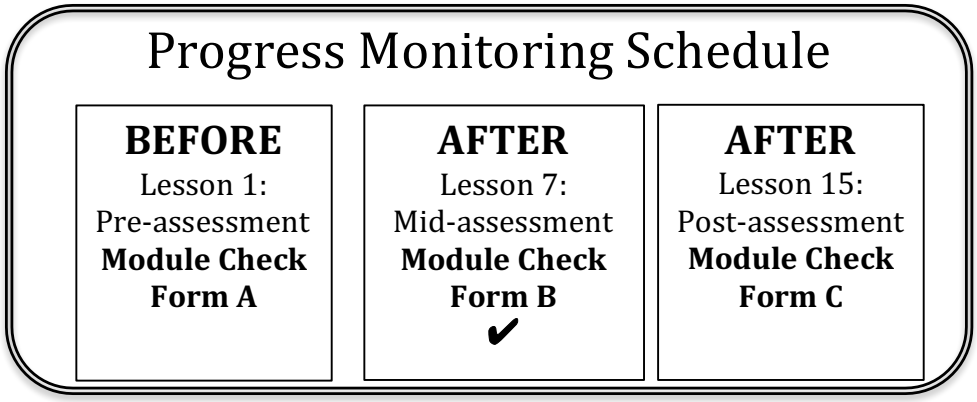
Clue 1: The difference is close to 5.

Clue 2: Both numbers in the problem are decimals.

Clue 3: Both numbers have 2 decimal places to the right of the decimal point.

What could be the 2 numbers I subtracted?

As time permits, have the students share their responses and how they solved.



Lesson 8: Subtracting Decimal Numbers

Lesson Objectives	Students estimate the difference of decimal numbers. Students subtract decimal numbers. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Difference, expanded form, estimate, front-end estimation, benchmark decimals	
Misconceptions	Students may not align the appropriate place-value digits. Students may instead align from the right-most digits or the left-most digits, resulting in an incorrect difference.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Decimal Cards (see pages 129-131 of Teacher Masters)	• Student Booklet • Red colored pencil

Warming Up

Students will review expanded form.

Cut the Decimal Cards apart (see pages 129-131 of Teacher Masters). Have them available for students. Display a number in expanded form.

Select students to come to the front of the class, select a card, and create the number represented by the expanded form.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Show the first number in expanded form.

> **Here is a decimal written in expanded form. I will ask some of you to come to the front and select the appropriate numbers to represent the expanded form.**

Call 5 students to the front (one will hold the decimal point.) Have each student select a number or decimal point, have tem stand in front of the class showing the class their decimal number, and have students in the class decide if the students holding the cards have shown the correct decimal (45.72).

Continue showing expanded forms and have students represent them. Other expanded form expressions can be used as appropriate.

Learning to Solve

TEACHER NOTES

In this lesson, students first estimate a difference, then subtract. They are asked to note the similarities and differences between subtracting with whole numbers and subtracting with decimal numbers. You may want students to check their answers by adding, which also gives them extra practice in addition.

1. Students will subtract decimal numbers.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the problem, 45 and 72 hundredths – 21 and 3 tenths. Do not solve the problem; instead, estimate only. Write your estimate on your Learning to Solve sheet.

Allow students time to work.

What was your estimate? What method did you use to solve? *(front-end estimation, benchmark, or other methods)*

Did anyone get a different estimate? What method did you use to estimate the answer?

If students have different estimates, write them on the board.

Are any of these estimates unreasonable? Why?

As we saw with addition, estimates are useful in helping predict the size of our answer. Let's subtract now to find the actual difference.

What is an important idea we learned with addition that we should consider as we subtract? *(accept reasonable answers, such as you need to add or subtract the same place values and the problem can be rewritten so place-value positions line up)*

A way to help us keep the same place values together is to the rewrite the problem vertically and align the place-value positions.

Model writing the problem vertically on the board.

Now the place-value positions line up. Using what you know about subtraction of whole numbers, subtract the 2 numbers.

Allow students time to subtract the numbers. If you notice many students having difficulty, you may want to do the problem together.

What is the difference of 45 and 72 hundredths and 21 and 3 tenths? *(24.42)*

How is subtracting decimals like or different from subtracting whole numbers? *(answers may vary, such as you may have to*

regroup to subtract; with decimals, you have to align the place-value positions)

How could you check your answer? *(by using addition)* **Because addition and subtraction are inverse operations, let's use addition to check our difference.**

Work through the addition problem with students to check the answer.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to estimate and then compute the answers to the problems on the activity sheet. Remind students to estimate first.

2. Ask pairs to share their process and solutions with the class. Watch for computational errors that may occur with regrouping.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

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Wrapping It Up

Display the Wrapping It Up in the Teacher Masters. Have students turn to the Wrapping It Up in their Student Booklets. Read the problem.

> **Jared said, "It is possible to subtract 2 decimal numbers that have 2 digits to the right of the decimal point and get a difference that has only 1 digit to the right of the decimal point."**
>
> **Do you agree with Jared? If so, give an example of a problem that proves Jared right. If you do not agree with Jared, explain why.**

Have the class discuss the problem, as time allows. You may want to use think-pair-share as a way to motivate the discussion. Answers may vary, such as I agree with Jared because $5.32 - 5.12 = 0.20 = 0.2$.

Decimals
Lesson 9

Lesson 9: Solving Contextual Problems by Subtracting Decimal Numbers

Lesson Objectives	Students solve contextual problems by subtracting decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)	
Vocabulary	**Minuend:** the number being subtracted from **Subtrahend:** the number that is to be subtracted	
Requisite Vocabulary	Difference	
Misconceptions	Students often search for key words to determine the operation to use with contextual problems. These key words include "left" and "take away." Students may also look at the magnitude of the numbers in the problems to decide which operation to use. Students should be encouraged to think about the actions or relationships in problems to determine which operation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Match it Up! Game Cards (see page 132 of Teacher Masters) • Match it Up! Game Sheet (see page 133 of Teacher

		Masters) • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Distribute Match It Up! Cards (see page 132 of Teacher Masters) and a Match It Up! Game Sheet (see page 132 of Teacher Masters) to each pair.

> **You have 8 cards. Each card has a decimal number on it. You will work with your partner to match the cards with the decimal number on your sheet so that it forms a correct subtraction problem. Each card will be used once and only once.**

Allow time for students to work. Have students share their matches.

Answers:

47.8	73.91	54.3	35.8
- 26.2	- 38.42	- 28.17	- 23.25
21.6	35.49	26.13	12.55

> **How did you decide where to place your cards?** *(Answers will vary, such as, some students may indicate they used guess-and-test.)*

Learning to Solve

Teacher Notes

Students will subtract decimal numbers to solve contextual problems. The focus is on the using part-part-whole problems with take-away, comparison, and missing-addend contexts.

Students will solve contextual problems by subtracting decimal numbers.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Ask a student to read the problem.

> **Listen as [student] reads the problem.**

Pause while the student reads.

> **What do we need to find?** *(How much more Cora weighs)* **How do we solve?** *(subtract)* **How did you decide that you needed to use subtraction?**
>
> **Before we subtract, let's estimate. Write your estimate.**
>
> **What was your estimate? What method did you use?**
>
> **What do we have to remember when subtracting decimal numbers?** *(align the place-value positions)* **Work with your partner to subtract.**
>
> **What is the difference?** *(5.46 pounds)*
>
> **Write 2 to 3 sentences explaining how you estimated and solved.**

Display the cheese problem from the Learning to Solve section on page 34, Lesson 6, in the Teacher Masters, so that students can compare.

> **We completed this problem about cheese in Lesson 6, on page 34 of your Student Booklet, by adding.**
>
> **But for our current problem about the twins in Lesson 9, we decided to subtract. Did the problem say that you had to select subtraction as the computation?** *(no)*
>
> **How is this problem similar to or different from the cheese problem we solved in Lesson 6?** *(answers will vary, such as, the cheese problem involved joining parts to find the whole, and this problem involves looking for the difference between 2 quantities)*

How could we check our work? What operation could we use? *(addition)* **Add the difference, 5 and 46 hundredths, to Nora's weight of 98 and 28 hundredths pounds. If we subtracted correctly, what should be the sum?** *(103.74 pounds)*

What is the sum? *(103.74 pounds)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the sheet.
2. Have pairs share their answers and their reasoning with the group.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 9A to provide extended practice before proceeding to Lesson 10.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Find the sum or difference for each problem.

Check whether any students struggle with addition and subtraction of decimal numbers. As time permits, give additional problems or have students select a different problem.

Decimals
Lesson 10

Lesson 10: Estimating Products of Decimal Numbers

Lesson Objectives	Students estimate products of decimal numbers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Factors**: numbers being multiplied together **Product**: the answer to a multiplication problem **Commutative property of multiplication:** states that factors can be multiplied in any order, resulting in the same product	
Requisite Vocabulary	Estimate	
Misconceptions	Many students overgeneralize the idea that multiplication "makes larger." Students may be surprised when they estimate products of numbers between 0 and 1 times numbers larger than 1. The product may be smaller than students expected.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 1 Deck of cards (number cards and Ace cards only)	• Student Booklet • Make Mine 1 Score Sheet (1 per pair, see page 134 of Teacher Masters) • Whiteboard • Dry erase marker

		• Red colored pencil

Warming Up

Group students into pairs. Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Display the directions for the game on the Make Mine 1 Score Sheet. The game requires a standard deck of cards, without the Jacks, Queens, and Kings.

Distribute 1 deck of cards; cut the Make Mine 1 Score Sheets (see page 134 of Teacher Masters) along the dotted line and give one to each student.

> **We are going to play Make Mine 1. Each pair has a deck of cards that has the kings, queens, jacks, and jokers removed. The number on the card represents the value. For example, a 7 represents 7. An ace represents 1 and a 10 represents 0. One person in your pair will be the dealer.** *(Teacher may want to designate dealer.)*
>
> **The dealer will deal 4 cards to each of you. You will use the numbers represented by those cards to make 2 addends that when added together will result in a sum close to 1. You will record those addends on the Make Mine 1 Score Sheet and find the sum. For your score, you will record how far you are from 1. For example, if you have a sum of 0.92, you will record 0.08. If your sum is 1.16, you will record 0.16. When you have played those cards, they are put on the bottom of the deck and you will get 4 new cards. We will play 5 rounds. At the end of 5 rounds, add your scores for each round to get your total score. The player with the lower score is the winner.**

Allow students time to play. At the end of play, discuss their strategies.

> **How did you decide where to place your cards?** *(Answers will vary, such as, some students may say they used estimation, others may have computed with a guess-and-test strategy.)*

Learning to Solve

Teacher Notes

Students will estimate the products of decimal numbers. This estimation technique will help students make sense of the products obtained from calculation. Students will reason about the relative size of a product, which should help them place the decimal point in the product without counting the number of places to the right of the decimal point in the factors. Sample language is provided to help students explain their rationale for estimating.

Students will develop a conceptual understanding of multiplication with decimal numbers.

Give each student a whiteboard. Write the following problem on the board:

$$\begin{array}{r} 45 \\ \times\ 14 \\ \hline \end{array}$$

We've been working with addition and subtraction, but today, we will begin to look at multiplication.

When we multiply, the numbers in the problem are called factors. What are the factors in this problem? *(45 and 14)*

Product is another name for the answer to a multiplication problem. Without actually doing the multiplication, how many digits do you think will be in the product? Write it on your whiteboard.

Have students show their answers. Select students to describe how they decided how many digits would be in the product.

How did you decide on the number of digits for the product? *(accept reasonable answers, such as multiplying a 2-digit number*

by a 2-digit number typically results in a 3-digit product or that when estimating, 50 times 10 results in a 3-digit number)

When I multiply a 2-digit factor by a 2-digit factor, will the product always be a 3-digit number? *(No)* **Can you think of an example of a 2-digit number multiplied by a 2-digit number that has a product that is only 2 digits?** *(at least one of the numbers is a decimal number)*

Discuss student answers.

Now, without doing the multiplication, estimate the product of the problem on the board. Write it on your whiteboard.

Have students show their answers.

What is your estimate? *(accept a range of 400 to 600)*

How did you estimate the product? *(answers will vary, such as 14 is close to 10, so 10 x 45 = 450 or 14 is close to 10 and 45 is close to 50, so 10 x 50 = 500)*

How was the method similar to or different from the way we estimated sums and differences? *(answers will vary, such as using rounding before computing)*

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Benchmark fractions, rounding, and front-end estimation can be used to estimate products, just like we did with sums and differences.

To estimate the product of 78 × 1 and 8 tenths, think about numbers the factors are close to. Let's start with 78. What number could we use that it is close to 78? *(accept reasonable answers, such as 75, 80, or 100)* **Write the estimate in the blank to the right of 78.**

What number could we use that is close to 1 and 8 tenths? *(accept reasonable answers, such as 2)* **Write the estimate in the blank to the right of 1 and 8 tenths.**

Remember, estimating is a tool to check our actual math. We want to pick numbers that are quick to compute and that we know we will be reasonable and as accurate as possible.

What would be the estimate of the product? *(accept reasonable answers, such as 160)*

Look at the next 4 problems. What do you notice? *(they have the same digits and the same operation but with a decimal point in some of the numbers)*

Let's do the first problem together. When estimating, you want to choose numbers that are easy to multiply in your head. For this first problem, I might just round 12 to 10 because 10 is close to 12 and multiplying by 10 is easy. So, 10 times 24 is 240. Another option is to round 24 to 25 and then multiply 25 by 10, making my estimate 250. Is there another way? *(accept reasonable answers, such as 30 x 10)*

Now, use what you know about benchmark decimals and other estimation methods to estimate each product.

Allow students time to work individually, in pairs, or as a class if students are having difficulty in finding an estimate.

If completing as a class, use the following script to help students explain how they estimate.

In the next problem, 24 × 1 and 2 tenths, an estimate might be to keep 24, and 1and 2 tenths is just a little more than 1 (a benchmark decimal number); therefore 24 × 1 = 24.

In the next problem, 2 and 4 tenths × 12, an estimate might be 20. 2 and 4 tenths can be thought of as 2 and 12 is close to 10. 2 x 10 = 20.

A different explanation for this same problem might be that 12 is just a little more than 10; 2 and 4 tenths is near the benchmark decimal of 2 and 5 tenths, so 10 times 2 and 5 tenths is 25. Or, an estimate might be to keep 12, and 2 and 4 tenths is closer to 2, therefore 12 × 2 = 24.

> **In the last problem, 24 hundredths x 12, you are looking at 12 equal-sized groups of $\frac{24}{100}$. The fraction $\frac{24}{100}$ is a very small number and is also less than 1, so the answer will be less than 12. Another way to estimate is to think of 24 hundredths as close to 25 hundredths, which is about one-fourth. One-fourth of 12 is 3.**

Have students share their responses to the last problem and discuss. Estimates will vary for all problems. Focus on valid estimation procedures and reasoning in your discussions.

> **Looking at these 4 problems, do you see any patterns that might help us when we estimate products?** *(accept reasonable answers, such as multiplying a number less than 1 by a number greater than 1 produces a smaller product than when multiplying 2 factors greater than 1)*

Students write comparison generalization statements in the Notes section of their Student Booklets.

> **The patterns you noticed about the estimates of products are generalizations. For example, you noticed that when you multiply a number less than 1 by a number greater than 1, you get a smaller product than when multiplying 2 factors greater than 1.**

Have students turn to the Generalization page in the Notes section of their Student Booklets. Write the generalizations on the board or in the Teacher Masters Generalization sheet for students to copy in their Student Booklets.

> **Turn to the Generalization sheet in the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships. Write this generalization: When you multiply a number less than 1 by a number greater than 1, you get a smaller product than when multiplying 2 factors greater than 1. In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.**

If students found more generalizations, write them on the board or in the Teacher Masters Generalization sheet for them to copy, provide their representations, and write their equations.

Multiple estimation strategies may be used. As with addition and subtraction, students may use front-end, compatible numbers, benchmark decimals, or rounding to determine an estimate for the factors. Their choice of a strategy will affect the estimate they find for the product.

Practicing Together

There is no Practicing Together section for this lesson to allow for expanded discussions in the Learning to Solve section.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping it Up

Write each problem on the board. Write only 1 problem at a time. Ask students to quickly write an estimate of each product on their whiteboards. Have them show their answers, and write some of their solutions on the board; discuss as time allows.

$11 \times 2.1 =$ _____ *(accept a range of 20 to 25)*

$1.93 \times 4 =$ _____ *(accept a range of 6 to 10)*

$9 \times 0.09 =$ _____ *(accept a range of 0 to 2)*

Decimals
Lesson 11

Lesson 11: Multiplying Decimal Numbers

Lesson Objectives	Students multiply decimal numbers. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Product, estimate, factor	
Misconceptions	Many students overgeneralize the idea that multiplication "makes larger." Students may be surprised when they estimate products of numbers between 0 and 1 times numbers larger than 1. The product may be smaller than students expected.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Blank Estimation Sorting Cards (see page 135 of Teacher Masters) • Index cards	• Student Booklet • Estimation Sorting Cards (10 cards per student to start; see page 136 of Teacher Masters) • Estimation Sorting Sheet (1 per student, see page 137 of Teacher Masters) • Index card • Red colored pencil

Warming Up

Group students into pairs. Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming sheet in their Student Booklets. Display the directions for the game on the Warming Up sheet.

Distribute a set of 10 Estimation Sorting Cards and an Estimation Sorting Sheet (see pages 135 and 137 of Teacher Masters) to each pair. This game includes 10 game cards. Teachers can make additional cards based on how students perform. Blank cards are provided (see page 136 of Teacher Masters); or teachers may have students create cards for game use.

> **You have a set of Estimation Sorting Cards and an Estimation Sorting Sheet. There are 3 columns on the sheet. Each card has a computation problem. Your task is to decide in your pair if the sum, difference, or product is closer to 10, 25, or 50. You cannot use a calculator or paper and pencil to find the answer. You need to use any of the strategies you have developed to estimate the answer and decide where to place the card. You have 4 minutes to make your decisions for the 10 cards, so try to make your estimate and place your card on the sheet in about 30 seconds or less.**

At the end of time, have students share how they sorted the cards. Discuss the strategies they used to determine the card placement.

Answers:

About 10: 43.73 – 32.9; 3.25 + 8.12; 31.6 x 0.31
About 25: 87.62 – 62.38; 14.85 + 7.15; 96.76 x 0.28
About 50: 21.96 x 2.3; 14.3 x 3.5; 16.887 + 35.001; 74.9 – 26.001

> **How did you decide where to place your cards?** *(Answers will vary. Some students may have used front-end estimation, rounding, or benchmark decimals.)*

Learning to Solve

Teacher Notes

Students will multiply multi-digit numbers. A quick review may be needed. Students will also be asked to explain the mathematics, so reviewing correct mathematical vocabulary, such as "factor" and "product" also might be needed.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the first problem and estimate the product.

Allow students time to work.

How did you determine your estimate? *(answers may vary, such as rounding or front-end)* **Did anyone get a different estimate?**

If students have different estimates, write them on the board.

Are any of these estimates unreasonable?

What is the range of our estimates? *(the estimates should be between 1,300 and 2,000)*

Multiply to find the actual product.

Allow students time to perform the multiplication.

What is the product? *(1,716)* **How did you solve?**

How do the estimate and the product compare? *(answers will vary, such as they don't because I estimated incorrectly)*

Look at the next problem. How is this problem related to the problem we just discussed? *(1 factor, 132, is the same, and the other factor has the same digits but includes a decimal point)*

How do you think the product of this problem will compare to the product of the first problem we did? *(it will be smaller because we are multiplying by a number that is closer to 1 than 10)*

What would be an estimate of the product of this problem? Write it in your Student Booklet.

Allow students time to work.

You may want to also encourage students to consider other ways of describing their estimates. For example, 47 is close to 50 and 3 and 8 tenths is close to 4. As students progress through the other problems, you may want to also include reasoning such as, 38 hundredths is close to 0.

Additionally, you may want to have students compare the number of digits to the right of the decimal point in the factors to the number of digits to the right of the decimal point in the product.

What estimate did you find? *(accept a range of 130 to 200)*

What do you notice about the estimate of the first problem and the estimate of this problem? *(accept reasonable answers, but emphasize that the product is smaller because 1 factor is smaller)*

Look at the product of the first problem, 132 × 13. How could we use that product to find the product of 132 × 1.3? *(accept reasonable answers, such as the product has the same digits, but the placement of the decimal point makes the product smaller)*

You said the digits were the same but that in the second factor, instead of 13, it is 1 and 3 tenths. We know that based on our estimate, the product should be between 130 and 200. Do you think we can use the same product as a guide for finding the product in this problem? *(yes)* **Why?** *(the product has the same digits)*

Let's look at this problem the same way we did with the first fraction problem.

For the first problem, 132 × 13, do you see any decimal points? *(no)* So does our product need a decimal point? *(no)*

Now look at the next problem. Does 132 have any digits to the right of the decimal point? *(no)* How many digits to the right of the decimal point in 1 and 3 tenths? *(1)*

In thinking about the product, 1,716, will this be the same digits in the product of 132 × 1 and 3 tenths? *(yes)*

Do we need to add a decimal point? *(yes)* Where do we place the decimal point? *(after the 1, before the 6)* What is the product of 132 × 1 and 3 tenths? *(171.6)*

Look at the next problem, 1 and 32 hundredths × 1 and 3 tenths. What do you notice about this problem in comparing it to the previous 2? *(same digits, but the numbers are smaller because of the placement of the decimal point)*

First, let's estimate. I will use rounding and think of the problem as 2 times 1, so my product will be close to 2. The first product was 1,716, the next product was 171 and 6 tenths, and now my estimate is 2. Is this a reasonable estimate? *(yes)* Why? *(accept reasonable answers, such as the product has the same digits, and I rounded the product to 2; or, I think 1.32 can be rounded 1; 1.3 also can be rounded to 1; 1 time 1 equals 1, so I estimated 1)*

What digits will be in the product? *(1, 7, 1, and 6)* Will this product have a decimal point? *(yes)* How do you know? *(answers may vary, such as both of the digits have decimal points, so the product probably has a decimal point)*

Our estimate was a product close to 2. Where might I place the decimal point in the product so that it is close to 2? *(between the 1 and 7)* The estimate helps us know where to place the decimal point.

Let's check that product by looking at each decimal number. How many digits to the right of the decimal in 1 and 32 hundredths? *(2)* How many digits to the right of the decimal in

1 and 3 tenths? *(1)* **How many digits should be to the right of the decimal point in the product?** *(3)*

What is the product of 1 and 32 hundredths × 1 and 3 tenths? *(1.716)* **Compare this product to the estimate. Is it close?** *(yes)* **Is it reasonable?** *(yes)* **Why?** *(1.716 is close to 2)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the problems on the Practicing Together sheet in the Student Booklet. Have pairs explain their solutions to one another.

2. As you monitor while students work, remind them that they can use the product of the whole-number factors to determine the product of the related problems. Also, as you listen to students as they communicate, check for their use of mathematically precise language.

3. As you discuss students' solutions, reinforce the relationships between (a) the estimates and the product, and (b) the number of digits to the right of the decimal point in the factors and the number of digits to the right of the decimal point in the product.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Give each student an index card. As an "exit slip," ask students to estimate the product of the following 2 decimal numbers.

> **On your index card, write your estimate of the product of 19.875 × 92.478.**

Collect the index cards as they exit the room or have students share their response if time permits. Accept reasonable answers in the range of 1800 to 2000.

Decimals
Lesson 12

Lesson 12: Solving Contextual Problems by Multiplying Decimal Numbers

Lesson Objectives	Students solve contextual problems by multiplying decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	**None**	
Misconceptions	Students often search for key words to determine the operation to use with contextual problems. For multiplication, these key words include "altogether," "equal-sized groups," and "total." Students may also look at the magnitude of the numbers in the problems to decide which operation to use. Students should be encouraged to think about the actions or relationships in problems to determine which operation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Multiplication Mania Game Sheet (1 per student, see page 138 of Teacher Masters) • Index card (1 per student

		or per pair) • Red colored pencil

Warming Up

Group students into pairs. Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming sheet in their Student Booklets. Display the directions for the game on the Warming Up sheet.

Distribute a deck of regular playing cards (minus face cards) and the Multiplication Mania Game Sheet (see page 138 of Teacher Masters) to each pair.

Distribute a deck of cards to each pair and a Multiplication Mania game sheet to each student.

> **We are going to play Multiplication Mania. Each pair has a deck of cards that has the kings, queens, jacks, and jokers removed. The number on the card represents the value. For example, a 7 represents 7. An ace represents 1 and a 10 represents 0. I will draw one card at a time from the deck. You will place that number in one of the blanks to create decimals numbers that when multiplied will get a product close to the number in the middle of your game sheet. You may place it anywhere on your sheet but once you place your number, you may not change it. When all of the blanks are filled, you will use your calculator to find the exact product for each of the close numbers. For your score, you will record how far you are from each of them and record it in the score column. You will add each of those scores to get your Total Score. The person with the lowest score wins.**

Call one number at a time. Allow students time to place their number. When all of the blanks are filled, remind students how to score. Then, discuss their strategies for placing the numbers.

> **How did you decide where to place your numbers?** (*Answers will vary, such as, some students may have used place value concepts or estimation of products.)*

Learning to Solve

TEACHER NOTES

Students will apply the skill of multiplying decimal numbers to solve contextual problems. The focus is equal-sized groups and arrays.

Students will solve contextual problems by multiplying decimal numbers.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve in their Student Booklets.

Have a student to read the problem.

Listen as [student] reads the problem.

What do we need to do to solve the problem? *(multiply)* **Why?** *(she makes the same amount of money each hour and there are 5 hours, equal-sized groups)*

Whenever we solve problems like this, we first want to think about an estimate. Write your estimate of 8 and 50 hundredths × 5 in your Student Booklet.

Allow students time to work.

What is your estimate? Why do you think that is reasonable? *(answers will vary, such as 45, because 8 and 50 hundredths is close to 9, and 9 times 5 equals 45)*

When we multiply decimals, what do we need to remember? *(do not need to line up the decimal point, use the estimate to understand the size of the product, then add decimal point after finding the product)*

Use what you know about multiplying decimals to find the product. Be ready to explain how you multiplied.

Give students time to find the product.

When students are finished, ask a student to share the solution process. Write the student explanation of the process on board.

> **How much money will Jess make after babysitting for 5 hours?** *($42.50)* **Is this reasonable?** *(yes)* **How did you multiply?** *(answers may vary, such as multiplied as whole numbers and then added decimal point after finding the product)*
>
> **How does the product compare to your estimate?**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the Practicing Together sheet in their Student Booklet. Students should estimate the answer first and then find the exact calculation.

2. Have student partners show their solutions to the problems, 1 at a time, and explain their procedure to calculate the estimate and the product.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 12A to provide extended practice before proceeding to Lesson 13.

Wrapping It Up

Distribute an index card to each student or to a pair of students. They will create a word problem that involves multiplication.

> **On your index card, write a word problem using 2 decimal numbers. The solution process must use multiplication. Solve the problem on the back of the card.**

Allow students time to work. If time permits, you can have students solve each other's problems and compare their answer to the one on the back of the card. Then collect the index cards.

Decimals
Lesson 13

Lesson 13: Dividing Decimal Numbers

Lesson Objectives	Students divide decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Dividend**: the number being divided, the whole amount **Divisor**: the number or quantity to be divided into the dividend; the number of groups or the number in a group **Quotient**: the answer to a division problem	
Requisite Vocabulary	Estimate	
Misconceptions	Students often confuse place value in division. The rules of moving the decimal point so many places in the dividend and the divisor can be confusing.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Calculator	• Student Booklet • Whiteboard • Dry erase marker • Calculator • Decimal Cards (1 set per pair, see page 139 of Teacher Masters) • Less Than the Dividend/ More Than the Dividend

		Sheet (see page 140 of Teacher Masters) • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Have a calculator available to check answers with the class.

> **Before we work on these problems, let's review division terms.**
>
> **The answer to a division problem is called the quotient. Look at the division problem at the top of the sheet. What is the quotient of 6 divided by 3?** *(2)* **Write "quotient" on the line below the 2.**
>
> **The dividend is the number being divided, the whole amount. In our problem, 6 is the dividend. Write "dividend" on the line below the 6.**
>
> **The divisor, or the 3 in this problem, is the number or quantity to be divided into the dividend. It tells the number of groups or the number in a group. Write "divisor" on the line below the 3.**
>
> **Now look at the problems.**

Read the directions to students and allow time for students to complete the sheet.

> **If you think the quotient is greater than the dividend, circle "greater." If you think the quotient is less than the dividend, circle "less." Be sure you can explain why you selected your answer.**

Give the students time to work, then ask individual students to share their answers.

> **Why did you select that answer? Does anyone have a different answer? How did you select your answer?**
>
> **Let's check our answers with a calculator.**

Work until students agree on the answers as a class.

Do you notice any patterns? *(when dividing a positive number by a number less than 1 and more than 0, the quotient is greater than the dividend; if the divisor is greater than 1, the quotient is less than the dividend.)*

Learning to Solve

TEACHER NOTES

Students have had experience dividing fractions. The patterns in dividing fractions are similar to those in dividing decimal numbers. As you go through the lesson, look for opportunities to link the division of fractions to the division of decimal numbers.

In this lesson, students use methods to divide whole numbers to divide decimal numbers. Students also use estimates to refine quotients.

1. Students will discuss a division generalization for dividing decimal numbers. Then they will estimate quotients and divide decimal numbers.

   In Warming Up, we talked about a pattern when dividing decimals numbers? What was that? *(When dividing a positive number by a number less than 1 and more than 0, the quotient is greater than the dividend; if the divisor is greater than 1, the quotient is less than the dividend)*

   Turn to the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships. Write this generalization on the Generalizations page: When dividing a positive number by a number less than 1 and more than 0, the quotient is greater than the dividend; if the divisor is greater than 1, the quotient is less than the dividend. In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Give each student a whiteboard. Write the problem 22.5 ÷ 3 on the board.

> **Write this problem on your whiteboard. What would you estimate the quotient to be?** *(estimates should be close to 7—a reasonable range is 6 to 8)*

Ask a student for his or her estimate and record it on the board.

> **Did anyone get a different estimate?**

If students have different estimates, write them on the board

> **Are any of these estimates unreasonable? How do you know?** *(answers may vary, depending on the estimate's reasonableness)*

> **Use what you know about division to divide 22 and 5 tenths by 3. Solve the problem on your whiteboard.**

Allow students time to divide. Have a student demonstrate his or her method for division.

> **Did anyone divide the numbers differently?**

> **Is there more than 1 way to find the quotient?** *(yes)* **How do you know?** *(answers may vary, such as repeated subtraction)*

Students may share algorithms they have learned in their core or general education classes.

> **When we multiplied numbers, we estimated first and then determined how many digits are to the right of the decimal point for the product.**

> **This same process works with division of decimal numbers. First, estimate. Next, calculate as if the numbers are whole numbers. Finally, refine your answer by looking at the dividend and divisor to determine where to place the decimal point.**

> **Our estimate for 22 and 5 tenths ÷ 3 was 7. Now we will solve but ignore the decimal point.**

What is 225 ÷ 3? *(75)* **Would 75 be a reasonable quotient for 22 and 5 tenths ÷ 3?** *(no)*

Is the answer, 7 and 5 tenths, closer to our estimation? *(yes)* **So where will we place the decimal point?** *(between the 7 and 5 to make 7 and 5 tenths)* **In our example, the dividend had a decimal point that helps us decide where to put the decimal point in the quotient.**

2. Write the problem 565 ÷ 0.5 on the board.

Let's work through another example together. In this problem, the decimal point is in the divisor, rather than the dividend.

First, we will estimate the quotient. In this problem, we are dividing 565 into groups of 5 tenths. Do you think there will be more than 565 groups of 5 tenths or less than 565 groups of 5 tenths? *(more)* **Why?** *(answers will vary. For example, students may notice that 5 tenths is half of 1. 565 ÷ 1 would be 565 but the number of groups will be double that. They may also notice that when dividing by a number less than 1 and greater than 0, the quotient is greater than the dividend)*

When we divide, the divisor needs to be a whole number. Think of this process similar to finding a common denominator with fractions. The only way we can eliminate the decimal point is to multiply by 10. What is 5 tenths times 10? *(5)*

Just like when we find a common denominator with fractions and multiply the numerator by the same number as the denominator, we need to multiply the dividend by the same number as the divisor. What is 565 times 10? *(5,650)*

What is the quotient of 5650 ÷ 5? *(1,130)* **Because neither the dividend nor devisor now have decimals, the quotient in our example is a whole number.**

Write the problem 5.65 ÷ 0.5 on the board.

We will do one more. In this problem, the decimal point is in both the dividend and the divisor. Write the problem on your whiteboard.

First, we will estimate the quotient. In this problem, we are dividing 5.65 into groups of 5 tenths. Notice that we are using the same digits as the previous problem. Do you think there will be more than 5.65 groups of 5 tenths or less than 5.65 groups of 5 tenths? *(more)* **Why?** *(answers may vary, similar to the previous problem)*

What is the first thing we have to do? *(the divisor needs to be a whole number, so we multiple 5 tenths times 10, which equals 5)*

Just like the previous problem, we need to multiply the dividend by the same number as the divisor. What is 5.65 times 10? *(56.5)* **Now we will solve but ignore the decimal point. What is the quotient of 565 ÷ 5?** *(113)* **Would 113 be a reasonable quotient for 56 and 5 tenths ÷ 5?** *(no)* **So where will we place the decimal point?** *(between the 11 and 3 to make 11 and 3 tenths)*

In this example, the dividend had a decimal point that helps us decide where to put the decimal point in the quotient.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to solve the problems on the sheet.
2. Select pairs to share their answers and the process they used. After each pair has shared, ask whether anyone solved it a different way or got a different answer. If so, ask that pair to share. Compare the 2 groups' methods. How are they alike? How are they different?
3. Ask students whether the generalization about the relationship of the quotient to the dividend continues to be true.

How does the generalization that we made about the relationship between the size of the dividend and the quotient help you? *(answers will vary, such as we can check the reasonableness of our answers)*

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Give each pair of students 1 set of Decimal Cards (see page 139 of Teacher Masters) and a Less Than the Dividend/More Than the Dividend Sheet (see page 140 of Teacher Masters). They will place the cards in 2 different groups, Less than the Dividend or More than the Dividend.

In your pair, solve each problem on the Decimals Cards. You will place each Decimal Card in one of two different groups on the Less than the Dividend/More than the Dividend Sheet, based on the quotient. Do not use pencils or calculators; use the generalization and ideas we found in the lesson. Be prepared to explain how you decided.

Allow students time to work. When completed, and as time permits or as appropriate, have students share their responses.

Decimals
Lesson 14

Lesson 14: Solving Contextual Problems by Dividing Decimal Numbers

Lesson Objectives	Students solve contextual problems by dividing decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Quotient, dividend, divisor	
Misconceptions	Students often try to find a key word in a contextual problem to determine which operation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Deck of cards with face cards removed (4 of each)	• Student Booklet • Find a Place Decimals Game Sheet (1 per pair, see page 141 of Teacher Masters) • Red colored pencil

Warming Up

Distribute the Find a Place Decimals Game Sheet, 1 per pair (see page 141 of Teacher Masters). Give the directions for students to play. You can display the directions from the Teacher Masters and/or play a round or so to demonstrate the play of the game. Tell the students they are going to play Find a Place with Decimals and have them place the Find a Place Decimals Game Sheet between them.

1. You will play in pairs. The person on the left is Player A. The person on the right is Player B.

2. The goal of the game is to create a decimal number, using the digits 0 through 9, that is as close as possible to the target decimal number in the center of the page: 0, 1, 5, 10.

3. To create the decimal numbers, I will draw a card from this deck and show it to you. The deck contains the 2 through 9 cards. The 10 card represents 0 and the ace represents 1. There are four of each number in the deck.

4. The first card I draw will be for Player A. Player A may put the number on the card in any place value position that is blank on his/her side of the Find A Place Game Sheet.

5. Player B will get the next card. That player will place the number in any place value position that is blank on his/her side of the game sheet.

6. I will keep drawing cards until all of the blanks are filled.

7. Once you place a number in a box, you may not change the number or move it.

8. You must play the number on your turn. In other words, you cannot save the number and play it later.

Scoring Directions

1. To score, you and your partner will subtract the target number and the number you created and place the score in the

box next to the target number on your side of the game sheet. The difference will always be positive, so you should subtract the smaller number from the larger number.

2. After you find all the differences, add them. That will give you your total score, or points.

3. The person with the least number of points wins.

Have students play the game. You may want students to discuss the strategies they used for placing their cards. You can replay the game again if time allows.

Learning to Solve

Students will practice estimating and then solving word problems with division.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Working with your partner, read the problem and estimate the answer.

Provide time for students to work.

What was your estimate? *(accept a range of 60 to 80 miles per hour)* **How did you make your estimate? What method did you use?** *(answers will vary, depending on the approach used, such as front-end, decimal benchmarks, rounding)*

Think about how can you solve the problem, find the solution, and then write 1 or 2 sentences that describe how you solved it.

Give students time to solve. As you monitor, if students do not know how to approach the problem, give them a simpler problem. For example, tell students that you can ride your bike 10 miles in 2 hours, how many miles can you ride in 1 hour? What if you rode 20 miles in 2 hours, how many miles can you ride in 1 hour? What operation did they use to find the answer?

As students answer the following questions, write the responses on the board.

> **What was the average miles per hour?** *(72 miles per hour)* **How did you solve?** *(answers may vary, such as dividing the number of miles by the number of hours to drive)*
>
> **Compare your estimate to your answer. Was your estimate reasonable? How do you know?**
>
> **At that rate, how far could you travel in 5 hours?** *(360 miles)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the problems on the sheet.

> **Working with your partner, first estimate your answers to these problems. Then, find the solution. Finally, use your estimate to determine whether your calculation is reasonable. Be prepared to explain how you found your estimate and your solution.**

As you monitor students working, if students do not know how to approach a problem, give them a simpler problem.

2. Have student pairs share their answers and explain the method they used to solve a problem and why.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 14A to provide extended practice before proceeding to Lesson 15.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Read the directions and provide time for students to work.

> **Write "T" for true if you think the answer is correct or "F" for false, if you think the answer is incorrect. The digits in the answer are correct, but the placement of the decimals may be incorrect. If the answer is incorrect, explain why.**

As time permits, have the students share their answers.

Decimals
Lesson 15

Lesson 15: Solving Contextual Problems With Decimal Numbers

Lesson Objectives	Students add, subtract, multiply, and/or divide decimals to solve contextual problems. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	None	
Misconceptions	Students may try to find key words that will give them clues about which computation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Deck of cards with face cards removed (4 of each)	• Student Booklet • Calculator (optional) • Find a Place Decimals Game Sheet (1 per pair, see page 141 of Teacher Masters) • Red colored pencil

Warming Up

Distribute the Find a Place Decimals Game Sheet, 1 per pair (see page 141 of Teacher Masters). Give the directions for students to play. You can display the directions from the Teacher Masters and/or play a round or so to demonstrate the play of the game. Tell the students they are going to play Find a Place with Decimals and have them place the Find a Place Decimals Game Sheet between them.

1. You will play in pairs. The person on the left is Player A. The person on the right is Player B.

2. The goal of the game is to create a decimal number, using the digits 0 through 9, that is as close as possible to the target decimal number in the center of the page: 0, 1, 5, 10.

3. To create the decimal numbers, I will draw a card from this deck and show it to you. The deck contains the 2 through 9 cards. The 10 card represents 0 and the ace represents 1. There are four of each number in the deck.

4. The first card I draw will be for Player A. Player A may put the number on the card in any place value position that is blank on his/her side of the Find A Place Game Sheet.

5. Player B will get the next card. That player will place the number in any place value position that is blank on his/her side of the game sheet.

6. I will keep drawing cards until all of the blanks are filled.

7. Once you place a number in a box, you may not change the number or move it.

8. You must play the number on your turn. In other words, you cannot save the number and play it later.

Scoring Directions

1. To score, you and your partner will subtract the target number and the number you created and place the score in the

box next to the target number on your side of the game sheet. The difference will always be positive, so you should subtract the smaller number from the larger number.

2. After you find all the differences, add them. That will give you your total score, or points.

3. The person with the least number of points wins.

Have students play the game. You may want students to discuss the strategies they used for placing their cards. You can replay the game again if time allows.

Learning to Solve

Teacher Notes

Students will use decimal computations to solve contextual problems. Some students may try to find key words to help them decide which operation to use. Discourage students from looking for key words; instead, have them focus on relationships and actions. It may help for students to solve easier alternative problems first and then the actual problems.

Depending on computational skills, students may use a calculator. Another option is to have students use a calculator to check computations.

Students will calculate estimates and solutions to contextual problems that involve addition, subtraction, multiplication, and division of decimal numbers.

We aren't always told whether to add, subtract, multiply, or divide when we are solving problems. Sometimes, in our real life, we have to make those decisions ourselves.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. You may want students to work in pairs or to solve as a whole class. Ask a student to read the problem.

> **Listen as (student) reads the problem.**
>
> **Work with your partner to solve the problem. Remember to estimate your answer first and then find the solution. Be prepared to explain how you arrived at your answers.**
>
> **What information do you know?** *(how many pounds of supplies she bought and the cost per pound)*
>
> **What are you trying to find?** *(the total money she spent)*
>
> **Picture yourself at the store buying these items. If you know the amount per pound and you know the number of pounds you bought, how would you determine the cost?** *(multiply the number of pounds by the cost per pound and add sums, or add the sums and multiply the pounds)*
>
> **Let's estimate. About how much did Selena spend on garden supplies?** *(accept a range of $90 to $110)*
>
> **Solve the problem.**

Allow students to complete the problem.

> **How much did she spend on her garden supplies?** *($99.15)*
>
> **How did you solve it?** *(answers may vary, such as added the sums and multiplied the pounds)* **Did anyone solve differently?**
>
> **How does the answer compare to your estimate?**

Have a student read problem 2.

> **Listen as (student) reads the next problem.**
>
> **How many minutes did Del talk altogether?** *(33 minutes)*

The first 12 minutes cost how much? *($2.20)* **How many more minutes did he talk?** *(21 minutes)* **How much does each minute after 12 minutes cost?** *(35 cents)*

Work with your partner to solve the problem. Remember to estimate first and then solve.

When students have finished solving the problem, select some students to share their solution and the solution method.

How much did Del spend? *($9.55)*

How did you solve the problem? *(answers may vary, such as subtracted 12 from 33 – 12 (21), multiplied 21 times 35 cents equals $7.35, added that to $2.20 for $9.55)*

Did anyone solve it a different way?

How did you use your estimate to check the reasonableness of our answer?

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Students will identify patterns and rules for writing decimals.

Read the directions and then have students work in pairs to complete the problems. For each item, ask the following questions. See Teacher Masters for answers to first two questions for each item. Answer may vary for how they found the patterns or rules

What are the next 3 numbers?

What is the pattern?

How did you find the pattern or the rule?

Did anyone find the rule differently?

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

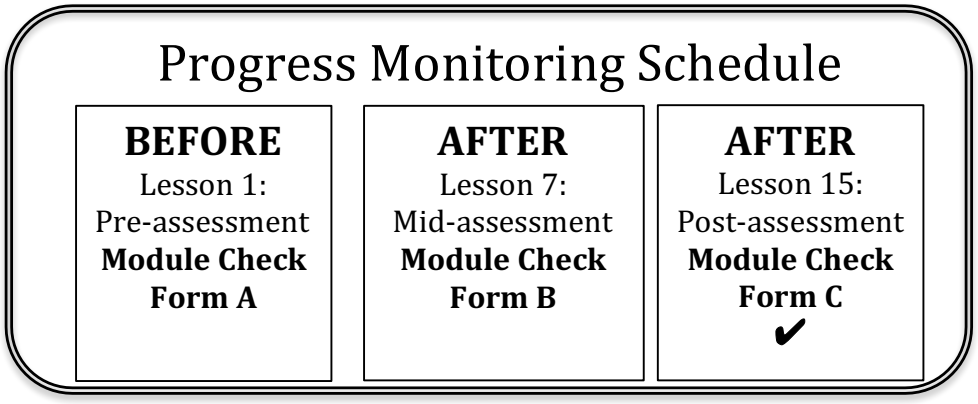
1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the concept map that was made in the first lesson.

Ask students what they would change, add, or delete from the map. Give them time to decide.

Have students share the changes they would make. As they share, make the changes to the concept map.



Appendices

Decimals

Decimals
Lesson 1A

Lesson 1A: Relating Place Value of Whole Numbers and Place Value of Decimals

Lesson Objectives	Students determine the place value of decimals up to thousandths. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Decimal number, numerator, denominator, decimal fraction	
Misconception(s)	Some students think the decimal point marks a symmetrical location in a decimal number. Instead, the decimal point separates the whole number values that are less than 1.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil

Warming Up

Students review whole-number place value concepts.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

For each whole number, fill in the blanks.

After students have finished, ask a student to write the answers on the board.

Does anyone have a different answer?

Learning to Solve

Teacher Notes

When reading decimals, refrain from using the word "point." For example, read 444.4 as "444 **and** 4 tenths," not "444 point 4." Reading decimals with "and" emphasizes the whole number and parts of the whole.

1. Students identify decimal place value.

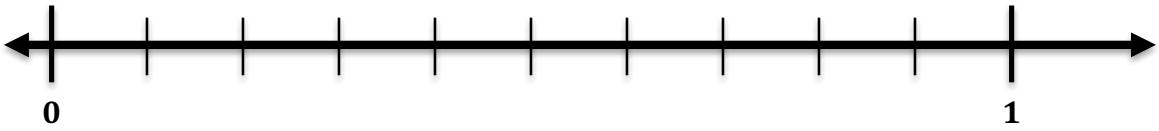
   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Display the first number line.

   The whole on this number line is the length between 0 and 1. Trace the whole with your finger.

   We want to partition the whole into 10 equal-length pieces. How many hash marks should I make? *(9)*

   Partition number line into ten equal-length parts.



What number represents the length between each hash mark? ($\frac{1}{10}$)

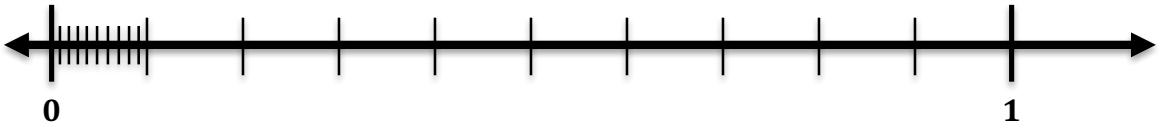
What do we call a fraction that has "1" as the numerator? *(a unit fraction)*

Write "$\frac{1}{10}$" under the first hash mark. What number represents the second hash mark? *($\frac{2}{10}$)* **We can say that this is 2 one-tenths. Write "$\frac{2}{10}$" under the second hash mark.**

How do you label the rest of the number line? Label your number line.

What if I partitioned each $\frac{1}{10}$ length into 10 equal-length parts?

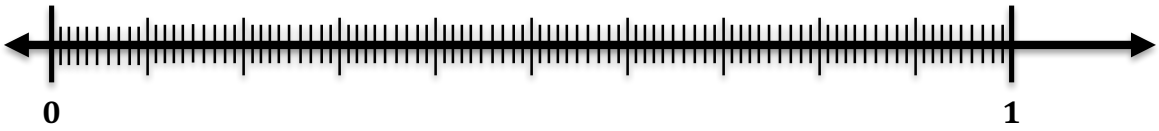
Partition the space between 0 and the first hash mark with 10 smaller hash marks.



If we partition each space between the hash marks with 10 equal-length parts, how many equal-length parts would there be between 0 and 1? *(100)*

What would be the size of each part? *(1 one-hundredth)*

Display the number line partitioned into hundredths from the answer key.



What if I partitioned the distance between 0 and $\frac{1}{100}$ into 10 equal parts? What would be the size of each part? *($\frac{1}{1,000}$)*

So far, we have partitioned 1 whole into $\frac{1}{10}$s, then $\frac{1}{100}$s, and then $\frac{1}{1,000}$s by partitioning each part into 10 equal parts. Does this pattern ever end? *(no)*

Why? *(distance can be partitioned into infinitely smaller lengths)*

2. Students connect decimal fractions to traditional decimal notation.

Decimal fractions can be written in a way that is not a fraction. Instead of writing fractions, we can represent them using a decimal point.

The portion of the lesson that uses 444 .4 as part of the discussion is a repeat from lesson 1. You may want to review this portion of the lesson with students using the following script.

Display and point to the number 444.4 in your Teacher Masters.

This number is 444 and 4 tenths.

Point to the hundreds place.

This 4 is in what place value? *(hundreds)* **Write the number 100 above the 4.**

What is its value? *(400)* **Write the number 400 below the 4.**

Point to the tens place.

This 4 is in what place value? *(tens)* **Write the number 10 above the 4.**

What is its value? *(40)* **Write the number 40 below the 4.**

Point to the ones place.

This 4 is in what place value? *(ones)* **Write the number 1 above the 4.**

What is its value? *(4)* **Write the number 4 below the 4.**

What is the relationship between 100 and 10? *(you can multiply 10 by 10 which equals 100 or divide 100 by 10 which equals 10, hundreds place value position is 10 times larger than the tens place value position, tens place value position is one-tenth of the hundreds place value position)*

Write "multiply by 10 or divide by 10" on the board. Write the examples as the script progresses.

What is the relationship between the tens place value position and the ones place value position? *(you can multiply 1 by 10 which equals 10 or divide 10 by 10 which equals 1, tens place value position is 10 times larger than the ones place value position, ones place value position is one tenth of the tens place value position)*

Point to the tenths place.

If the pattern is to multiply or divide by 10 as you move to a new place value, what is the place value for this 4? *(4 tenths)*

The 4 is in the tenths place because when we divide the ones place by 10, we get $\frac{1}{10}$, or tenths. Write the word tenths above the last 4.

Point to the tenths place again.

What is another way to write 4 tenths? ($\frac{4}{10}$)

Write "$\frac{4}{10}$" on the board. Be sure to say "4 tenths" as you point to 0.4 and again as you point to $\frac{4}{10}$ on the board.

4 tenths can be written as zero point four or $\frac{4}{10}$. Write the fraction $\frac{4}{10}$ under the last 4.

From this example, we can see that a part of a whole can be written as a decimal number or a fraction.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Follow the directions and solve each problem.

Have students complete individually, in pairs or small groups, or as a whole class.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Fill in the blanks with the correct answers.

As time permits, go over students' answers.

Lesson 3A: Comparing and Ordering Decimals

Lesson Objectives	Students compare and order decimals. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	**None**	
Misconception(s)	Students often think that the longer a decimal number, the larger the number—for example, that 0.4675 is greater than 0.65. Students may also think that the shorter a decimal number, the larger it is—for example, that 0.3 is larger than 0.81 because "a tenth is larger than a hundredth."	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard or equivalent	• Student Booklet • Whiteboard • Dry erase marker • Red colored pencil

Warming Up

Students review comparing and ordering fractions.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklet

> **Put the list of fractions in order from greatest to least. Think about how you will defend your answer.**

Allow students to work.

> **What is the correct order? How did you determine the order? What method did you use?**

Learning to Solve

Teacher Notes

Be sure to read the decimal point as "and" and not "point." For example, read 6.219 as "6 and 219 thousandths." Reading this way emphasizes place value and clearly indicates the whole and the parts.

Students often mix up writing numbers from greatest to least and writing numbers from least to greatest. Remind students to refer back to the problem to make sure that they have written the numbers in the order specified.

1. Students connect decimal fractions to traditional decimal notation.

   Give each student a whiteboard. Write the number 6.219 on the board.

   > **At the bottom of your whiteboard, write this number. Above each digit, write the name of the place value.**

   Give students time to complete. Have them turn to their neighbor and compare answers.

Ask students for each place value and write it above each digit on the board. Help students compare and order decimals.

> **Look at 6 and 219 thousandths.**
>
> **Is it closer to 6 or 7?** *(6)* **How do you know?**
>
> **Is it closer to 6 and 2 tenths or 6 and 3 tenths?** *(6.2)* **How do you know?**
>
> **Is it closer to 6 and 21 hundredths or 6 and 22 hundredths?** *(6.22)* **How do you know?**

Write "6.65" on the board.

> **Look at 6 and 65 hundredths.**
>
> **Is it closer to 6 or 7?** *(7)* **How do you know?**
>
> **Which number is greater, 6 and 219 thousandths or 6 and 65 hundredths** *(6.65)*

Write "6.65 > 6.219" on the board.

> **Why?** *(because 6.65 is closer to 7 and 6.219 is closer to 6)*

Write "6.27" on the board.

> **Look at 6 and 27 hundredths.**
>
> **Is it closer to 6 or 7?** *(6)* **How do you know?**
>
> **Is it closer to 6 and 2 tenths or 6 and 3 tenths ?** *(6.3)* **How do you know?**
>
> **Which number is greater, 6 and 219 thousandths or 6 and 27 hundredths?** *(6.27)* **How do you know that 6 and 27 hundredths is greater? What about the fact that there are 3 digits to the right of the decimal point in 6 and 219 thousandths?** *(both numbers have the same value in the ones and tenths place, but in 6.219, there is only 1 hundredth, while in 6.27, there are 7 hundredths)*

Write "6.27 > 6.219" on the board. Underneath it, write "6.219 < 6.27."

Read the compare statements. Do both of these statements show that 6.27 is greater than 6.219? *(yes)*

If I write either of these statements, I am correct in showing that 6 and 27 hundredths is greater than 6 and 219 thousandths.

6.27 > 6.219 **6.219 < 6.27**

Display Learning to Solve sheet of the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklet.

Look at the decimal numbers: 4 and 38 hundredths and 4 and 362 thousandths. Decide on a method that you could use to determine which decimal number is greater. Describe your method in the space provided.

Ask students who used different methods to describe their process.

2. Students order decimal numbers.

Using our ideas about comparing decimal fractions, we can put a list of decimal numbers in order. Let's look at the list of decimal numbers in your Student Booklet and work together to put them in order from greatest to least.

Write the list on the board:

3.82 11.1 3.8065 5 3.011

What is the first thing we could do to begin putting these decimal numbers in order from greatest to least? *(look at the numbers and think about what whole number they are closest to)*

Underneath the list in your Student Booklet, write the whole number that each decimal number is closest to.

Write the list again and underneath each decimal number, write the whole number that students say each decimal number is closest to.

What whole number is each decimal number closest to? *(4, 11, 4, 5, 3)*

From determining what whole number each decimal is closest to, we can now put almost all 5 decimal numbers in order from greatest to least. Which decimal number is greatest? *(11.1)* **Why?** *(11 is the greatest whole number)* **11.1 is the greatest decimal number in the list, so I will write it first.**

Based on the whole numbers, what would be the second greatest decimal number? *(5)*

How can 5 be a decimal number? I don't see a decimal point. *(with any whole number there is an understood decimal point to the right followed by 0s)*

My list is now: 11.1, 5.

We next have 2 decimal numbers that are close to 4. Let's just leave a space for those and come back to them.

What is the smallest decimal number? *(3.011)*

Why? *(it is closest to 3, which is the smallest whole number)*

My list is now 11.1, 5, ____, ____, 3.011

We have 2 decimal numbers remaining to put in order, 3.82 and 3.8065. Decide which of these decimal numbers is the greater and be able to tell me how you decided.

Give students time to determine which decimal number is greater and have them explain their answer.

What is our final order for this list of decimal numbers from greatest to least? *(11.1, 5, 3.82, 3.8065, 3.011)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklet.

Students may work individually, in pairs or small groups, or as a class.

> **Do the four problems on the sheet. Problem 1 asks two questions, problem 2 asks three questions. Then do problems 3 and 4.**

Discuss the answers as a class.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Wrapping It Up sheet in their Student Booklet.

> **Write 3 sentences to explain how you would decide whether 15.34 is greater than or less than 15.304.**

Ask students to read their response if time allows.

Decimals
Lesson 14A

Lesson 14A: Dividing Decimal Numbers

Lesson Objectives	Students divide decimal numbers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Dividend, divisor, quotient, estimate	
Misconception(s)	Students often try to find a key word in a contextual problem that will tell them what operation to use.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Deck of cards with face cards removed (4 of each)	• Student Booklet • Find a Place Decimals Game Sheet (1 per pair, see page 141 of Teacher Masters) • Index card for exit ticket • Red colored pencil

Warming Up

Distribute the Find a Place Decimals Game Sheet, 1 per pair (see page 141 of Teacher Masters). Give the directions for students to play. You can display the directions from the Teacher Masters and/or play a round or so to demonstrate the play of the game. Tell the students they are going to play Find a Place with Decimals and have them place the Find a Place Decimals Game Sheet between them.

1. You will play in pairs. The person on the left is Player A. The person on the right is Player B.

2. The goal of the game is to create a decimal number, using the digits 0 through 9, that is as close as possible to the target decimal number in the center of the page: 0, 1, 5, 10.

3. To create the decimal numbers, I will draw a card from this deck and show it to you. The deck contains the 2 through 9 cards. The 10 card represents 0 and the ace represents 1. There are four of each number in the deck.

4. The first card I draw will be for Player A. Player A may put the number on the card in any place value position that is blank on his/her side of the Find A Place Game Sheet.

5. Player B will get the next card. That player will place the number in any place value position that is blank on his/her side of the game sheet.

6. I will keep drawing cards until all of the blanks are filled.

7. Once you place a number in a box, you may not change the number or move it.

8. You must play the number on your turn. In other words, you cannot save the number and play it later.

Scoring Directions

1. To score, you and your partner will subtract the target number and the number you created and place the score in the

box next to the target number on your side of the game sheet. The difference will always be positive, so you should subtract the smaller number from the larger number.

2. After you find all the differences, add them. That will give you your total score, or points.

3. The person with the least number of points wins.

Have students play the game. You may want students to discuss the strategies they used for placing their cards. You can replay the game again if time allows.

Learning to Solve

Students solve problems involving division of decimal numbers.

Display the Learning to Solve sheet in the Teacher Resource. Have students turn to the Learning to Solve sheet in their Student Booklets.

Have students form groups of 4. Assign each student in the group 1 of the problems to solve.

Each of you will solve 1 problem. You will estimate the answer first. Then you will find the solution. You need to be able to explain how you calculated the answer. When you finish, discuss your solution with your group.

When all groups are finished, have the students who solved problem 1 sit together, those who solved problem 2 sit together, those who solved problem 3 sit together, and those who solved problem 4 sit together. Have each group compare answers and methods.

Ask whether any group has not received consensus on the answer. Their methods may vary, but the answer should be the same. If any groups disagree on an answer, have the class discuss the problem and model the solution process for students.

Practicing Together

There is no Practicing Together section. Spend the extra time working in groups on the Learning to Solve problems.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

On an index card, write a story problem involving division with a quotient that is larger than the dividend and the divisor. Show the solution to the problem.

If time allows, have some students read their problem aloud or have students trade with others and solve. Ensure the quotient is greater than the dividend and divisor. You may want to use this as an exit ticket.