Expressions and Equations 1

Teacher Lessons

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Expressions and Equations 1
Lesson 1

Lesson 1: Patterns and Generalizations

Lesson Objectives	Students create a generalization in their own words to describe a pattern. Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Pattern**: an arrangement of objects or numbers following a specific rule	
Requisite Vocabulary	**None**	
Misconception(s)	Students may attend to only the first 2 terms in a pattern. If the pattern is numerical, this misconception could lead to an additive rule rather than a multiplicative rule.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Square color tiles (at least 20 per student pair) • Calculator

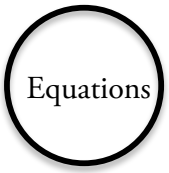
Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Preassessment **Form A** ✔	Lesson 7: Midassessment **Form B**	Lesson 15: Postassessment **Form C**

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Draw a circle on the board with "Equations" in the middle.



Today, we will start by thinking about equations. Write everything you know about equations in your Student Booklet.

Ask for student responses. Make a concept map on the board as students share ideas. Ask for students to clarify or provide examples to aid in the brainstorming of ideas.

What are some important ideas? What is an equation? Can you add an example to any of your words?

Take a picture of the concept map when students finish sharing their ideas and/or display for the duration of this module. The concept map will be used again at the end of this module. Displaying during the course of the module will allow both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

TEACHER NOTES

Students begin identifying patterns in this lesson. The identification of patterns is the first step in making generalizations that can be described by using a variable. The focus is on describing the pattern using natural language; variables will be introduced in later lessons.

1. Students will recognize a pattern and use words to describe the pattern.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Give each student pair 20 (or more) square tiles.

   > **Searching for and describing patterns is an important part of mathematics. Sometimes, geometric shapes can be arranged to create a pattern. We can also use numbers to create a pattern.**
   >
   > **Today, you will work with a partner to find and explain some patterns using geometric shapes. The first problem is called the Staircase Problem on the Learning to Solve page.**
   >
   > **You may want to use the square tiles to create the pattern. You can record the information in a table.**

   Allow students time to work. Display the table on the second page and ask a student to enter the missing values in the Number of Square Tiles column. Tell the other students to check their answers against what is being entered to see whether they agree.

   When the student has entered all the values, ask the class whether they agree or disagree with any of the entries. Discuss any variations in answers.

What patterns do you notice from the table or from the staircases you made? *(Answers will vary. For example, the height of the staircase in the middle is the same as the number of steps. Encourage multiple patterns.)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity. Have calculators available for students to use.

With your partner, answer the questions. Watch for patterns as you work.

When finished, ask student pairs to give their answers. As they share their answers, ask how they decided.

How did you find your answer? What patterns did you use?

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Laura wrote a number pattern: 3, 8, 13, 18, 23, 28, . . .**
>
> **How would you describe the pattern Laura used? In your own words, write a description of the pattern.**

If time permits, ask students to read their description of the pattern.

Expressions and Equations 1
Lesson 2

Lesson 2: Patterns

Lesson Objectives	Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Pattern	
Misconception(s)	Students may think that all patterns must be numerical in nature.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Package of toothpicks • Calculator

Warming Up

Teacher Notes

Watch for students who string expressions together. For example, 16 + 5 = 21 + 16 = 37. If students do this, model how to write the equations as in the Teacher Masters. You may have to explain that whatever is in the first expression in an equation (16 + 5) must equal the last expression (37) for the equation to be true.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Read the problem carefully and solve.

Ask a student to give the answer and explain how he or she determined the solution.

Learning to Solve

Teacher Notes

Describing a pattern using their own words helps students link the physical model of a geometric pattern to an algebraic representation. This is moving students closer to using a variable to describe the pattern or represent it symbolically.

Students will recognize and use their own words to describe a pattern.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the diagram in problem 1. The directions tell you to find the number of toothpicks, without counting. What that means is you should use any patterns you notice in the

diagram to do the counting. You should not be counting each toothpick individually.

With your partner, complete the toothpick problem. Then for number 2, identify a pattern and write a description of the pattern, using your own words. Write what makes sense to you and what will help you explain it.

Give the students time to complete the second problem, then move to number 3.

For number 3, what is another way to determine the number of toothpicks needed? How are the methods similar and different? Write your answers.

Give the students time to complete the second problem, then move to number 4.

Now look at problem 4; find the number of toothpicks, again without counting. *(pause)*

When finished, follow the instructions on the sheet for number 5.

Allow students time to solve. For each question, have a student pair provide their answer. Then ask for responses from a student pair who used a different method to arrive at their solution to the question. As they share their methods, note the pattern. Some students may suggest methods that do not work for more squares or for an even number of squares.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity. Have calculators available for students to use.

When students are finished, have student pairs write their problem on a whiteboard and exchange it with another student pair for them to solve.

If time permits, have students their problems with the entire class or in a group of 4. Discuss any interesting problems or problems where the "solving" pair did not agree with the pair that created the problem.

Trying It on Your Own

This lesson has no Trying It on Your Own section.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Using your own words, describe any patterns in the table.

Have a student provide his or her pattern. Ask whether other students developed a different pattern.

Expressions and Equations 1
Lesson 3

Lesson 3: Patterns That Use Variables

Lesson Objectives	Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Variable**: a letter or symbol than can represent a specific value or a generalized quantity **Area:** a measure or count of the number of square units contained in a polygonal region **Perimeter:** a measure or count of the number of length units on the boundary of a polygon	
Requisite Vocabulary	Pattern, expression	
Misconception(s)	Students often think that a variable represents an object rather than a quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • 10 red trapezoids and 32 green triangles from pattern blocks (per student pair)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Work with your partner to answer the questions. You may want to use the green triangle from the pattern blocks to help you answer the questions.**

Have students share their answers. They can use the pattern blocks to support their answers. Emphasize that when they are measuring perimeter with the triangles, that the measurement is based on the length of the side of the triangle. The area measurement is based on the area of the face of the triangle.

Teacher Notes

For problem 2, students may state that the area of a rectangle is 2. This is true if the area is measured in square units. However, the question asked for the area in triangular units. Therefore, students need to determine how many triangular area units are in the rectangular region.

Learning to Solve

Teacher Notes

Students often think of a variable as an object, rather than a quantity. Emphasize in this task that the variable represents the **number of** trapezoids, and not a trapezoid.

Students may not notice that the number of trapezoids does not increase consecutively in the table. The values skip from 6 to 10.

1. Students will recognize and use their own words and variables to describe a pattern.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Look at the first problem. We will measure the perimeter of the trapezoid. The perimeter is the number of length units on the boundary. You can think of perimeter like the length of fence around a yard. We will use the side length of the triangle piece to find the perimeter of the trapezoid.**
>
> **What is the perimeter of 1 trapezoid, using the length of 1 side of a triangle as our length unit?** *(5—trace the sides or use a triangle piece to show the 5 for students)* **Write "5" in the blank to describe the perimeter.**
>
> **We will use the area of the face of the triangle piece to find the area of the trapezoid. Area is the number of area units in a shape. Usually, we use square units to find area but in this problem, our area unit is the area of the triangle. We are trying to find how many triangles cover the area of the trapezoid. What is the area of the trapezoid in triangular area units?** *(3 triangular area units)* **Write "3" in the blank to describe the area.**
>
> **Now add another trapezoid to make a trapezoid train. First, let's find the perimeter. What is the perimeter of 2 trapezoids?** *(8)* **What is the area of the 2 trapezoids?** *(6)*
>
> **Complete the table with your partner. What is the largest trapezoid train you will make?** *(10 trapezoids)* **As you complete the chart for perimeter and area, look for patterns. Do not complete the last column in the table.**

Allow time for students to complete the table, finding area and perimeter up to 10 trapezoids. Have a student display his or her table. Students should make any corrections needed. Some students may not have

noticed that the number of trapezoids used in the table is not consecutive numbers.

> **What patterns did you notice for perimeter and area?** *(answers will vary—students may say that the area is 3 times the number of trapezoids and that the perimeter is 3 times the number of trapezoids plus 2)*

Write the patterns they find on the board or document camera. As they share their patterns, have other students check to see if it fits for their table entries.

TEACHER NOTES

They may need additional support for finding patterns for perimeter. To help them find the pattern, have them use the figures. As they add a trapezoid, they may notice that the lengths of the 2 outer ends remains the same, but 2 ends are covered up. The 2 that remain the same represent the 2 in the plus 2. In the same way, each additional trapezoid adds 3 more length units.

The last column in the table asks for *n* trapezoids. We can think about a pattern that we can use to find the area and perimeter of any number of trapezoids. In this table, *n* represents the number of trapezoids. We will use your patterns that lets us find the perimeter or area for any number of trapezoids.

Let's think about area and write an expression that we can use to find area in the second row of our table. One of the patterns you noticed was that the area is 3 times the number of trapezoids. Using *n* to represent the number of trapezoids, I can write 3*n*, so 3 times the number of trapezoids is the area. Write "3*n*" in the last column for the area.

Try it. What was the area of 6 trapezoids? *(18)* **What is 3 times 6?** *(18; you may want to try other number of trapezoids as a check)*

Now let's think about perimeter. Look at your train in the first row of the table. The ends or sides of your train are constant, meaning that there are always 2 length units on the end, no matter how many trapezoids we have. When we add a new trapezoid, how many new triangular pieces are needed to measure the perimeter? *(3)*

As you identify the perimeter pattern, use the pattern blocks to show the pattern by placing 2 triangles at the ends and then showing the additional 3 units that are added each time. If this was done in the prior discussion on patterns that were noticed, skip to the discussion of using *n*.

When we had 1 trapezoid, the perimeter was 2 + 3. What can we write when we have 2 trapezoids? *(2 + 3 + 3)* **What could you write if add another trapezoid?** *(2 + 3 + 3 + 3)*

Using *n* as the number of trapezoids, we want to show 3 times *n* or 3 times the number of trapezoids plus 2. Write the expression "3*n* + 2" in the last column on your table.

Let's try it to check to see if the expression works for any number of trapezoids. What is the perimeter of 10 trapezoids? *(32)* **Using our expression, 3 times 10 is...?** *(30)* **What is 30 + 2?** *(32, you may want to try other number of trapezoids using the general expression)*

Look at problem 4 on the next page. With your partner, find the perimeter and area of a trapezoid train containing 45 trapezoids. You can use the expressions we just created or the patterns you found.

Allow students time to complete. As students share their answers, ask them to explain how they used the expression or pattern.

What is the area of a 45-trapezoid train? *(135 area units)*

What is the perimeter of a 45-trapezoid train? *(137 length units)*

Practicing Together

This lesson has no Practicing Together section.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 3A to provide extended practice before proceeding to Lesson 4.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Resource Booklet. Have students turn to the Wrapping It Up sheet in the Student Booklet. Read the problem.

> **Gwen wrote "$w + 10$" on the board. What do you think she meant by this expression?**

Have students share the ways they could interpret this expression.

Lesson 4: Order of Operations

Lesson Objectives	Students evaluate expressions, using order of operations. Students make sense of problems and persevere in solving them. (SMP 1)	
Vocabulary	**Order of operations:** the rules to evaluate expressions containing different operations **Exponent:** a quantity to which a base is raised, indicating the number of times the base is used as a factor	
Requisite Vocabulary	Expression	
Misconception(s)	Students often think that addition must be done before subtraction, regardless of the order of the computations in an expression. The same is true for multiplication and division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard

Warming Up

Display the Warming Up sheet in the Teacher Masters. Give each student a whiteboard. Read the problem.

> **Solve this problem on your whiteboard.**

Provide time for students to work.

> **What is the answer?** *(6)* **How did you solve?** *(Answers may vary, such as 5x divided by 2 = 15; then multiply each side of the equation by 2, so 5x = 30; then divide each side of the equation by 5, so x = 6)* **Did anyone get a different answer or use a different method to solve?** (*some students may work backwards, others may use guess and check. Other methods are possible.)*

Learning to Solve

Teacher Notes

Do not talk about order of operations or write any grouping symbols. Have students simplify problems as they are written, based on their best judgment.

Computations within addition and multiplication can be done in any order, not necessarily from left to right. For example, 8 + 5 + 2 could be solved by first making 10 and then adding 5. This is because properties of addition or multiplication allow for regrouping. However, multiplication must be computed before addition per the order of operations.

If a student misses a step or evaluates an expression out of order, have the student rewrite how to follow each step and/or underline or highlight each computation as it is solved.

> Watch for students reading exponents incorrectly. For example, 3^4 should be read as 3 to the 4^{th} power or 3 raised to the 4^{th} power.

1. Students will evaluate expressions, using order of operations.

Give each student a whiteboard. Write "8 + 4 ÷ 4" on the board.

> **Simplify the expression 8 + 4 ÷ 4 by doing the computations. Write your answer on your whiteboard. Show how you found your answer.**
>
> **What was your answer?** *(answers may vary, depending on whether students used order of operations; expect them to find 9, but some may find 3.)* **How did you simplify it?**

Select students who added first, resulting in an answer of 3, and another student who divided first, resulting in an answer of 9.

> **We have 2 different answers, depending on how we chose to simplify the expression. To avoid confusion, mathematicians have agreed to an order of operations. "Order" means to do something in steps or to follow a sequence.**
>
> **The word "operations" refers to mathematical operations. What operations were in the expression we just evaluated?** *(addition and division)* **What are other operations we have used in math?** *(subtraction, multiplication, exponents)*
>
> **The order of operations is a specific order to perform operations in math.**

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **The box contains the order of operations. What step is first?** *(simplify expressions within grouping symbols)* **What are some grouping symbols that could be used?** *(Answers will vary. For example, parentheses, brackets, braces, the bar in a fraction)* **Look**

back at our expression on the board, 8 + 4 ÷ 4. Are there any grouping symbols? *(no)*

What is the second step? *(compute all exponents, or powers)* **An example of a number involving an exponent is 3 to the power of 2, more often called 3 squared. 3 is the base number. The exponent 2 tells us to use the base number as a factor twice, so 3 × 3.**

Write on the board: 4^3.

Think about 4 cubed. What is the base? *(4)* **How many times do we use 4 as a factor?** *(3)* **What is 4 × 4 × 4?** *(64)*

Are there any exponents in our expression on the board? *(no)*

What is step 3? *(compute multiplication and division)* **You compute any multiplication or division computations first, before addition and subtraction. Also, you compute in order from left to right. Look at our expression, 8 + 4 ÷ 4. What should we have computed first?** *(4 ÷ 4)* **What is 4 ÷ 4?** *(1)*

What is step 4? *(compute addition and subtraction)* **The last step is to perform any addition or subtraction computations. As with multiplication and division, you compute in order from left to right. The next step is 4 plus 4, which is equal to 8. In our expression, what is the last step we need to do to simplify or evaluate it?** *(8 + 1)* **What is 8 + 1?** *(9)* **What does 8 + 4 ÷ 4 equal?** *(9)*

Look at number 1 on the Learning to Solve sheet. We will do this problem together. The first step is to compute any operations in the parentheses. Are there any parentheses? *(no)* **What is step 2?** *(compute all exponents, or powers)* **Are there any exponents?** *(yes)* **What is 3^2?** *(9)*

Write a 9 below 3^2 and then write the remaining expression, 9 + 4 × 5. What do we do next? *(multiply)* **How do you know?** *(step 3 of the order of operations)*

What is 4×5? *(20)* **Write a 20 below 4×5. Now the expression is $9 + 20$. What is the last step?** *(add)* **What is $9 + 20$?** *(29)*

Using the order of operations, work with your partner to evaluate the other 2 expressions.

Ask student pairs to come to the board and show how they arrived at their solution. Focus on the order of operations to justify the simplified result.

Practicing Together

TEACHER NOTES

Order of operations can be flexible. Student answers may vary if they choose to multiply/divide or add/subtract in different order. Computations within a particular operation can be done in any order, not only from left to right. For example, 8 + 5 + 2 could be solved first by making 10 and then adding 5.

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete the activity sheet. Explain that problem 1 provides an example of what they are to do in the remaining problems Reinforce the idea that computations within multiplication/division and addition/subtraction can be done in different orders (either multiplication or division can be computed first; either addition or subtraction can be computed first), not just left to right.

Ask student pairs to come to the board and show or explain how they arrived at their answer.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Show the worked example. Have students analyze the work and decide if they agree or disagree with it. If they agree, why do they agree? If they disagree, what error was made?

> **Look at the Wrapping It Up page. Work with a partner to analyze Norah's work. Do you agree or disagree with how Norah used order of operations? Why? Justify your answer.**

As time allows, discuss student responses.

Lesson 5: Evaluating Expressions, Using Order of Operations

Lesson Objectives	Students evaluate expressions, given rational numbers. Students make sense of problems and persevere in solving them. (SMP 1)	
Vocabulary	**Commutative property of addition:** for any real numbers a and b, $a + b = b + a$ **Commutative property of multiplication:** for any real numbers a and b, $a \cdot b = b \cdot a$ **Associative property of addition:** for any real numbers a, b, and c, $a + (b + c) = (a + b) + c$ **Associative property of multiplication:** for any real numbers a, b, and c, $a \times (b \times c) = (a \times b) \times c$ **Distributive property of multiplication over addition:** if a, b, and c are real numbers, then $a \times (b + c) = (a \times b) + (a \times c)$	
Requisite Vocabulary	Order of operations, exponents, expression	
Misconception(s)	Students often think that addition must be done before subtraction, regardless of the order of the computations in an expression. The same is true for multiplication and division because of the order of the description in the order of operations.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent)	• Student Booklet • Whiteboard • Sticky notes (3 per student)

	• Projector (or equivalent)	

Warming Up

Display the Warming Up Sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

What is order of operations? *(the order in which to simplify expressions containing more than 1 operation)*

What is step 1? *(simplify expressions within grouping symbols)* **On the line make a picture—for example, make parentheses to help you remember. What are other grouping symbols?** *(braces, brackets, or the fraction bar)*

What is step 2? *(compute all exponents, or powers)* **Write a base number and exponent or power.**

What is the third step of order of operations? *(compute multiplication and division)* **How do we know which operation to complete first?** *(whichever comes first, going from left to right)* **Make a picture or symbol, like the multiplication and division symbols, to help you remember this step.**

What is the fourth and final step when simplifying an expression with more than 1 operation? *(compute addition and subtraction.* **How do we know which operation to complete first?** *(whichever comes first, going from left to right)* **Make addition and subtraction symbols to help you remember.**

Learning to Solve

Teacher Notes

Computations within a particular operation can be done in any order, not only from left to right. For example, 8 + 5 + 2 could be solved by first making 10 and then adding 5. If a student misses a step or evaluates an expression out of order, have the student rewrite how to follow each step and/or underline or highlight each computation as it is solved.

1. Students will review the associative, commutative, and distributive properties of addition and multiplication.

Give each student 3 sticky notes and a whiteboard.

> **On the 3 sticky notes, write the numbers 4, 6, and 5. We will use these 3 numbers to review properties of multiplication and addition.**
>
> **Place the sticky notes 4 and 6 on your board and write an addition symbol between them, making 4 + 6. What is 4 + 6?** *(10)* **Does it matter what order we add the numbers? Why?**
>
> **What if we changed the addition symbol to multiplication? What is the answer?** *(24)* **What order did you use for multiplying?**
>
> **Now use the order of 4, 5, and 6 for your sticky notes. Put an addition symbol between each of these. What is the sum? (*15)* Did everyone add 4 + 5 first, and then add 6? (*Watch for students that did not do this order.)***
>
> **Could I have added 4 + 6 first to make the addition easier by finding a 10? (*Yes)* Did anyone add the numbers in a different order? (*Answers will vary, for example, a student may add 5 + 6 first)***
>
> **Now replace the addition symbol with multiplication. Multiply the 3 numbers. What is your product? (*120*) Did everyone multiply 4 x 5 first? (*Answers will vary. Some students may have multiplied 5 x 6 first).***
>
> **For addition and multiplication, we can change the order of the numbers we are adding or multiplying. This is the commutative property. This property allows us to reverse the order of the numbers we are adding or multiplying without changing the sum or product. We have two commutative properties; the commutative property of addition and the commutative property of multiplication.**

Is there a commutative property for subtraction? *(no, the order for subtracting cannot be reversed without changing the difference)* **What about division?** *(no, changing the order for dividing changes the quotient)*

Have students give examples of subtraction and division to illustrate why they do not have a commutative property associated with them.

Students often confuse order in division and subtraction. For example, some students will say 4 divided by 8 when they mean 8 divided by 4. Watch for students who make that error and ask them to indicate the appropriate order.

2. Students will change groupings to change totals.

Students can continue to use the whiteboard or write the expressions in the Notes section of their Student Booklet. Write each expression on the board as the lesson progresses.

Put all 3 numbers on your whiteboard, with a plus sign between them. Add parentheses around 4 and 6, making (4 + 6) + 5. The parentheses made a grouping. According to order of operations, what do we compute first? *(4 + 6)* **What is the sum of the entire problem?** *(15)*

Erase your parentheses and place parentheses around 6 and 5, so 4 + (6 + 5). What is the sum? *(15)*

Keep the parentheses and change the addition symbols to multiplication. What is the answer? *(120)* **Does it matter how we group or ungroup the 3 numbers?** *(no)* **Why?** *(accept reasonable answers that help explain the associative property)* **The property that allows us to regroup in addition or multiplication is called the associative property. We have the associative property of addition and the associative property of multiplication.**

What if we had both addition and multiplication? Would adding grouping or parentheses change the total? Let's try it. Write "(4 + 6) × 5." What do you compute first? *(4 + 6)* **Why?** *(computing amounts within grouping symbols is the first step of*

order of operations) **Simplify or evaluate the expression. What is the result?** *(50)*

Keeping the symbols the same, group 6×5, so you have $4 + (6 \times 5)$. What do we compute first? *(6×5)* **Simplify the expression. What is the result?** *(34)*

Write the expression $4 + 10 - 5 + 7$. Add parentheses so that your result when you do the computations is 2.

Allow students time to solve. As you monitor students working, watch for students who incorrectly apply the order of operations.

Where did you place parentheses? *(around the 5 + 7)* **If we placed them around the 4 + 10, what would the answer be?** *(16)*

What if we did not add any grouping symbols and just solved, using order of operations? What is the result? *(16)* **How we group the computations affects the answer.**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the problems on the sheet.
2. Review answers and ask students how they solved each problem.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets. Read the problem and then allow time for students to work.

> **Use at least 3 numbers and at least 2 computations to write an expression that results in an answer of 24.**

If time permits, have students share their answers.

Expressions and Equations 1
Lesson 6

Lesson 6: Evaluating Expressions

Lesson Objectives	Students evaluate expressions. Students make sense of problems and persevere in solving them. (SMP 1) Students look for and make use of structure. (SMP 7)	
Vocabulary	**Substitute:** replace a variable with a value	
Requisite Vocabulary	Commutative property, associative property, distributive property of multiplication over addition, variable, expression, equation	
Misconception(s)	Students often think that addition must be done before subtraction, regardless of the order of the computations in an expression. The same is true for multiplication and division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 3 dice	• Student Booklet • Whiteboard • Cross It Out! Gameboard (1 per pair; pages 105-109 of Teacher Masters)

Warming Up

Play Cross It Out! Distribute a game board (pages 105-109 of Teacher Masters) to each pair of students.

> **We are going to play a game called Cross It Out! The goal is to use 3 numbers to make the numbers from 1–10.**
>
> **To play, roll 3 dice to get 3 numbers. Record the numbers rolled on the dice. You must use all 3 numbers from the roll of the dice. You may not use any other numbers. You can use any operation, such as addition, subtraction, multiplication, and division. You can use any operation you want.**
>
> **Let's try a practice round. Say I rolled 2, 3, and 6. I can make 1 by adding 2 + 3, then subtracting from 6. I will record my equation to show what I did by writing 6 – (2 + 3) = 1. On my game board, I would cross 1 out. I can make 5 by subtracting 6 – 3, then adding 2. I would record 6 – 3 + 2 = 5. You will keep trying to cross out all the numbers from 1 to 10.**
>
> **When you think you cannot find any more numbers, record the sum of the numbers that are remaining on the board and not crossed off. For example, if you did not cross off 1, 3, and 4, your score would 1 + 3 + 4, or 8.**
>
> **You will play as many rounds as there is time. The pair with the lowest score after the game ends is the winner.**

Make sure students understand the rules. Then, roll the dice. Record the roll and have students record on their sheet. Allow time for students to play the round. Then roll the dice again. Continue for as many rounds as time allows.

Learning to Solve

Students will evaluate expressions containing a variable.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. As you read, point to the different equations.

> **Equations show 2 expressions that represent the same quantity. The equals sign is used between the 2 expressions to show that they are equal, or equivalent. An expression is different from an equation. It represents a quantity and the value of the quantity changes when we change the value of the variable. This relationship can be evaluated when we substitute values for the variable, but an expression does not use an equal sign.**
>
> **Let's look at 4 different ways to write a multiplication equation. For example, we might use shapes to represent the factors. We can use the multiplication sign, or parentheses, or a dot between the factors.**
>
> **Write 1 of the equations on your sheet. What could the values be that would make the equation true?** *(Answers will vary. Possible combinations of answers include 1 and 20, 2 and 10, 4 and 5, 40 and 0.5, 50 and 0.4, 60 and $\frac{1}{3}$)*

Ask students to read the equation they wrote and the values of the variable.

> **In these equations, variables or shapes were used to represent a value. A variable represents 1 value, many values, or no value. When we multiplied the value of the variables, the product was always 20, but we could assign different values to each variable.**
>
> **In $m(n)$, m can be 5 and n can be 4. Or, using the product of 20, the m might have a value of 10 and n would have a value of 2.**
>
> **Look at the expressions at the bottom of your Learning to Solve sheet. Each expression contains 1 variable. For these expressions, substitute the value 4 for the variable in each problem, and then evaluate each expression.**

Have students complete this activity individually. When they are finished, have students give an answer and explain the process they used.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

> **Both you and your partner will choose a value, or unique number, between 5 and 10 to represent the variable *x*; write them at the top of the appropriate column. Evaluate each expression, using the number you chose, and fill in the table.**

When they finish, have student pairs supply their answers to fill in the table.

Discuss how a variable can represent many different values.

> **What patterns do you see in the table?**
>
> **Are your patterns different from your partner's patterns? Why?**

When finished, have the students do problem 4 and discuss.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets. Read the problem and allow students time to work.

> **Greg evaluated the expression below ($5f + 2 - 8$) and said the value of the expression was 19. What value did he use for the variable f?** *(5)*

If time permits, ask students to give their answer and explain how they determined the value of the variable.

Expressions and Equations 1
Lesson 7

Lesson 7: Equivalent Expressions

Lesson Objectives	Students identify and evaluate equivalent expressions by substituting values for the variable. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7)	
Vocabulary	**Equivalent expressions**: 2 or more expressions that equal the same value for *all* replacements of the variables **Equation:** the same amount or quantity on both sides of an equal sign **Equal:** the relationship between quantities that are the same amount or equivalent	
Requisite Vocabulary	Variable	
Misconception(s)	Students often think that addition must be done before subtraction, regardless of the order of the computations in an expression. The same is true for multiplication and division because of the order of the description in the order of operations.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 3 dice	• Student Booklet • Whiteboard • Cross It Out! gameboard (1 per pair; pages 105-109 of Teacher Masters)

Warming Up

Play Cross It Out! Distribute a gameboard (pages 105-109 of Teacher Masters) to each pair of students.

> **We are going to play a game called Cross It Out! The goal is to use 3 numbers to make the numbers from 1–10.**
>
> **To play, roll 3 dice to get 3 numbers. Record the numbers rolled on the dice. You must use all 3 numbers from the roll of the dice. You may not use any other numbers. You can use any operation, such as addition, subtraction, multiplication, and division. You can use any operation you want.**
>
> **Let's try a practice round. Say I rolled 2, 3, and 6. I can make 1 by adding 2 + 3, then subtracting from 6. I will record my equation to show what I did by writing 6 – (2 + 3) = 1. On my game board, I would cross 1 out. I can make 5 by subtracting 6 – 3, then adding 2. I would record 6 – 3 + 2 = 5. You will keep trying to cross out all the numbers from 1 to 10.**
>
> **When you think you cannot find any more numbers, record the sum of the numbers that are remaining on the board and not crossed off. For example, if you did not cross off 1, 3, and 4, your score would 1 + 3 + 4, or 8.**
>
> **You will play as many rounds as there is time. The pair with the lowest score after the game ends is the winner.**

Make sure students understand the rules. Then, roll the dice. Record the roll and have students record on their sheet. Allow time for students to play the round. Then roll the dice again. Continue for as many rounds as time allows.

Learning to Solve

Students will write partner equations to show equivalency.

Have students complete this part on a whiteboard or in the Notes section of the Student Booklet. Write equations on the board as the lesson progresses.

> **We use the equal sign in many equations. What does "equal" mean?** *(the same amount or quantity on both sides of an equals sign)* **The equals sign means that 2 quantities represent the same amount or value. It does not just mean that the answer comes next. When we evaluate or simplify expressions, the equals sign acts like a balance to show the same value on either side of it.**
>
> **Write the expression 8 × 4 on your whiteboard or in the Notes section of your Student Booklet. What does it equal?** *(32)* **Write "32" below the expression. Now write an equals sign and another expression that also equals 32. This tells us that the expressions are equivalent, or represent the same amount. Each expression is equal to 32.**

Have students write and share their expression with peers. Call on a few students, writing their expression on the board.

> **What expression did you write? Did anyone write a different expression? How did you decide which expression to write?**

Write the equation $7 \times 8 + 2 = 6 \times 9 - 3$ on the board.

> **Copy this equation. Is this equation true? Evaluate each side of the equation to see whether both sides equal the same quantity.**
>
> **Is this equation true?** *(no)* **Why?** *(the expression on the left equals 58 and expression on the right equals 51)* **This is not a true statement because the values, 58 and 51, are not the same, so they are not equal. Another symbol that is used in mathematics**

is the not equals sign. To show this sign, draw a line through the equals sign to show that 2 expressions are not equal.

Write a new expression on the right of the equal sign to make this balanced or to show a true equation, $7 \times 8 + 2 =$.

Now write a new expression on the left of the equal sign to make this balanced or a true equation, $= 6 \times 9 - 3$.

Allow a few minutes for students to write equivalent expressions. Have students share their expressions, writing them on the board and/or sharing with peers.

How do you know that the expressions on the 2 sides of the equals sign are equivalent? *(answers may vary, such as there is the same amount or quantity on both sides of an equal sign)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students complete the Practicing Together sheet in pairs or in small groups. The second part of the activity sheet is for students to write expressions and exchange with other groups.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 7A to provide extended practice before proceeding to Lesson 8.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

In your own words, write your definition of the equals sign.

As time allows, have students share their definitions.

Progress Monitoring Schedule

BEFORE	**AFTER**	**AFTER**
Lesson 1: Pre-assessment **Module Check Form A**	Lesson 7: Mid-assessment **Module Check Form B** ✔	Lesson 15: Post-assessment **Module Check Form C**

Expressions and Equations 1
Lesson 8

Lesson 8: Fact Families

Lesson Objectives	Students solve equations, using fact families or related equations. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Fact family:** a set of 4 (or fewer) equations of related addition/subtraction or multiplication/division facts.	
Requisite Vocabulary	Equations	
Misconception(s)	When students use fact families, they may use the commutative property inappropriately with subtraction or division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Fact Family Sort cards (page 110 of Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Look at the tiles on the balance. The balance shows that the quantity on the left is the same as the quantity on the right. What equations could you write that would represent the relationship shown? *(Answers may vary. For example, 2x + 5 = 11 or x + x + 5 = 11)*

Use the balance scale to decide the value of the variable to make it equal or balanced. How did you decide what the value is? *(Answers may vary. For example, some students may use the equation or others may remove and group tiles on balance scale. Either method should so that x = 3)*

Could a balance also show two quantities that are not equal? *(Yes)* **How would the balance show two quantities that are not equal?** *(Answers will vary, such as the balance will be "higher" on one side.)*

What values could the *x*-tiles represent that would result in a relationship that is not equal? *(Answers will vary, such as the x could be any value other than 3.)*

As time permits, you may want students to give examples of values that will result in a not equal relationship. As they give their examples, discuss which quantity is greater than or less than the other and how that would affect the scale.

Learning to Solve

Teacher Notes

Fact families help students relate algebraic equations to their experiences in number. Feel free to use terms that students are familiar with and use in core math instruction, such as "number family," "number teams," or "fact teams."

1. Students will review fact families and determine which operations are related.

   Have students turn to the Notes section of their Student Booklets or have them use a whiteboard. As you say each equation, write it on the whiteboard or document projector.

   Write "2 + 3 = 5." What is the sum? *(5)* **What are the addends?** *(2 and 3)*

   Using 2, 3, and 5, what is another addition equation we can write? *(3 + 2 = 5)*

   What property did you use to write that equation? *(The commutative property of addition)*

   We can also write 2 subtraction equations, using just these numbers. What are the 2 subtraction equations? *(5 – 3 = 2 and 5 – 2 = 3)* **Write them.**

   Write this equation, 5 times 4 equals 20.

   Is there another multiplication equation that could be written, using these 3 numbers? *(4 • 5 = 20)* **What property did you use to write this equation?** *(The commutative property of multiplication)*

   What 2 division equations are related to the multiplication equation? *(20 ÷ 5 = 4 and 20 ÷ 4 = 5)* Write these equations.

   Addition and subtraction are related—they are inverse operations. Multiplication and division are related—they are inverse operations.

   Understanding how addition and subtraction and multiplication and division are related can help us find the value of a variable to make an equation true.

2. Students will use fact families to solve for the unknown in algebraic equations.

Write "$5x = 105$" on the board. Write the other fact family equations as the lesson progresses. Have students use a whiteboard or the Notes section of their Student Booklets.

> **Sometimes, we know the value of a variable because it is in an equation that we have memorized—for example, $5x = 20$. You probably know what number multiplied by 5 would give a product of 20. What is the number?** *(4)*
>
> **Other times, fact families can help us determine the value of the variable.**
>
> **Look at the equation, $5x = 105$. Is this a fact that you have memorized?** *(most students do not know the value of x quickly)*
>
> **There are 3 values: 5, x, and 105. What is another multiplication equation we can write, using just these 3 values?** *(x(5) = 105)* **Can this help us find the value of x?** *(answers will vary, but most students will indicate that this equation is not helpful)* **Why?**
>
> **What are the 2 division equations we can write with these quantities?** *(105 ÷ 5 = x and 105 ÷ x = 5)* **Write the 2 equations. Can 1 of these equations help us to find the value of the variable?** *(yes, 105 ÷ 5 = x)* **Why?** *(the value of the variable can be found by dividing and the unknown is the quotient)*
>
> **What is the value of x?** *(21)* **Let's check our work. Rewrite the original equation, substituting 21 for x, and solve. What is 5 times 21?** *(105)*

Write "$142 \div x = 71$" on the board.

> **Write the next equation, $142 \div x = 71$. Write the equations in the fact family, using the 3 values.**

Have students share the equations they wrote.

> **Which equation would be the most helpful in finding the value of the variable?** *(142 ÷ 71 = x)* **Find the value of the variable.**

Allow students time to write and solve.

> **What is the value of *x*?** *(2)* **How did you find its value?** *(divided)* **Did anyone find the value of *x* differently?**
>
> **If I wrote "71*x* = 142," would this have changed how you solved to find the value of the variable?** *(no)* **Why?** *(because the equation is related to the division equation 142 ÷ 71 = x)*

Practicing Together

Have students do the Fact Family Sort activity. Cut apart the cards (page 110 of Teacher Masters) and have them ready for pairs to work with.

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work in pairs to sort the cards by fact family. When they have sorted them, ask pairs to share a fact family. Have the other pairs determine if they agree with the fact family or not. If not, why not?

After the cards are sorted, determine which one would be the most helpful in finding the value of *x* in each set.

Use that equation to find the value of *x*.

Ask students to share their answers. Discuss how they determined the value of *x*.

When completed, have the students to the multiple choice problem 2 and share their answers.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

In the Notes section of their Student Booklets, have students describe how to use fact families to find the unknown, as they did in Practicing Together.

If time allows, discuss the use of fact families and how inverse operations are related and can assist in solving for the unknown.

Lesson 9: Writing Expressions From Word Problems

Lesson Objectives	Students write algebraic expressions from a mathematical phrase. Students write expressions containing a variable from a word problem. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Expression, fact family, sum, difference, increased by, decreased by, product, quotient, addend	
Misconception(s)	Students often believe that they can "solve" an expression. An expression can only be evaluated when a value is given to the variable but we do not refer to this as "solving."	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Matching Cards (1 set per pair; page 111 of Teacher Masters)

Warming Up

Have students turn to the Notes section of their Student Booklets.

In mathematics, we can write numbers or use symbols to show relationships or describe a quantity or situation. I will give you a relationship that describes a quantity. You will write a mathematical expression that uses a variable and other symbols.

Write the expression that represents the quantity 8 more than 29.

What did you write? *(8 + 29 or 29 + 8)*

What is the sum? *(37)*

Write the expression that represents the quantity 3 times larger than 12.

What did you write? *(3(12) or any equivalent expression that uses other multiplication symbols)*

What is the product? *(36)*

Write the expression that represents the quantity 14 less than 50.

What did you write? *(50 – 14)*

What is the difference? *(36)*

Learning to Solve

Teacher Notes

Students will write algebraic expressions to represent mathematical and real-life situations. Watch for students who use

the words *expression* and *equation* inappropriately. Students often interchange the terms.

Some students may also look for "key" words that tell them what to write. It is important not to give them particular key words as these are not always consistent in other contexts.

1. Students will read a word description of a mathematical situation and write the algebraic expression.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Look at number 1: The product of 6 and *n* decreased by 4. First, what is the variable, or the unknown? *(n)* **What are the relationships, or operations, in the statement?** *(multiplication and subtraction)* **What is the multiplication relationship?** *(the product of 6 and n)* **How would you represent the multiplication?** *(6n or any equivalent representation)*

   The subtraction relationship involves the product of *6n* and 4. How would you represent the subtraction? *(6n – 4)* **Write the expression. What did you write?** *(6n – 4)*

   Look at number 2: *t* cubed increased by 3. What operations will this expression have? *(powers or exponents, addition,)* **What words in the description help you decide what operations to use?** *(cubed, increased by)* **The situation involves 2 addends. How would you express the first addend?** *(t^3)* **Write it.**

   How would you show the second addend? *(3)*

   What is the completed expression? *($t^3 + 3$)*

2. Students will write expressions containing at least 1 variable from word problems.

   You have been reading mathematical situations to write the expressions. Expressions can also be written from word problems. You can also write equations from word problems.

You might see these tasks on a test that asks you to write how you would solve, without actually solving the problem.

Look at the first word problem.

Select a student to read the problem.

Follow along as [student] **reads.**

Pause for the student to read.

What is the unknown? *(the number of apples Jeffery picked)* **We do not have the value for the number of apples Jeffery picked. We can write a variable to represent this number. Choose a variable to represent this number.**

Write "number of apples Jeffery picked = x."

Do we know how many apples Brian picked? *(no)* **Because we do not know the number of apples he picked, but we know the relationship of the number he picked to the number Jeffery picked, we can write that relationship. Write "$x - 32$" under the problem.**

Look at the second problem.

Select a student to read the problem.

Follow along as [student] **reads.**

Pause for the student to read.

What is the unknown? *(the number of shells Hazel collected)* **Use a variable to represent the number of shells Hazel collected.**

Write "number of shells Hazel collected = z."

Write the equation.

What did you write? *(73 + z = 219 or any equivalent form such as 219 – 73 = z)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students do the Matching Cards activity in pairs (1 set of cards per pair, page 111 of Teacher Masters).

   Now you will work in pairs and do the Matching Cards activity. There are two sets of cards. On one set is card with an equation written on it; on the other set, each card has a statement that goes with an equation. Working together, match each statement to its equation.

2. Review the answers, having students write the expressions and matching statements on the board or on document camera.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets. 1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets. Read the problem.

Write an algebraic expression to represent the difference of 8 and w.

If time permits, ask students to read their expression or show what they have written.

Expressions and Equations 1
Lesson 10

Lesson 10: Writing and Solving Equations

Lesson Objectives	Students write and solve equations with a variable. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Solution to the equation:** the value that when substituted for the variable creates a true statement	
Requisite Vocabulary	Equation, expression	
Misconception(s)	Students may indicate that the "answer" comes after the equals sign. They may also confuse the "solution of an equation" with the number, if there is one, that comes after the equals sign.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	Student Booklet Make It a Rectangle cards, 1 set per pair (page 112 in Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Read the problem.

> **Yesterday, I brought 4 granola bars to school for a snack. A teacher gave me some more granola bars. What expression could I write to show this relationship?**

Allow students time to write an expression. Have some students share their expressions. Students should identify a variable to represent the number of granola bars you were given by writing, for example, j = the number of granola bars the teacher gave me. The expression is $4 + j$, or any equivalent expression.

> **What if the teacher gave me 7 granola bars? How many would I have? If we substitute 7 for the variable j, we can evaluate the expression to find that I have 11 granola bars.**
>
> **Select another value for your variable. Substitute it into the expression. Write the numerical value you get.**

Allow students time to do the substitution. Have students share the value they got for the algebraic expression. As each student shares, have students determine the value that was substituted for the variable to get the value. Have the student who shared the value indicate whether it is correct.

> **What value did** [student] **substitute for the variable to get this result? How did you decide?**

Continue with as many examples as time permits.

Learning to Solve

Teacher Notes

Students will move from writing and evaluating expressions to solving equations in this lesson. Reinforce the correct usage of *expression* and *equation*. Some students may be confused by having a value come after that equals sign that is not the "answer."

Asking students to find the 'solution to an equation' for some means that you find an answer. Stress that finding the solution to an equation means that they are determining the value of the variable that makes the equation true. They may confuse this with the number that comes after the equals sign.

Students will read a word problem, write an equation, and then solve for the variable.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the first problem on your sheet.

Select a student to read the problem.

Follow along as [student] **reads.**

Pause for the student to read.

What is the unknown? *(the number of days she will run)* **Until we solve the problem, we do not know how many days she will need to run to reach 48 total miles. How can we find the number of days?** *(accept reasonable answers—most students will suggest multiplication or division)*

We can write an equation to represent the problem. How many miles does Evelyn run each day? *(4)* **So we know 4 miles**

each day, but we do not know how many total days. I will let *y* represent the number of days.

Write "*y* = the number of days Evelyn runs."

How many total miles will Evelyn run for this problem? *(48)* Write "= 48." What operation would show 4 miles a day for a certain amount of days equals 48? *(multiplication)* **How do you know?** *(accept reasonable answers)*

What operation is related to multiplication that we can use to help us solve? *(division)* **Think about fact families. Write the other equations for this fact family, using *y* as the unknown value.**

Select and circle an equation to use to solve. How many days does she have to run to reach 48 miles ran? *(12)*

Write "*y* = 12 days."

Look at the next problem.

Select a student to read the problem.

Follow along as [student] reads.

Pause for the student to read.

First, select an equation. Then, write no more than 3 sentences about why you chose that equation.

What is the variable? What is the unknown for this problem? *(number of cookies per box)* **What do we know?** *(6 boxes of cookies and 72 total cookies)*

What equation did you select? *(6c = 72)* **Why?**

If we knew the value of *c*, the number of cookies in each box, we can multiply that by the number of boxes to find a total number of cookies. We do not know the value of *c*. How can we find it? *(by division—some students may recognize by inspection that 6 • 12 = 72)* **Divide 72 by 6.**

How many cookies were in each box? *(12)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work together to solve the problems on the activity sheet.
2. Ask for students to show their equations and describe how they solved. Watch for students to use fact families.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 10A to provide extended practice before proceeding to Lesson 3.

Wrapping It Up

Give each pair of students a set of Make It a Rectangle cards (page 112 in Teacher Masters). They are to match the edges of each card, an algebraic expression and a word description, so that they have a rectangle.

> **You will work in pairs to play Make It a Rectangle. In this game, you will match the edges of the cards so that an algebraic expression and a word description match up. When you finish, you should have a rectangle that measures 3 cards by 4 cards.**

Have one of the pairs share their work with the other students and provide error correction as needed.

Expressions and Equations 1
Lesson 11

Lesson 11: Solving Equations By Inspection

Lesson Objectives	Students solve equations by inspection. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Solve by inspection:** use knowledge of basic facts to solve an equation	
Requisite Vocabulary	Equation, expressions, variable, solution to an equation	
Misconception(s)	Students often think the number that comes after the equal sign is the solution to the equation.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Look at the maze. Using *n* as the variable, write an algebraic expression for each word phrase. Then, find the path from the start to the end, and draw a line through the word phrases that involve multiplication.**

Allow students for time to work. When done, have students share their expressions and the path that they took.

When students finish their explanations, display the completed page of the Warming Up sheet in the Teacher Masters. Have students compare their expressions and the path that they took to the one shown.

Learning to Solve

Teacher Notes

Students have been solving equations since the early elementary grades. This lesson provides opportunities for students to use their knowledge of basic facts to solve simple equations by inspection (that is, by looking at equations and recognizing basic facts).

1. Students will solve 1-step equations by inspection only.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   > **We have been writing algebraic phrases and solving word problems from equations and expressions. Today, we will solve equations by inspection. That is, we will use knowledge of basic facts to solve an equation. What does it mean to solve an**

equation? *(find the value of the variable that makes the equation a true statement)*

What is a variable? *(an unknown)* **The variable, or unknown, can be any letter that represents a quantity.**

Look at the first expression, k + 8. What is the variable? *(k)* **Underneath this expression is a question: What number added to 8 is 17?**

Can you solve this by inspection or knowing addition facts? *(yes)* **What is the value of k?** *(9)*

Try the next 2 problems, solving by inspection and using your knowledge about basic facts.

Allow time for students to work.

What is the value of t? *(11)*

What is the value of z? *(15)*

Now look at number 4. When a number is next to a variable, what is the operation? *(multiplication)* **Read the equation.** *(6r = 42)*

What number multiplied by 6 equals 42? *(7)*

2. Students will solve an equation by covering up and then inspecting.

Let's try an equation that contains 2 operations. Look at number 5. Read the equation. *(2 + 4g = 22)*

First, cover up the multiplication. Now think, 2 plus another number equals 22. What number plus 2 equals 22? *(20)*

So, 2 + 20 = 22. Does everyone agree with this statement? *(yes)*

We can't just write that g equals 20 because g is multiplied by 4. 4 times what number equals 20? *(5)* **What is the value of g?** *(5)*

Check your work. Rewrite the equation with 5 instead of g.

Make sure that students write "2 + 4(5) = 22."

Is the equation true for this value of the variable? *(yes)*

The solution to the equation is $g = 5$.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work together to complete the four problems on the activity sheet.
2. Have students share their answers and describe how they solved the problems.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section in their Student Booklets.

Write an equation whose solution is 2. *(Answers will vary. For example, 2x = 4; x + 3 = 5. Watch for students that write x + 1 = 2.)*

Expressions and Equations 1
Lesson 12

Lesson 12: Making Tables

Lesson Objectives	Students use tables to solve equations. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Expressions, order of operations, variable, solution to an equation	
Misconception(s)	Students sometimes think the solution to an equation is the value of an expression in an equation rather than the value of the variable that makes the equation true.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Complete the sheet as you progress through the lesson.

> **We will review order of operations. We can use the acronym GEMS to help us remember the order. We'll change the wording a little from what we used in previous lessons so they fit GEMS. What is the first step of order of operations?** *(Grouping symbols)* **Write it.**
>
> **What is step 2?** *(Exponents or powers)* **Write them.**
>
> **What are step 3?** *(Multiplication and division)* **Write them. How do I know which to do first?** *(perform them from left to right, whichever comes first)*
>
> **What is the last step of the order of operations?** *(Subtraction and addition)* **Write them. How do we know which to complete first?** *(perform them from left to right, whichever comes first)*
>
> **Evaluate the next 3 expressions, using the order of operations.**

Allow students time to complete the problems.

Review the answers when students are done. Ask students to share the process they used. Compare their process to GEMS.

Learning to Solve

Teacher Notes

Using a table to solve an equation helps students see the relationships between the expression on the left side of the equation and the expression on the right side of the equation. Notice that in a table, you can see the relationship between the 2

expressions when the values substituted for the variable do not create a true equation.

Students will use a table to find the solution of an equation.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Look at number 1. We want to use the Guess and Check strategy to find the solution to the equation $2h - 8 = 84$. A table can help us find the value of h that will make the equation a true statement. Look at the 3 columns.**
>
> **The first column shows the value of the variable h. We are trying to find values that we can substitute for h that will make $2h - 8$ equal to 84. The second column gives the value of $2h - 8$ when we substitute a value for h. The third column is our check column. When the second column and the third column both show 84, we have found the solution to the equation.**
>
> **I am guessing 10 for the value of h. When 10 is substituted into the expression, I can check that $2h - 8$ is equal to 12. That is way too small. Next, I am guessing 30. When I substituted 30 into the expression to check, $2h - 8$ is equal to 52, which is still too small. What is another value we can substitute for h?** *(accept any number)*

Have students suggest (guess) values for h. If the values they suggest are less than 30, discuss why options greater than 30 might be more appropriate. However, for some students, using any value they suggest and evaluating $2h - 8$ would help them see that values less than 30 will not work. Similarly, values much larger than 30 may not be appropriate.

Place the value in the first column. Have students evaluate $2h - 8$. Compare the result to 84. Adjust the next value to get closer to the solution.

The table shows that when we substitute 46 for h, the value of the expression on the left side is 84. The value of the expression on the right side is 84. When the 2 values are the same, we know we have used the Guess and Check strategy to find the solution to the equation.

Let's try another equation and table. Look at problem 2. Talk with the person next to you. Decide what might be a reasonable value to begin checking in our table.

Allow students time to talk to a partner.

Have a pair share a value. Ask them to explain how they decided whether the value was reasonable. Place the value in the table. Have students evaluate $2x + 3$. Check the value to see whether it equals 27. If not, have students select another value. Continue in this way until they have found a value that makes the equation a true statement.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the activity sheet.
2. Review the answers as a group, asking students to explain how they completed the tables.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students write a response to the following in the Notes section of their Student Booklets or discuss as a class.

> **We have used the Guess and Check strategy to solve equations by using inspection, fact families, and tables. Write your preference for 1 of these three in the Notes section of your Student Booklet. Then write why it is your favorite.**

Lesson 13: Solving Equations

Lesson Objectives	Students solve word problems containing a variable by inspection. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equation, expression, solution to an equation	
Misconception(s)	Students often think that addition must be done before subtraction, regardless of the order of the computations in an expression. The same is true for multiplication and division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Make it a Rectangle (page 110 in Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Students should understand that the equation used to find the value of a variable is the one where the variable stands alone next to the equals sign, for example, 125 – 14 = *y* or *y* = 125 - 14.

> **Write the fact family for each problem. Circle the equation you will use to find the value of the variable that makes the equation a true statement. Solve it.**

Give students time to work.

> **What fact family did you write? Which equation would most help you solve for the value of the variable?**

Discuss the solution to each problem.

Learning to Solve

Teacher Notes

Fact families help students relate the move to algebraic equations to their experiences in number. Feel free to use terms students are familiar with and use in core math instruction, such as *number family*, *number teams*, or *fact teams*.

Students will read a word problem, write an equation or equations, and solve.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as you progress through the lesson.

> **Look at the first problem on your sheet.**

Select a student to read the problem.

> **Follow along as** [student] **reads.**

Pause for the student to read.

> **What are we trying to find?** *(the amount of money Tori made pet sitting)* **What equation can we write to show the problem?** *(12(8) = x)* **What does the variable represent?** *(the amount of money Tori made)*
>
> **The first equation we wrote is a multiplication equation. Could we use another equation to solve this problem?** *(accept other answers, writing equations on board)*
>
> **How much money did Tori make?** *($96)*
>
> **Look at the next word problem.**

Select a student to read the problem.

> **Follow along as** [student] **reads.**

Pause for the student to read.

> **What are we trying to find?** *(the number of stamps Juan used)*
>
> **What equation could you write to show this relationship?** *(accept any equations equivalent to 54 – x = 36)*

If students have difficulty, use the script below.

> **How many stamps did Juan start with?** *(54)* **How many did stamps did he use? Do we know?** *(no)* **What operation would show how many stamps Juan used?** *(subtraction)* **What will the variable represent in the problem?** *(the number of stamps Juan used)* **What is an equation that could be used to solve?**

Have students write as many equations as they can that could be used to solve the problem. Ask the following questions, writing other possibilities on the board, such as $54 - x = 36$, $36 + x = 54$.

Why is there more than 1 equation that can be written? *(Answers will vary, such as, the relationship of the number of stamps he had and the number of stamps he used can be represented in different ways.)*

How many stamps Juan use? *(18)* **Did using a different equation change the answer?** *(no because they all represent the same relationship)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work together to complete the activity sheet.
2. Review the answers as a class, having students explain different ways to solve.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Give each pair of students a set of Make It a Rectangle cards (page 110 in Teacher Masters). They are to match the edges of each card, an algebraic expression and a word description, so that they have a rectangle.

> **You will work in pairs to play Make It a Rectangle. In this game, you will match the edges of the cards so that an algebraic expression and a word description match up. When you finish, you should have a rectangle that measures 3 cards by 4 cards.**

Have one of the pairs share their work with the other students and provide error correction as needed.

Expressions and Equations 1
Lesson 14

Lesson 14: Translating Into Expressions and Equations

Lesson Objectives	Students write and solve equations containing a variable from a word problem. Students write an expression from a verbal phrase. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equation, expression, fact family, solution of an equation	
Misconception(s)	Some students think that key words can be found in all word problems and will provide a clue as to what operation will be appropriate.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Words to Symbols, 1 set per pair (pages 113-114 of Teacher Masters).

Warming Up

Distribute 1 set of Word to Symbol cards to each pair (pages 113-114 of Teacher Masters). Have them match the word statement to the algebraic expression. This is a review from previous lessons but provide a segue into solving word problems with algebraic equations.

> **Each pair has a set of cards. The cards labeled with an E are expression cards. The cards labeled with a W are the word cards. In your pair, you are to match the word statement that goes with the expression.**

Allow time for students to make the matches. Have pairs share the matches they made. Discuss the matches as needed or appropriate.

Learning to Solve

Teacher Notes

Fact families are used to solve word problem situations in this lesson. They help students understand relationships described in the problems as well emphasize the inverse relationships of operations. If you prefer that your students use a table, you can substitute a table solution for the fact families.

Students will read a word problem, write an equation, and solve for the variable.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as you progress through the lesson.

> **Look at the first problem on your sheet.**

Select a student to read the problem.

> **Follow along as** [student] **reads.**

Pause for the student to read.

What are we trying to find? *(the amount of money he made mowing lawns)* **Talk with a person sitting next to you. Is the amount of money he made greater than $20 or less than $20? How do you know?** *(allow students to guess with an explanation. Students may say that it has to be greater than $20 because he makes that much with just one lawn.)*

We can write an equation to help us solve and to understand the relationships that are given in the problem. What information do you know? *(the amount he charges for each lawn mowed; the number of lawns he mowed)*

We can represent the amount of money he makes, or our unknown amount, with a variable. I'm going to use *x*. We always identify what the variable represents so in this case, *x* = the amount of money he makes.

Think about the relationship among the number of lawns mowed, the cost of each one, and the total he earned. If we know the number of lawns he mowed and the cost for each lawn, how would we find the total cost? *(multiply the number of lawns times the cost for each lawn)* **What equation could we write with the information we have?** *(6(20) = x)*

How can we solve this equation? *(accept reasonable answers)* **Think about fact families. What would the other members of the fact family be?** *(6 • 20 = x, x ÷ 20 = 6, x ÷ 6 = 20)*

Which equation in the fact family would help us solve for *x*? *(6 • 20 = x is a logical choice)*

In this equation, we can find the product of 6 • 20. What is the quotient? *(120)*

Write "*x* = $120." What does $120 mean? *(the amount of money James made mowing lawns)*

Let's try the next problem.

Read the problem.

What relationships do we know from the problem? *(answers will vary. For example, Jenna's numbers of runs scored across all games would be added first, then that sum would be divided by the number of games she played)* **What is the unknown or what are we trying to find?** *(Jenna's number of runs per game)* **Pick a letter to represent our variable, or the unknown value. I will use *x*. Write *x* = Jenna's number of runs scored per game.**

We know that Jenna's total runs scored will be summed by adding together number of runs she scored across all of the games she played in. How can we represent that? *12 + 0 + 6 = 18* **Write that. We also know that this sum will be divided by the total number of games she played, which was how many games?** *(9)* **How do you know?** *(6 + 2 + 1 = 9)* **Write it. What equation could we write to find how many runs per game Jenna scored?** *(18 ÷ 9 = x)* **Write that.**

Now solve to find Jenna's number.

What value did you find for *x* that made the equation true? *(x = 2)* **What does 2 represent?** *(the number of runs Jenna scored per game)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work together to complete the problems on the activity sheets.

2. Review the answers as a class, sharing how students solved each problem.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students discuss or write their thoughts in the Notes section of their Student Booklets.

> **How can fact families help us solve for the variable in the equation $32 - x = 14$?**

Lesson 15: Solving Equations

Lesson Objectives	Students write equations from word problems and solve with a variety of methods. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equation, expression, solution of an equation	
Misconception(s)	Students may have difficulty in representing relationships that involve multiple values.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Complete the sheet as you progress through the lesson.

Select a student to read the problem.

> **Follow along as** [student] **reads the problem.**

Pause for the student to read the problem.

> **We can solve this problem by writing an equation. What is the unknown?** *(the number of students who listed "watching TV")* **Let's use x to represent the number of students who like to watch TV, but we could use any letter. Write an equation.**

Give students time to write an equation.

> **What did you write?** *(answers may vary, such as, $3x - 12 = 249$ or $249 = 3x - 12$)* **How are the equations related?** *(answers may vary, such as, students may say that the equations belong to the same fact family or they may comment that they are equivalent. The equations represent the symmetric property of equality.)*
>
> **Solve for x to find the number of students who voted for TV.**

Ask students to answer these questions based on the equation they wrote.

> **What is the value of x?** *(87)* **How did you solve to find the value of the variable?** *(answers may include inspection, cover and then calculate, fact families, or tables)* **Did anyone get a different answer? Does this answer make sense? Why?**

Learning to Solve

There is no Learning to Solve in this lesson. Students have been applying their understanding of expressions and equations to word problems. This lesson extends that understanding and provides additional practice support.

Practicing Together

Teacher Notes

Encourage students to use other strategies in addition to the ones given here. Individual students may have a preference for some techniques over other methods.

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the activity sheet.
2. Review the answers and have students discuss how they solved the problems.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.

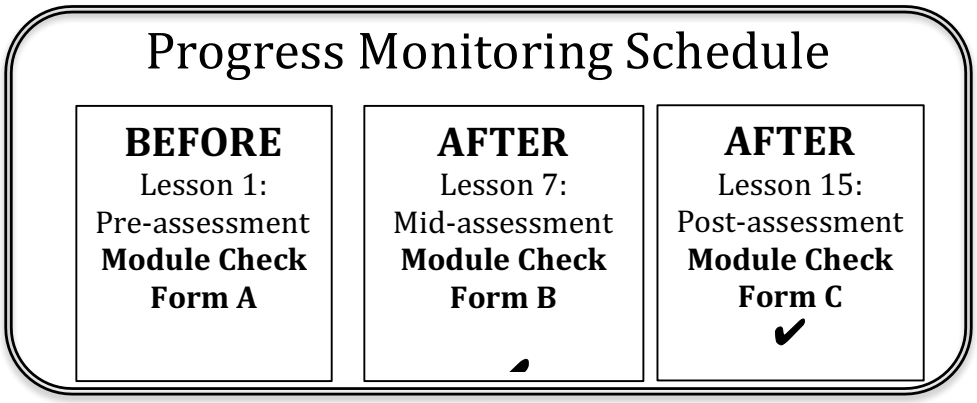
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

This is the last lesson in the module. Display and revisit the concept map made in the first lesson for the module. Give students time to talk with a partner about revisions or additions. Have students share their changes while you make them on the map.

> **This is the concept map that we made in our first lesson of this module. Talk with your partner to decide whether there are things you would like to revise or add to the map.**



Appendices

Expressions and Equations 1

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Expressions and Equations 1
Lesson 3A

Lesson 3A: Patterns That Use Variables

Lesson Objectives	Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Variable:** a letter or symbol than can represent a specific value or a generalized quantity **Expression:** Numbers, symbols and operators (such as +, ×, ÷, and –) grouped together to represent a quantity **Equation:** A statement that shows that two quantities are equal; this is represented with an equals sign.	
Requisite Vocabulary	Pattern	
Misconception(s)	Students often think that a variable represents an object rather than a quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Cubes, such as base-10 units (50 per student pair)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Look at each problem. An equation is a statement that shows that two quantities are equal; it will have an equals sign. Find the unknown or missing value that makes the equation true. We used a line, a circle, a square and a letter to show where the missing number is in the equation. These are different representations for a value that makes the equation true.**

Provide time for students to work.

When finished, ask students to give their answer to a problem and explain how they arrived at their answer.

> **How did you decide what value would make the equation true?**

Students should not be expected to use algebraic means to solve the problems. Instead, they should use number sense, numerical reasoning, and logical thinking to find the missing values.

> **What did you notice about the values on each side of the equal sign?**

Stress that each side of the equals sign should represent the same amount.

Learning to Solve

Teacher Notes

Students may think of a variable as an object, not as a value or part of a relationship. A variable can be used to represent an unknown value or describe a pattern.

Students can use any variable to represent the missing value. However, encourage them to select a variable that is not the first

letter of the unknown quantity. When students select this first letter, they confuse the fact that the variable is representing the unknown **value** and not the unknown object.

1. Students will review the symbols that represent the operation of multiplication and the symbols that represent the operation of division.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Look at problem 1.

   An expression is made up of numbers, symbols and operators (such as +, ×, ÷, and –) grouped together that show a quantity. It does not have an equals sign because the value of the expression can vary. Write the expression 3 times 4 as many ways as you can, using symbols.

   Ask students to share their expressions. Write all the expressions on the board.

   Look at problem 2.

   Write the expression 15 divided by 5 as many ways as you can, using symbols.

   Ask students to share their expressions. Write all the expressions on the board.

   Have students add to their activity sheet any expressions they did not write.

2. Students will recognize and use their own words and variables to describe a pattern.

   Have students work with a partner to complete the cube problem. Provide 50 cubes to each student pair to use (if they wish) in developing this pattern.

> **Complete questions A to D of the cube problem. Use your own words to describe the patterns you notice.**

Have students share their answers and their solution method. Write the answers on the displayed Learning to Solve sheet.

Next, you will rewrite a student's sentence from question D with variables and describe what you are doing. The following uses this example sentence: To find the number of concrete blocks needed to make a wall, multiply the number of stories by 4 and then add 2 less than the number of stories.

> **What quantities do we not know?** *(the number of concrete blocks to make the wall and the number of stories in the tower)*
>
> **Which quantity of the object depends on the value of the other quantity of the object?** *(the number of concrete blocks depends on the number of stories in the tower)*
>
> **Write a sentence that would tell someone outside of class how to find the number of concrete blocks needed to build a wall with towers of any number of stories.**

Pause for students to write.

> **One sentence could be: To find the concrete blocks needed to make a wall, multiply the number of stories by 4 and then add 2 less than the number of stories.**
>
> **In the Warming Up section, we used circles, squares, lines, and letters to represent the value that was unknown. These representations are appropriate, but I want you to write an expression that uses the letter *h* to represent the number of stories of the tower, which is the value we need to know to calculate the number of concrete blocks needed to make the wall. Write that now.**

Write the expression as you link the mathematical representations to the word description.

Our words said to multiply the number of stories by 4. We can represent that by $4h$. Then, we said to add, so we will put a plus sign.

Write "$4h +$."

Next, we add 2 less than the number of stories. We can represent that with $h - 2$. The whole expression is $4h + (h - 2)$. Write it.

TEACHER NOTES

Be sure to use "substitute" rather than "plug in" when you find new values.

Display the table on the next page of the activity sheet.

We can use this expression to find the number of concrete blocks needed for any number of stories. Let's try it. If the building is 15 stories high, how many concrete blocks are needed? What would we do first? *(substitute 15 for h)* **Our new expression is $4 \times 15 + (15 - 2)$. What is 4 times 15?** *(60)* **What is $15 - 2$?** *(13)* **What is $60 + 13$?** *(73)* **How many concrete blocks are needed for a building that is 15 stories high?** *(73 concrete blocks)*

In your Student Booklet, work with your partner to complete the table. Then, work together to write an expression that represents your description of the pattern.

Have student pairs write their expressions on the board. Discuss any differences among expressions.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Work with a partner to answer the questions in Practicing Together. Be prepare to explain how you chose your answer.

Have student pairs read their expressions and explain how they determined them.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section in the Student Booklet.

In the toothpick problem from an earlier lesson, the sentence that described the pattern to determine the number of toothpicks needed to make a certain number of squares was: Multiply the number of squares by 3 and then add 1.

Write an expression to represent this pattern. Be sure to add what the variable represents.

(3r + 1 where r represents the number of squares)

Have students share their pattern. Make sure they indicate what the variable represents.

Expressions and Equations 1
Lesson 7A

Lesson 7A: Algebraic Expressions and Equations

Lesson Objectives	Students write expressions from diagrams. Students evaluate equivalent expressions with variables. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7)	
Vocabulary	**Equivalent**: equal, or the same as **Equivalent expressions:** 2 expressions that represent the same value	
Requisite Vocabulary	Expression, equation, equals, variable,	
Misconception(s)	Students often confuse the equals sign as meaning "the answer comes next" rather than representing a relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Algebra tiles

Warming Up

TEACHER NOTES

Many students exhibit a misconception in understanding the equals sign. Writing equations in different orders (for example, 14 = 6 + 8), as well as discussing the role of the equals sign, is important to correct the misconception.

In the Warming Up problem 6 + 8 = ___ + 5, watch for students who write a 14 in the blank. Many students think of the equals sign as meaning "the answer comes next." In this situation, many students will first think to add 6 + 8 and then add 5 later or to the answer, much like 6 + 8 = 14 + 5 = 19 in a string of equalities. A string of equalities like this is incorrect and should be avoided.

Review the meaning of the equals sign.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **The equals sign is used to show that 2 expressions are equal or represent the same value. What does "equal" mean?** *(the same amount or quantity on both sides of an equals sign)* **We can think of the equals sign like a balance.**
>
> **Look at the first equation, 6 + 8 = [blank] + 5. What number can we write on the line to make this statement true?** *(9)* **Write it.**
>
> **Now look at the second equation, [blank] + 9 = 15 – 3. What number do we write on the line to make this a true equation?** *(3)* **Write it.**

Now, write 2 different numerical expressions that represent the same value. We can write those expressions in an equation. The equals sign lets us know that the 2 expressions represent the same value.

Select 1 to 3 students to share their equivalent expressions. As they share, have the other students check to be sure that the expressions they wrote are equivalent. Discuss any that are not.

Learning to Solve

Teacher Notes

Students often think that the value of a variable is dependent upon its placement in the alphabet. For example, if the variable is *b*, its value must be less than the value of the variable *m*, because *m* occurs later in the alphabet.

Students will write expressions from diagrams.

A variable can represent 1 value, many values, or no value. "Equivalent" means "equal, or the same as." On the Warming Up sheet, we wrote equivalent expressions. They did not contain the same numbers, but they had the same value when we evaluated them. Today, we will determine whether expressions are equivalent by using different methods.

Give each student or student pair algebra tiles.

Algebra tiles are tools that can help us explore expressions and equations by creating a model. The yellow squares represent 1. We will not use the red side of the small squares. The green rectangles represent *x*. And, like the small squares, we will not use the red side.

If we want to model 4, we would use 4 yellow squares.

Have students model 4 with you.

How do you think we might model 2*x*?

Have students work with their partner to decide how to create that model. Some students may want to use 2 yellow squares and an *x* tile. The correct model should be 2 of the green rectangles.

How could we model 3*x* + 2?

Have students work with their partner to decide how to create that model. You may want students to share some of their models. Compare and contrast what students created. The model should be 3 green rectangles and 2 yellow squares.

Display the Learning to Solve sheet in the Teacher Masters. Complete as you progress through the lesson. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the diagrams of the algebraic expressions, numbers 1 and 2. The small square represents 1, and the rectangle represents a variable, or an unknown value. They represent the same tiles we used.

Create this diagram, using your algebra tiles.

What expression could you write that would represent these tiles? *(x + 6)* **Write the expression *x* + 6 to represent this picture. Can we use any letter to represent the variable?** *(yes)* **Why?** *(variables represent values, so the letter doesn't matter)*

Look at the next diagram. What expression could you write to represent these tiles*?* *(3x, or x + x + x)* **Why do you think this diagram can represent either expression?** *(answers will vary, such as 3 of the rectangles are put in a group)*

Look at numbers 3 and 4. Using your own tiles, 1 partner will create the diagram in the left box, and the other partner will create the diagram in the right box. The diagrams in the boxes represent equivalent expressions.

Provide time for partners to build the diagrams.

In number 3, what is the expression represented by the diagram on the left? *(5x or x + x + x + x + x)* **What does 5*x* equal?** *(15)*

If 5x is equal to 15, what must *x* equal? Think, 5 times what number equals 15? *(3)* **What are some ways to find the value of *x*?** *(accept reasonable answers, such as count by 5s or divide 15 by 5)* **Are there any other numbers that we could use to make this statement true?** *(no)*

Now look at number 4. What equation can we write? *(7 + x = 16)* **Write it.**

How can we identify the value of *x* that will make this equation true? *(accept reasonable answers, such as counting on from 7 to 16, or 16 – 7, or recalling the basic fact to answer 7 plus what number equals 16)*

What is the value of *x* in this example? *(9)*

The value of a variable must make the equation a true statement.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work together to complete the activity sheet.
2. Review the answers, asking for students to share and work out the equivalent expressions written.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

> **We have used 3 vocabulary terms in this lesson: "variable," "expression," and "equation." Write an example or a picture to describe each of these terms.**

If time permits, ask students to give an example or share a picture.

Expressions & Equations-Part 1
Lesson 10A

Lesson 10A: Translating into Expressions and Equations

Lesson Objectives	Students write and solve equations containing a variable from a word problem. Students will write an expression from a verbal phrase. Students make sense of problems and persevere In solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Expression, equation	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Make It a Rectangle cards, 1 set per pair (page 112 in Teacher Masters)

Warming Up

Give each pair of students a set of Make It a Rectangle cards (page 112 in Teacher Masters). They are to match the edges of each card, an algebraic expression and a word description, so that they have a rectangle.

> **You will work in pairs to play Make It a Rectangle. In this game, you will match the edges of the cards so that an algebraic expression and a word description match up. When you finish, you should have a rectangle that measures 3 cards by 4 cards.**

Have one of the pairs share their work with the other students and provide error correction as needed.

Learning to Solve

Students will read a word problem, write an equation and then solve for the variable.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as the lesson progresses.

> **Look at the first problem on your sheet. Follow along as** [student] **reads. Alicia's 85 on her social studies test was 37 points less than twice the grade on her math test. What was the grade on her math test?**
>
> **What are we trying to find, what is the unknown?** *(the grade on her math test)* **Is the grade on her math test less than or greater than the social studies test?** *(allow students to guess either)* **Why do you think it is greater or less than?** *(accept reasonable explanations, does not necessarily need to be the definitive answer)*
>
> **We can write an equation to help us find her grade. I am going to write down 85 first and then an equal sign. I know that 85 is the same as 37 points less and twice the grade on the math test. Write 85 = .**

Now think about the algebraic expression, 37 points less than twice the grade on her math test. What is the unknown value? *(score on the math test)* I am going to use w to represent the value of the score of the math test.

Twice the grade on the math test—how do we write this operation? *(2m)* Twice tells me multiplication by 2. Write $2m$ after the equal sign. What are we missing in the numerical equation? *(37 points less)* How can we show 37 points less? *(subtraction)* Next to $2m$, write – 37. Our algebraic equation is $85 = 2m - 37$.

How can we solve this equation? *(accept reasonable answers)*

For now, let's ignore the multiplication by 2. Think about fact families. What if the equation was $m - 37 = 85$. How would we solve? *(85 + 37 = m)* What is 85 + 37? *(122)*

Let's think about this equation differently, write $2 \times m = 122$. Thinking about fact families, write the other equations using these 3 values? *($m \times 2 = 122$, $122 \div 2 = m$, $122 \div w = 2$)*

Which of those equations can help us solve? *($122 \div 2 = m$)* What is $122 \div 2$? *(61)*

Write $m = 61$. What does the 61 mean? *(that is the score on her math test)* Did she do well on her math test? *(no)*

Let's try the next problem. Frank received 366 votes for student council president, which were 12 more than 3 times the amount of votes received by Quinn. How many votes did Quinn receive?

First, let's set up our equation. What is the unknown, what are we trying to find? *(number of votes Quinn received)* Pick a letter to represent our variable or the unknown value. I will use x to represent the number of votes Quinn received.

We know that Frank earned 366 votes. This is our ending value. How can we write 12 more than 3 times the amount of

votes? *(3x + 12)* **Write the equation, $3x + 12 = 366$.**

Double check our equation. 3 times x or 3 times the number of votes for Quinn + 12 is equal to the 366 votes received by Frank. Is that right? *(yes)* **We used the information about the amount of votes for Frank to find a way to determine the amount of votes for Quinn.**

Now we can solve. First, let's ignore the multiplication by 3 and just write $x + 12 = 366$. This is just one part of the quantity. Using our fact family knowledge, how can we solve for x? *(366 – 12 = x)*

What is 366 – 12? *(354)* **So we know that we have 354 votes, so our new equation is $3x = 354$.**

We are closer to finding out how many votes Quinn received. Think about fact families, how can we solve $3x = 354$? *(354 ÷ 3 = x)* **How do you know?** *(answer should mention fact families and the 3 values and the fact that multiplication and division are inverse operations)* **What is 354 ÷ 3?** *(118)* **What does 118 mean or represent?** *(the number of votes received by Quinn)* **Write $x = 118$.**

We can check our work. We solved for x, the number of votes.

Now let's see if 3 times 118 + 12 is equal to 366. Work out the problem. What is 3 times 118? *(354)* **What is 354 + 12?** *(366)* **So, $3x + 12$ is the same value as 366. We know that our solution is correct.**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students complete the Practicing Together sheet in their Student Booklets. Review the answers as a class, sharing how students solved each problem.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students discuss or write their thoughts in the Notes section of their Student Booklets.

> **How can fact families help us to solve for unknown variables, like in $32 - x = 14$?**

Have a couple of student share their work with the other students. The see if any students write something different.