Expressions and Equations 2

Teacher Lessons

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Expressions and Equations 2
Lesson 1

Lesson 1: Distributive Property

Lesson Objectives	Students use the distributive property to simplify algebraic expressions. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Distributive property of multiplication over addition:** for all real numbers a, b, and c, $a(b + c) = ab + ac$. **Distributive property of multiplication over subtraction:** for all real numbers a, b, and c, $a(b - c) = ab - ac$. **Variable:** a letter that represents 1 or more values.	
Requisite Vocabulary	Order of operations, simplify, evaluate	
Misconception(s)	Students often multiply the first term in the parentheses but neglect to multiply the second term. If subtraction is involved within the parentheses, they often confuse the subtraction sign with a directional symbol indicating a negative number.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Chart paper and marker	• Student Booklet

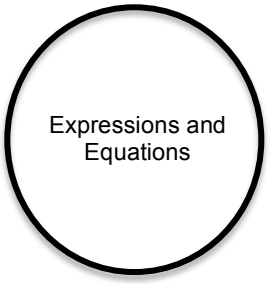
Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Preassessment **Form A** ✔	Lesson 7: Midassessment **Form B**	Lesson 15: Postassessment **Form C**

Warming Up

Have students turn to the Warming Up sheet in their Student Booklets. Students will first write their own ideas on this page and then add peer responses.

Draw a circle on the board with "Expressions and Equations" in the middle.



Today, we will start by thinking about what you know about expressions and equations. Write everything you know about expressions and equations in your Student Booklet.

Ask for student responses. Create a concept map on chart paper as students share ideas. Ask for students to clarify or provide examples to aid in the brainstorming of ideas.

What are some important ideas? What is an expression? What is an equation? Can you add an example to any of your words?

When students finish sharing their ideas, display the completed concept map for the duration of this module. The concept map will be used again at the end of this module. Displaying during the course of the module will allow both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

Teacher Notes

When using the distributive property to simplify an expression that involves subtraction, have students draw upon what they know about integer subtraction and rewrite the equation as an addition expression before they multiply. For example, the expression $3(y - 8)$ would be rewritten as $3(y + (-8))$. This method eliminates confusing the directional symbol (–) in the second expression with an operation symbol (–) in the first expression.

Using PEMDAS as the acronym for order of operations may cause confusion about grouping symbols. The P represents parentheses but, in the order of operations, other grouping symbols are used, such as brackets, braces, and the bar of a fraction.

1. Students will use the distributive property of multiplication over addition and the distributive property of multiplication over subtraction with real numbers and variables.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   You will use order of operations to simplify, evaluate, or solve each expression.

What is the order of operations? *(grouping symbols, exponents, multiplication and division, addition, and subtraction)* **After we do any operation in parentheses or other grouping symbols, what is next?** ***(exponents)*** **What operations are next?** *(multiplication and division)* **Would you do multiplication or division first?** *(whichever occurs first from left to right)* **What are the last operations to solve or evaluate?** *(addition and subtraction)*

Have students complete the problems (A and B pairs of problems within each item) individually, in pairs, or as a class.

What do you notice about each pair of problems and your results when you simplified? For example, look at both solutions for 1, 2, and 3. How could you describe the answers? *(they are equivalent)* **Why do you think 1A and 1B were the same, or equivalent, answers?** *(answers may vary. These problems represent the distributive property of multiplication over addition or subtraction.)*

Problems 4 and 5 resulted in 2 different answers for A and B. The numbers are the same in the problems. Why do you think the results were different? *(answers may vary. The distributive property does not apply in these problems.)*

Focus students on the structure. They should see that problems 1–3 involve multiplication and addition.

When we simplify expressions that have the form like problems 2 and 3, we are using a special property. It is called the distributive property of multiplication over addition. In 2A, what is being added? *(1 and 5)* **What is being multiplied?** *(6 and the sum of 1 and 5)*

How are problems 2 and 3 different from problem 1? *(answers may vary—for example, for problem 1, the operation is subtraction in the parentheses rather than addition. Students may also notice that problem 3 involves multiplication by a negative integer.)* **In problem 1, this property is called the**

distributive property of multiplication over subtraction. In 1A, what is being subtracted? *(6 – 4)* **What is being multiplied?** *(2 and the difference of 6 and 4)*

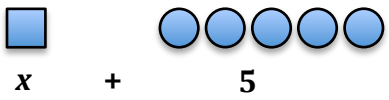
Why did you not get the same results on 4A and 4B? *(answers may vary—for example, 4A and 4B involve multiplication with no addition)* **Why did you not get the same results on 5A and 5B?** *(answers may vary—for example, 5A and 5B involve division)*

2. Students will use the distributive property to simplify algebraic expressions.

For the first 3 problems, we worked only with numbers and applied the distributive property. Now, we will apply the distributive property to expressions that have a variable. A variable is a letter or a shape that represents a value. Follow along on the second page of your Learning to Solve sheet.

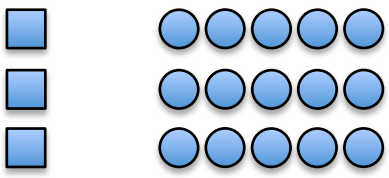
We will simplify 3(x + 5) by using both pictorial and symbolic representations, meaning pictures and numbers.

We can represent x + 5 by using a square for x and 5 circles for 5.



In 3(x + 5), what does the 3 outside the parentheses indicate? *(3 groups of x + 5).* Write 3 groups of x + 5.

We can show 3 groups of x + 5 like this.

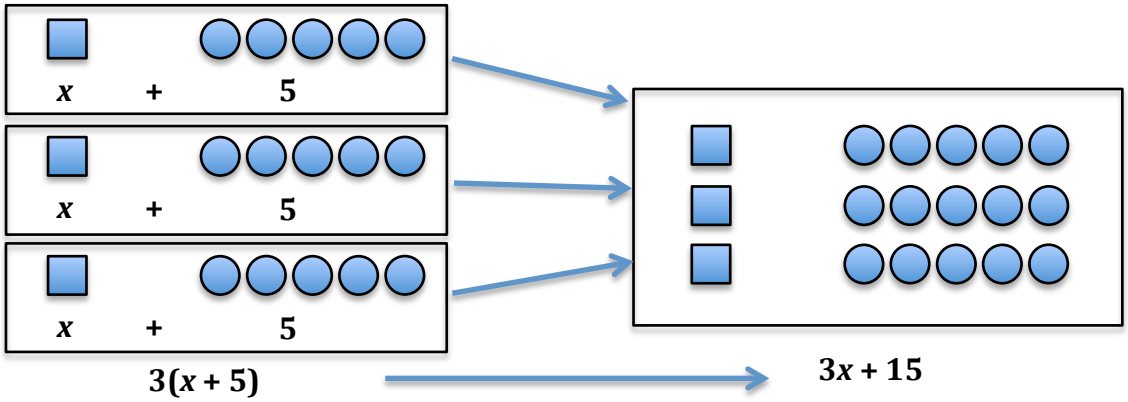


How many *x*'s do we have? *(3)* We can show that by writing 3x. *Write 3x.*

How many circles do we have? *(15)* Write 15.

How would we write the sum? *(3x + 15)* Write 3*x* + 15.

Show students the model and the symbolic representation. It is important to connect the representations.



Look at the symbolic representation and talk to your neighbor about how you can apply the distributive property without drawing a picture for 3(*x* + 5) = 3*x* + 15.

Allow students to discuss with a peer.

What is the procedure or the steps to solve? What do we need to multiply?

Make sure that students discuss the procedural aspects of multiplying both terms in the parentheses. Students should be moving toward a generalization that $a(b + c) = ab + ac$.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. Students may work with a partner.

Simplify each algebraic expression.

After students have solved the problems, have student pairs show on the board the method they used. When students have written their method on the board, have them explain what they did.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets. Read the problem and provide time for students to work.

Ask a student to write his or her answer on the board. Ask whether anyone has a different answer.

Expressions and Equations 2
Lesson 2

Lesson 2: Solving Multi-step Problems

Lesson Objectives	Students use the distributive property to solve multi-step problems. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP 4) Students use appropriate tools strategically. (SMP 5)	
Vocabulary	**None**	
Requisite Vocabulary	Order of operations, distributive property of multiplication over addition (subtraction)	
Misconceptions	In problems that involve situations, students often misapply the order of operations or try to use 1 operation that involves all of the numbers in the problem by looking for key words.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the problems.

Pause for students to complete.

For problem 1, how did you solve it? Who solved it differently? In Problem 2 there is a lot of information! How did you decide what was important? (*for the first question***,** *there are different ways to solve the first question, and the second question may or may not have anyone answer (we can't predict).*

Learning to Solve

Teacher Notes

In this lesson, students will solve problems that require multiple steps. For example, some problems require students to use multiplication and addition. Students will have to use order of operations to determine which operation to perform first.

As students create the expressions to solve the problems, watch for students who look for "key" words. Discourage the key word strategy.

1. Students will solve multi-step problems.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in the Student Booklet. Select a student to read.

Follow along as [student] reads the problem.

Pause while the student reads.

Solve the problem on your own in your Student Booklet. Then, decide whether there is another way to solve it.

Have students show their solution on the board. The problem can be solved by using the distributive property. The 2 solution methods you may find are:

1. $8(1.25) + 8(0.75) = 10 + 6 = 16$

2. $8(1.25 + 0.75) = 8(2) = 16$

If students do not show the second method, share it as a way a student solved it in another class.

Compare the 2 methods we could have used to solve. What do you notice about the 2 methods? How are they alike? Is 1 method more efficient than the other method? Why? *(the second method uses the distributive property and allows you to multiply a much easier number after you do the addition)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work in pairs, in small groups, or as an entire class. If students struggle with the computations, allow the use of a calculator.

Check for understanding by having pairs share their work.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets and solve the problem.

Ask a student for his or her expression. Ask whether any students have a different expression.

Expressions and Equations 2
Lesson 3

Lesson 3: Rewriting Expressions in Different Forms to Determine Equivalency

Lesson Objectives	Students rewrite expressions in different forms to determine equivalency. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7)	
Vocabulary	**Equivalent expressions:** 2 algebraic expressions are equivalent when the value of 1 expression equals the value of the other expression for *all* replacements of the variables.	
Requisite Vocabulary	Expression, equivalent, substitute	
Misconceptions	Students sometimes believe that substituting 1 value into an expression is sufficient to determine equivalency.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Evaluate each expression, substituting 2 for the variable *u*.

As you monitor their work, watch for students who may be confused about terms such as $4u$. You may need to model how to evaluate these terms by describing that $4u$ means to multiply by 4 and by working together to evaluate the term.

How did you evaluate the expression? Did anyone solve it differently? How did you use order of operations?

Learning to Solve

TEACHER NOTES

Students sometimes believe that substituting 1 value into an expression is sufficient to determine equivalency. This lesson develops the notion that for expressions to be equivalent, every value substituted into them should result in the same value.

Students will rewrite expressions in different forms to determine equivalency.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

We often write expressions in different forms to fit our needs. Sometimes, we want the simplest or most efficient form and at other times we want a different form.

No matter the form we use, it is important to make sure that we are using equivalent forms. What does "equivalent" mean? *(accept reasonable answers, such as 2 things represent the same quantity)*

A way to determine the equivalency of 2 expressions is to substitute values for the variable. If the results of evaluating the expressions are the same, the expressions could be equivalent. Let's try this method to determine whether $6n$ and $4n + 2n$ are equivalent.

Students may select a different value for *n*; simply substitute the other value in the script. The example below uses the value 2. Model writing the expression with the value.

We will substitute 2 for n in each expression. What operation is $6n$? *(multiplication)* **What is 6 times 2?** ***(12)***

Now we can substitute 2 in the expression $4n + 2n$ or
$4(2) + 2(2) = 8 + 4 = 12.$

Both expressions are equal to 12 when we substitute 2 for n. Is substituting 1 value and getting the same result enough to prove that these 2 expressions are equivalent?

Give time for students to discuss. They may think that using 1 value to show equivalence is enough.

Let's try another value. This time substitute –1 for n.

Allow students time to complete.

What are the products? *(–6)*

Have each student select another value and substitute it for *n*. Have students share the values they selected and the result.

Why do you think both expressions gave the same result, no matter what value was substituted for the variable?

Have students talk with their partner and share their ideas. Students may notice that $4n + 2n$ can be simplified to $6n$. The expressions are equivalent. All values substituted for the variable result in the same value for both expressions.

2 algebraic expressions are equivalent when the value of 1 equals the value of the other for *all* replacements of the variables.

Look at the next set of expressions. Let's use that process to determine whether these expressions are equivalent. The value of each of these expressions is 4 when $x = -2$.

You may want to have students verify that each expression has a value of 4 when $x = -2$.

In your Student Booklet, evaluate the expressions for $x = 2$ and $x = \frac{1}{2}$.

Allow students time to evaluate the expressions. Move about the students and check for understanding.

Which expressions have the same value when you substitute 2 and $\frac{1}{2}$? *(2x + 8 and 2(x + 4))*

These expressions are equivalent. Can you show why with the distributive property? *2(x + 4) simplifies to 2x + 8)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Look at the expressions in the table on the Practicing Together sheet. Decide whether they are equivalent expressions. Be prepared to share your reasoning for your decision.

Check for understanding by having students share their work.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 3A to provide extended practice before proceeding to Lesson 4.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Decide how you would respond to the question. Be able to support your reasoning.

Have students explain, as time permits. You can also use this question as an exit ticket.

Expressions and Equations 2
Lesson 4

Lesson 4: Evaluating Multi-step Expressions

Lesson Objectives	Students evaluate multi-step expressions. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent expressions, distributive property of multiplication over addition (subtraction)	
Misconception(s)	Students may think that in the acronym PEMDAS for order of operations, they must do all multiplication before they do any division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Evaluate each expression by substituting 4 as the value for the variable.

Ask students to give their answer for the problems and explain how they evaluated.

Learning to Solve

Teacher Notes

The order of operations is: parentheses or other groupings, exponents, multiplication or division (working from left to right), addition or subtraction (working from left to right). However, keep in mind that this is not a strict order.

A student may say that you perform all operations in order from left to right; another student might neglect to include parentheses and exponents; and another student might indicate, for example, that you perform multiplication before division, even if the division operation is written before the multiplication operation.

Look for students who show equivalency as $3 + 5 = 8 + 2 = 10$. The expressions separated by the equal sign are not equal. Encourage students to write the expressions so that the first expression remains equivalent to all other expressions they write.

1. Students will solve multistep algebraic expressions.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   On your own, simplify or evaluate the expression.

$6 + 8 \div 2 \times 3 - 5$. *(13)*

Ask a student to write his or her solution on the board and explain how he or she found it.

Does anyone disagree with the student's solution? How did you solve or simplify?

Incorrect solutions:

1. Some students may have the answer of 16 if they perform the calculations from left to right.

2. Other students may have calculated 25 if they performed the division and then added 6 before multiplying.

3. Another incorrect solution, 14, is found by multiplying first and then subtracting 5 before dividing.

If no one indicates that they have evaluated the expression in 1 of the above incorrect ways, show these errors by using the following script.

Look at your Student Booklet. In another class, 3 students solved the problem in a different way.

One student solved it this way.

Read 1A.

Another student solved it this way.

Read 1B.

A third student solved it this way.

Read 1C.

Talk with your partner about what each student did. Then decide whether each solution is correct. If it is correct, justify or explain the correct process. If it is incorrect, decide what error the student might have made.

Discuss the problems. Ask a student pair to describe their decision. Make any incorrect use of order of operations explicit by comparing the process of each student to the order of operations.

> **When we work with algebraic expressions, it is important to remember the order of operations. In your Student Booklet, write how you remember the order of operations.**

Have a student read his or her description. Ask whether anyone has a different way to remember the order of operations.

> **In this example you have multiple ways that you can solve this without violating order of operations:**
>
> **$3 \times 2 + 8 \div 4 + 3 + 9$ You can add 3 and 9 first without changing the value of the expression.**
>
> **You can do the multiplication first, then the addition, then the division, then evaluate and not change the value of the expression.**

2. Students will evaluate multistep expressions for a range of values.

Call attention to the expression in problem 2.

> **What operations are in this expression?** *(division, multiplication, subtraction, and addition)*

Have students identify where each of the operations is in the expression. Some students may have to be reminded that an expression written in fraction form is another way of representing division.

> **According to the order of operations, which operation could we perform first when we evaluate the expression for a variable?** *(multiplication)*

You may want to restrict the values that are possible, such as a value for y that is between 0 and 10.

> **Now we'll evaluate the expression for the value of the variable.**

Give me a value for the variable *y*. *(pause)* Write it on the blank.

Using the value that the student gives, have students work with you to evaluate the expression. Feel free to change the order of the steps, as there is more than 1 way to evaluate the expression. You may want to ask students to give you the next step. Or you can have them do a step and then report the step they did.

For example, if students give 2 as the value of *y*, you can think of the steps in this way:

$$\frac{6y - 3}{3} + 2y$$

$$\frac{6(2) - 3}{3} + 2(2)$$

$$\frac{12 - 3}{3} + 4$$

$$\frac{9}{3} + 4$$

$$3 + 4$$

$$7$$

As time permits, have students give you other values for the variable.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. Pair the students.

Work with your partner to complete the table by evaluating each expression for the indicated value.

Check for understanding by having pairs share their work.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Have students answer the questions.

Expressions and Equations 2
Lesson 5

Lesson 5: Evaluating Multi-step Expressions

Lesson Objectives	Students evaluate multi-step expressions. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent expressions, distributive property of multiplication over addition (subtraction)	
Misconception(s)	Students may think that in the acronym PEMDAS for order of operations, they must do all multiplication before they do any division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Evaluate each expression for the given value

Ask students to give their answer for a problem and explain how they found it.

Learning to Solve

TEACHER NOTES

The correct order of operations is: parentheses or other groupings, exponents, multiplication or division (working from left to right), addition or subtraction (working from left to right). However, keep in mind that this is not a strict order.

A student may say that you perform all operations in order from left to right; another student might neglect to include parentheses and exponents; and another student might indicate, for example, that you perform multiplication before division, even if the division operation is written before the multiplication operation.

Look for students who show equivalency as 3 + 5 = 8 + 2 = 10. The expressions separated by the equal sign are not equal. Encourage students to write the expressions so that all expressions are equivalent.

1. Students will evaluate multi-step algebraic expressions.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Each expression in the table is equal to 14. Find the value of the variable that will make this statement true. We will do the first expression together.

To determine the value of the variable that will make
$7m - 14 = 14$, a true equation, we need to figure out what number multiplied by 7 and then subtracted by 14 equals 14?

I know that $28 - 14 = 14$, so $7m$ must equal 28. 7 times 4 equals 28, so m must equal 4.

Using this same process, find the value of the variable in the other expressions.

Have students work in pairs to determine the value of the variable that will make the remaining 3 expressions equal to 14.

Have student pairs give their answer to a problem and ask them to explain the procedure they used to find the value of the variable.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Work with your partner to complete the table by finding the value of the variable that will make the expression equal to 10.

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 5A to provide extended practice before proceeding to Lesson 6.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Write an expression that equals 10 when the variable is equal to 5.

Have students write their expression on the board.

Equations and Expressions 2
Lesson 6

Lesson 6: Solving Equations by Guess-and-Test

Lesson Objectives	Students solve equations using the guess-and-test method. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Solution of the equation:** A value that when substituted for the variable makes the equation true.	
Requisite Vocabulary	Expression, equation	
Misconception(s)	Students often confuse expressions and equations, not realizing that equations show that two expressions represent the same amount.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

For the 5 problems shown, find the value for the variable that makes the expression equal to the value given.

Ask students to give their answer for one of the problems and explain how they found it.

Learning to Solve

Teacher Notes

Students use guess-and-test to find values that make the equation true. Using this strategy focuses on students' intuitive understanding of equations and allows them to logically find a value that solves the equation.

1. Students will solve multi-step equations using the guess-and-test method.

   On the board, write the equation $2x + 4 = 12$. Give each student a whiteboard or have each one complete in the Notes section of the Student Booklet.

   $2x + 4 = 12$ *(x = 4)*

   Ask students to talk to their neighbor to determine the value of x that will make this equation true. They should write their solution on their whiteboard and be able to explain how they found it.

   Have student pairs explain their answers.

   Some of you chose a number, substituted it for the variable, and calculated the value of each side of the equation. If the value on both sides is the same,

then the value substituted for the variable is a solution of the equation.

We call this the guess-and-test method because you guess a solution and test it to see whether it is correct.

Display the Learning to Solve problems in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Use this method to solve $5m - 3 = 22$. First, I am going to guess 4 for m. Then I test. What is 5(4)? *(20)* **What is 20 – 3?** *(17)* **Write it in your booklet. 17 is not equal to 20, therefore, 4 is not a solution.**

Then I think, well, 17 is less than 22 so I need a larger number or value for m. I do not want to increase too much, so I will try 5. What is 5(5)? *(25)* **What is 25 – 3?** *(22)* **So, what is the correct value for m?** *(5)* **Write it in your booklet.**

You can use the same idea to find the missing term in an equation when we are given the value of the variable by using the guess-and-test method.

We will do the next one together. $3n$ – some number equals 16, for n equals 9. First, substitute 9 for n in the term $3n$. What does it equal? *(27)* **Now guess, what number subtracted from 27 is equal to 16?** Write your guess in your booklet. **Then test to see if you got the right answer** *(11).*

We are going to use this method to solve problem 2. First, substitute 1 for c. Now think, what number multiplied by 1 and then added to 8 will equal 17? *(9)* **How did you solve?**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

> **Working with your partner, find a value for the variable that will make the statement true in each problem.**

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Orren said that the solution to $8n - 21 = -5$ is $n = 2$. Is he correct?** *(yes)*

Have students write their answer on their whiteboard and show how they determined if $n = 2$ is the solution to $8n - 21 = -5$.

Lesson 7: Solving Equations Using a Table

Lesson Objectives	Students solve equations using a table. Students use appropriate tools strategically. (SMP 5) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Variable, equation, expression	
Misconception(s)	Students often confuse expressions and equations, not realizing that equations show that two expressions represent the same amount.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Use >, <, or = to make a true statement.

Have students give their answer and explain how they arrived at the answer.

Learning to Solve

TEACHER NOTES

The next strategy in learning to solve equations is to make a table. Students combine the guess-and-test strategy with a table to organize their data. As they create the table, they will notice that when they find the solution to the equation, the values for the variable less than or greater than the solution can create an inequality.

1. Students will solve equations using a table.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Another method for solving equations is to make a table.

   Let's use the equation $5b + 11 = 1$.

   We used guess-and-test as a way of solving equations in our previous lesson. To start the table, guess a number that you think is reasonable.

   Have a student give a guess. Record it in the table.

Now let's check. Thinking about order of operations, what do we compute first? (5 times the number) Then we add 11. What is the solution?

How close is the answer to 1? Do you think the guess was too high or too low? How do you know? Thinking about our solution, we can now adjust our guess to be either higher or lower.

Have students guess another value and then check.

Does the left side of the equation equal 1? You have two options if the guess is not correct.

Option 1: think how the guess can be adjusted either higher or lower.

Option 2: begin checking values in increments of 1 or 2. For example, if your guess was 1, the value of the left side of the equation would be 16. The left side is too high so the value must be less than 1.

Continue in this way until students find that the solution is $b = -2$.

Make sure students understand that the answer should be written in the form "$b = -2$." A possible completed table is shown below.

A table is useful because it helps us organize our information. As you work through the problems today, think about how a table can be useful and think about when a table would not be useful.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Working with your partner,

1. **Fill in the column headings**
2. **Try different values to find the value of the variable that makes the equation true.**

Once students have completed the table, have student pairs come to the board and fill in the tables with their results. Students will have different values for the variable. Ask them to explain how they decided on what value to use.

They will use the guess-and-test method to determine the values of the variable. The table organizes this process.

Trying It On Your Own

Display the Trying It On Your Own problem in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **So far, you have learned two methods for solving equations – the guess-and-test method and the table method. Compare these methods. Write 1 thing that is similar about these methods and 1 thing that is different.**

Have students read the similarity and difference that they wrote. Discuss as time allows.

AFTER this lesson: Administer Progress Monitoring
FORM B

Expressions and Equations 2
Lesson 8

Lesson 8: Solving Equations Using a Graph

Lesson Objectives	Students solve equations using a graph. Students make sense of problems and persevere in solving them. (SMP 1) Students model mathematics. (SMP 4) Students use appropriate tools strategically. (SMP 5) Students attend to precision. (SMP 6)	
Vocabulary	**Dependent variable:** a quantity whose value depends on another quantity **Independent variable:** a quantity that affects another variable (the dependent variable)	
Requisite Vocabulary	Equation, *x*-coordinate, *y*-coordinate	
Misconception(s)	Using the graphing method may expose some misconceptions that students have about graphing. For example, students may think that the ordered pair (3, 2), for example, gives one point on the *x*-axis at 3 and the other on the *y*-axis at 2.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Graphing calculators (if available)

Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Form A** ✔	Lesson 7: Mid-assessment **Form B** ✔	Lesson 15: Post-assessment **Form C**

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Look at the graph. You will need to graph the coordinate points. Point to the *x*-axis on the graph. Point to the *y*-axis. Look at the first coordinate. The first value is 7. This tells the distance from 0 on the *x*-axis. In what direction do we move from 0? *(right)* **Why?** *(the value is positive)* **What if the value is negative?** *(move to the left)*

What is the next coordinate? *(4)* **This tells the distance from 0 on the *y*-axis. In what direction do we move from 0?** ***(up)*** **Why?** *(the value is positive)* **From the *x* value, go up 4. This is the intersection of the coordinates. Place your point there to show where 7 and 4 intersect and label it with the letter A.**

Graph the remaining coordinate points and label them with the letter.

Have students come to the board and place one of the points on the grid.

Learning to Solve

TEACHER NOTES

Talk with students about the dependent and independent variables. The independent variable is typically represented by x and the dependent variable is typically represented by y.

1. Students will solve equations using a graph.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   We have used the guess-and-test method and have made a table to find the value of the variable that will make an equation true. Another method for solving equations is to draw a graph.

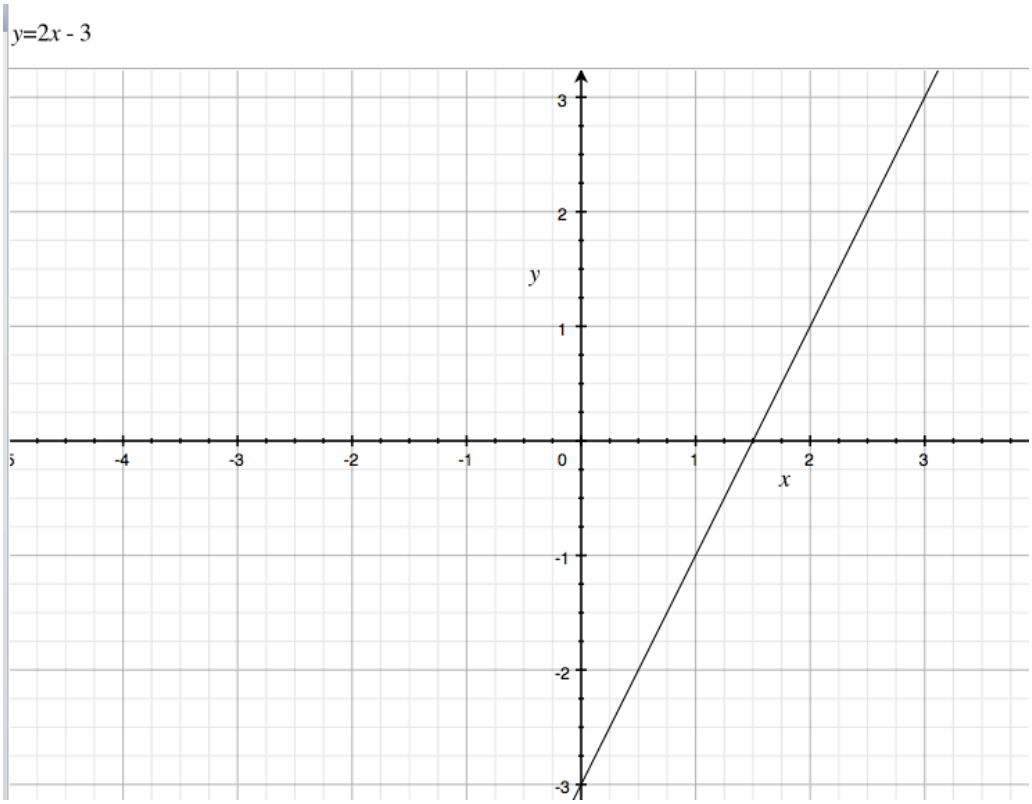
   Write the following problem on the board: $2x - 3 = 5$

   Problem 1: $2x - 3 = 5$

   Graphing Calculator Method (Preferable):

   If graphing calculators are available, use this method. To use the graphing calculators, students will graph the right side of the equation and the left side of the equation as two lines. They will look for the intersection of the two lines to find the solution.

   Graph of $2x - 3$



The intersection of the two lines occurs at (4, 5). This means that when $x = 4$, the left side and the right side of the equation represent the same amount. The solution to the equation is $x = 4$.

The x-coordinate represents the value of the variable substituted into the expression on the left side of the equation. The y-coordinate represents the value of the right side of the equation.

Try this method with the following equations as time allows:

$$2x + 3 = 7$$

$$2x - 5 = 3x + 14$$

Table and Graph Method:

If graphing calculators are not available, use this method. Demonstrate to students how to complete the table by entering one value for x at a time and evaluating the expression $2x - 3$ with the value of x.

Tell students that any values can be substituted for x.

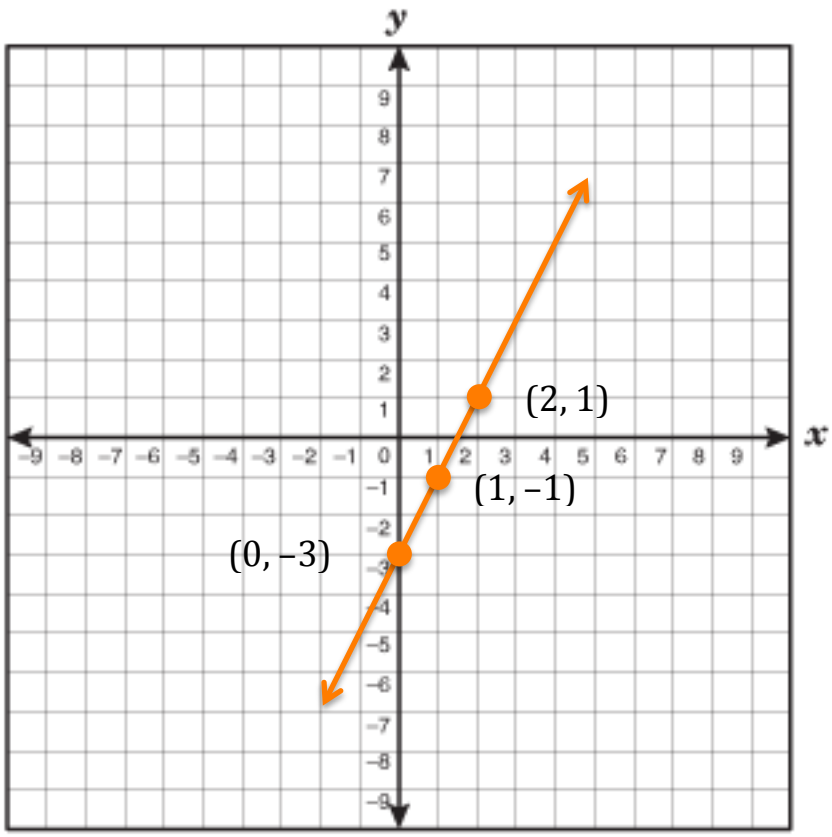
x	$2x - 3$
0	$2(0) - 3 = 0 - 3 = -3$
1	$2(1) - 3 = 2 - 3 = -1$
2	$2(2) - 3 = 4 - 3 = 1$

The value in the first column is the x-coordinate of a point. The value in the second column is the y-coordinate of the point.

You can continue to add as many ordered pairs as you want, by continuing to find values for x and substituting into the expression.

At this point, with 3 ordered pairs, students can draw a line. They will look for the ordered that has 5 has the y-coordinate.

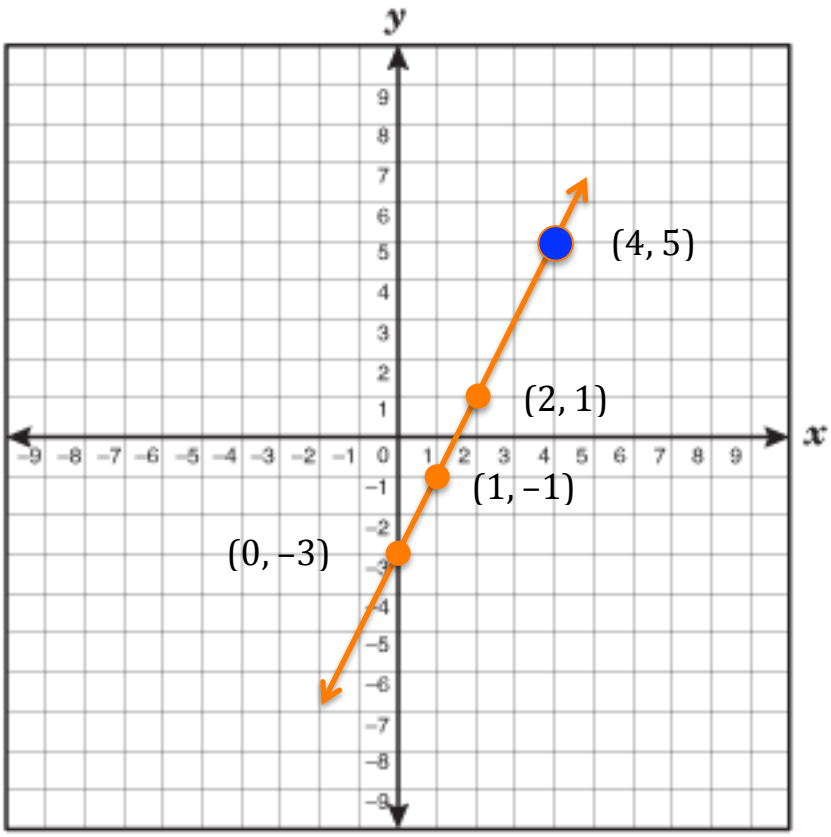
Graph the points on the coordinate grid in your Student Booklet. Connect them by drawing a line through the points.



How can we use this line that is the graph of the expression $2x - 3$ to determine the value of x that will make the equation $2x - 3 = 5$ true?

Have students look at their graph and see if they can determine what the value of x will be to make the expression $2x - 3$ equal to 5.

Students should conclude that for the expression $2x - 3$ to be equal to 5, the value of x must be 4. This can be found by looking at the line and determining what the x-coordinate will be when the y-coordinate is 5.



Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

If using graphing calculators:

> **Working with your partner, solve the problems using your graphing calculator.**

If not using graphing calculators:

> **Working with your partner, solve the problem using the graphing method.**

Have a student pair come to the board and complete numbers 1 – 4. Ask another student pair to answer number 5. Discuss the students' work.

Discuss the graphing techniques used as appropriate.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **By graphing a line for an equation, Sterling found that the coordinate point (3, 2) made the equation true. What might the equation be?** *(Answers will vary. For example,*
> *$2x - 4 = 2$, $3m - 7 = 2$, $x + 7 = 10$*

Have students write their equivalent equations on the board.

Expressions and Equations 2
Lesson 9

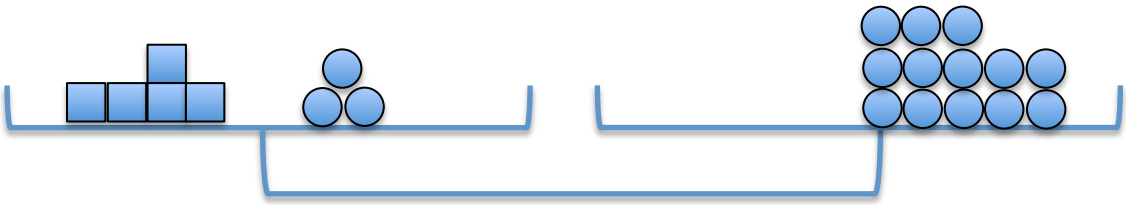
Lesson 9: Solving Equations Using a Diagram

Lesson Objectives	Students solve equations using a diagram. Students make sense of problems and persevere in solving them. (SMP 1) Students model with mathematics. (SMP 4) Students attend to precision. (SMP 6)	
Vocabulary	**Constant:** A number that contains no variables and has a fixed value that does not change, such as 5, –1, or 3.14	
Requisite Vocabulary	Variable	
Misconception(s)	Students may not recognize that in order to maintain equality, the expressions on either side of the equal sign must represent the same quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Chips or cubes	• Student Booklet • Two-color chips, cubes, or algebra tiles

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Using the diagram on a scale below, determine how many circles are equal to a box. Remove boxes and circles until you have only 1 box on 1 side of the scale. As you solve this problem, write the steps that you took. For example, for step 1, you could write, "I removed 3 circles from each side of the scale."



Pause for students to complete.

What is the answer? ***(1 box = 2 circles)*** **How did you solve? Did anyone solve it differently?**

Learning to Solve

Teacher Notes

The diagrams can be represented with algebra tiles if they are available. The same procedure would be followed without the tiles. However, if algebra tiles are used, you will want to focus students' attention on the area aspects of the tiles.

For equations that involve subtraction, use the definition of subtraction to rewrite the equation. For example, $3c - 8 = 16$ would become $3c + (-8) = 16$.

The symbols being used in the Learning to Solve section are helpful for students who are struggling. They allow the work to be organized easier than showing the 'steps' under the equation.

1. Students will solve equations using a diagram.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. If you have chips and cubes available, you may want to model this.

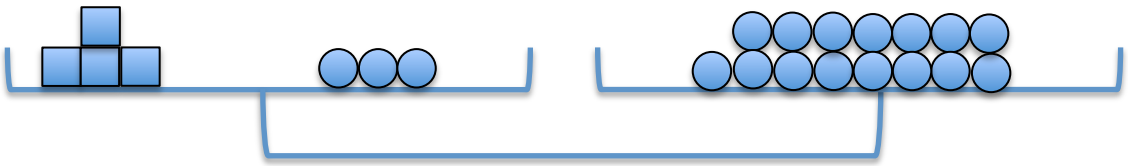
In the previous lessons, we have solved equations using guess-and-test, making a table, and graphing. Another method for solving equations is to draw a diagram.

Look at a diagram that helps us solve $4y + 3 = 15$.

This equation has both a variable and constants. What is a variable? *(a letter or symbol that represents a value)* **A constant is a number that contains no variables and has a fixed value that does not change. What are the constants in this equation?** *(3 and 15)*

In my diagram, a box represents the variable y and circles represent the constants, the values or numbers that do not change, 3 and 15.

$4y + 3 \quad = \quad 15$



Using my diagram, I am going to remove 3 circles from both sides of the scale by crossing them out. Cross out 3 circles on both sides. Remember, in order to maintain the equality, I have to keep the quantities the same.

How many squares are left on the left side? *(4)* **How many circles are left on the right side?** *(12)*

$4y = 12$

Removing 3 circles in the diagram from each side of the scale is the same thing as subtracting 3 from each side of the equation. This leaves 4 boxes, or $4y$ on the left side and 12 circles, or 12 on the right side of the balance. Write S_3 to the right of your equation. When we write S_3, this tells us that we subtracted 3 from both sides of the equation.

$4y = 12 \quad S_3$

How could we find out how many circles are equal to $1y$? *(accept reasonable answers such as the circles can be grouped into 4 groups, with each group matching 1 box or y)*

Model the drawing of the diagram has the script progresses.

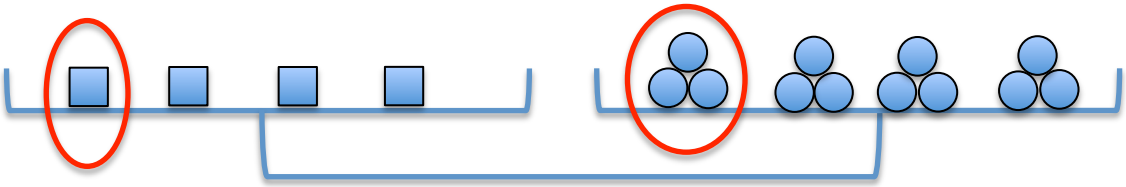
Because there are $4y$, let's place the blocks and circles on each side of the balance into 4 groups. Draw the 4 squares on the left side, 1 in each group. How many groups? *(4)*

We need 4 groups for the circles. How many circles will be in each group? *(3)* **Draw 3 circles in 4 groups.**

$4y \quad = \quad 12 \quad D_4$

The symbol D_4 means that we are dividing by 4. Write D_4 after the equation.

Now we can find the value of 1*y* by looking at how many circles are in each group. How many circles in each group? *(3)* **What is the value of *y*?** *(3)* **Write *y* = 3 under the diagram.**



$y \quad = \quad 3$

Look at the table on the next page. These symbols are used to show the operation or action we did to find the value of the variable that makes the equation true.

Show the symbols. Discuss as needed. The symbols should be intuitive for students.

Symbol	Meaning
A_3	Add 3
S_3	Subtract 3
M_3	Multiply by 3
D_3	Divide by 3
DPMA	Distributive Property of Multiplication over Addition

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Working with a partner:

1. **Draw squares to represent the variable and circles to represent the constant number in each expression of the equation.**
2. **Show each of your steps on the scale.**
3. **Write the symbolic representation of each step below the scales.**
4. **Solve for the variable.**

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Perform the operation indicated. Express your answer in simplified form.**

Review the answers and have students explain how they found the answer.

Expressions and Equations 2
Lesson 10

Lesson 10: Solving Equations by Working Backward

Lesson Objectives	Students solve equations using the working backward method. Students reason abstractly and quantitatively. (SMP 2) Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Constant, variable	
Misconception(s)	Students may want to add or remove the blocks or circles from one side only, rather than making the same move on both sides.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **In the pyramid, the number in a box is the sum of the two numbers in white boxes above it. Find the value of x that makes the bottom number correct.**

As you monitor student work, if they are having difficulty, you may want to help them do the first computations in the top row. Remind them that they may have to work forward and also work backward.

Have a student come to the board and fill in the pyramid. Ask another student to explain the process used to begin to solve this problem. Ask if anyone came up with a different solution.

Learning to Solve

1. Students will solve equations by working backward.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   > **Another method for solving equations is to work backward.**
   >
   > **Two less than the product of 9 times a number is equal to 16. What is the number?**
   >
   > **First, we are going to write an equation to help solve the problem. We can use any letter or variable to represent the unknown number or the number we are trying to find.**
   >
   > **One equation that can be written to show the word problem is**
   > $9m - 2 = 16$.

We are going to use a different method to solve for *m*. We are going to work backward. I am going to write the steps I used in the rectangles.

Model writing the steps in the rectangles.

First, we will start with 16 and undo the operations. What does it mean to undo the operations? *(accept reasonable answers. For example, students may indicate that if 2 was subtracted to get 16, then 2 should be added to 16.)*

Let's think, 9 times a number minus 2 equals 16. What number minus 2 equals 16? *(18)* **That means I have to start with 18. What number times 9 equals 18?** *(2)* **What is the value of *m*?** *(2)*

First, since subtracting 2 from 9*m* equals 16, 9*m* must be equal to 18. Write in the first rectangle, "Because subtracting 2 from 9*m* equals 16, 9*m* must equal 18."

Next, since the product of 9 and a number or *m* is equal to 18, *m* must equal 2. Write the second rectangle, "Since the product of 9 and a number *m* is equal to 18, *m* must equal 2."

Does solving by working backward remind you of any other method you've used? *(accept reasonable answers such as this method is similar to some of the thinking done in guess-and-test.)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Work with your partner. For each problem, complete following 3 steps:

a. Write an equation to help solve the problem (if needed).

b. In the boxes, write the working backward steps you use to solve the problem.

c. Write the solution to the equation.

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

I am thinking of a number. If I multiply the sum of my number and 6 by 3, and then add 9, the result is 33. What is my number? *(2)*

Have students come to the board and show the process they used to find their solution.

Expressions and Equations 2
Lesson 11

Lesson 11: Solving Equations

Lesson Objectives	Students solve equations. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Solution (to an equation)	
Misconception(s)	Students are often confused by the use of the term "solution to an equation." They mistakenly think that the solution is the constant that comes to the right of the equal sign.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve each equation using any method you prefer.

Pause for students to complete, and then have them discuss their answers.

As students are explaining their process, watch for those that describe working backward or any other strategy. When those strategies are described, emphasize them with the class by saying, "Jon used the working backward strategy."

What is the solution of each equation? How did you solve? Did anyone solve any of them differently?

Learning to Solve

Teacher Notes

Look for students who show equivalency as 3 + 5 = 8 + 2 = 10. The expressions separated by the equal sign are not equal. Encourage students to write the expressions so that the first expression remains equivalent to the last expression they write.

1. Students will solve equations.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Students may work with a partner, small groups, or can be completed with the entire class if more applicable.

   What 5 methods have we learned and practiced to use to solve equations? *(guess-and-test, create a table, draw a graph, draw a diagram, and work backward)*

For these problems, use the method given to find the value of the variable that makes the equation true.

Have student pairs give their solutions to the problems. Ask if any students found a different answer.

Practicing Together

There is no Practicing Together in this lesson as the Learning to Solve section was practicing the different methods of solving equations.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Write an equation whose solution is 3.

Expressions and Equations 2
Lesson 12

Lesson 12: Solving Inequalities

Lesson Objectives	Students solve inequalities and graph the solution on a number line. Students make sense of problems and persevere in solving them. (SMP 1) Students model with mathematics. (SMP 4) Students attend to precision. (SMP 6)	
Vocabulary	**Inequality**: a relationship between two quantities (or expressions) that are not equal. The relationship is shown by using $<$, $>$, $\neq$, $\leq$, $\geq$.	
Requisite Vocabulary	Profit	
Misconception(s)	As students are writing equations or inequalities to represent a word problem, they often write n = rope. When students do this, they are indicating the variable is an *object*, and not a *quantity*.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Write an algebraic expression that can be simplified to $48 - 8x$. For example, if I wrote $8(6 - x)$, how would I simplify? I would need to multiply 6 and x by 8. What is 8 times 6?** *(48)* **What is 8 times x?** *(8x)* **This expression would be simplified to $48 - 8x$.**
>
> **What is another expression that could be simplified to $48 - 8x$?**

This is a reversibility question that has many possible responses. All students should have a response to this question. Encourage them as they work (and time remains while others are finishing) to find more than one expression.

Ask students to give their expression. Have other students check the expressions that they can be simplified to $48 - 8x$.

Learning to Solve

Teacher Notes

Solving inequalities is often taught after solving equations. However, there is a relationship between the values that makes the equation true or not. By using equations as a springboard for inequalities, students have the opportunity to connect the two topics and enhance their understanding of number relationships.

When working with inequalities, avoid using the "alligator mouth" as a way of reading the inequality symbol. Focus on the language associated with each symbol.

1. Students will solve inequalities and graph the solution on a number line.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Follow along as [student] reads the first problem.

   If students are not familiar with the profit concept, you may want to clarify what profit means.

   What is the problem asking us to find? *(the number of feet of rope that a customer must buy for Sam to make a profit)*

   What expression would we write to represent the cost of rope at Sam's Fish Shop? *(2n + 5, be sure that students identify what the variable represents. They should write n = number of feet of rope.)*

   We are going to write the relationship between the cost and the amount of profit. We have been using an equal sign, but sometimes, an inequality is needed to show that two quantities or expressions are not equal.

   If we wrote an equal sign after 2n + 5, this would show that the relationship between the cost and the profit is the same or equal. An equal sign would mean that it costs $17 to make the rope and the profit is $17. If the cost and the profit are the same, would Sam be making any money? *(no)* **If you were making a product, do you want the cost to be greater than or less than your profit?** *(yes)* **Why?** *(accept reasonable answers)*

   How would we write the relationship to show the number of feet that needs to be sold to make a profit? *(2n + 5 > 17)* **Why?** *(Students should indicate that the amount of profit is determined by the amount of rope sold. If he 'breaks even,'* 2n *+ 5 = 17. When the values for n are found that make the left side of the inequality greater than 17, Sam will make a profit.)*

Have students work with their partner and use one of the previous methods for solving equations to solve the problem: guess-and-test, make a table, create a graph, make a diagram, or work backwards.

> **This relationship is called an inequality. With your partner, find values for *n* that will make the inequality, $2n + 5 > 17$ a true statement. This means that the value of *n* multiplied by 2 added to 5 is always greater than 17.**

Give students time to identify values for the variable. Have them share the values. Organize the values in a table.

> **What do you notice about the table?** *(values greater than 6 make the inequality true, 6 makes the two expressions (2n + 5 and 17) equal, values less than 6 make the left expression less than 17)*
>
> **Write the solution to the inequality.** *($n > 6$)*

Have a student read the inequality. Ask students to give you other values not in the table that would make the inequality true. For example, if *n* was 10, 8, 6.5, and so on. Record those values under the inequality. Using the number line in the Teacher Masters, have students graph all of the points that were given as you graph in the Student Booklet.

> **Graph the solution on the number line.**
>
> **How could we show all of the values on a number line that will make this inequality true, that would include all of the values larger than 6?**

Demonstrate shading the number line to the right of 6.

> **We know we cannot include 6 as part of the values that make the inequality true because 6 will make the two expressions equal. To show that 6 is not included as a value that makes the inequality true, we draw a circle around 6 on the number line. Do the same on your number line.**

Write the inequality $2n + 5 \geq 17$. Ask a student to read the inequality.

> **Follow along as [student] reads the inequality. *(pause)* How would the solution to this inequality be different from the one we solved?** *(6 can be a solution or a value that would make the inequality true)*

Have students graph values in the Student Booklet that would make the inequality true as you graph points that make it true in your Teacher Booklet.

> **In this case, we want 6 to be a value that can be substituted into the inequality because 6 will make the inequality true. To show that, instead of using an open circle, we will use a closed circle at 6 to show that 6 can be a solution to the inequality.**
>
> **How are the solutions to these inequalities different from the solution to an equation?** *(an equation typically has one solution, in the case of those they have been solving, while an inequality will have many solutions)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

> **Working with your partner, solve each problem by using any method you prefer.**

Ask student pairs to show how they represented the solution to the inequality on the number line.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Oscar is thinking about selling custom-made t-shirts for $8 plus $3 for each word printed on the shirt. He wants to sell each t-shirt for $20 or more. He wants to know how many words have to be printed on the shirt in order to sell them for $20 or more.**
>
> **He found an inequality to model the problem. He let x = the number of words. He wrote: $3x + 8 \geq 20$. He said that if $x \geq 4$, the inequality would be true. What does that mean?** *(it means that he has to print at least 4 words for the cost of the shirt to be $20 or more)*
>
> **Write your response in your Student Booklet.**

Have students read their written response.

Expressions and Equations 2
Lesson 13

Lesson 13: Solving Equations Involving the Distributive Property

Lesson Objectives	Students solve equations involving the distributive property. Students make sense of problems and persevere in solving them. (SMP 1) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Distributive Property of Multiplication over Addition (Subtraction)	
Misconception(s)	Students may only apply the distributive property to the first term in the parentheses. For example, given the expression $3(x + 4)$, they may only multiply 3 times x.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Simplify each expression.

Have students read each simplified expression.

Learning to Solve

TEACHER NOTES

In this lesson, students formalize the process of solving an equation. They apply properties of operations in more complex equations.

1. Students will solve equations involving the distributive property.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   We have learned multiple methods for solving equations. Any of these methods can be used to solve an equation such as $3(d + 2) = 12$.

   Let's use the diagram method first.

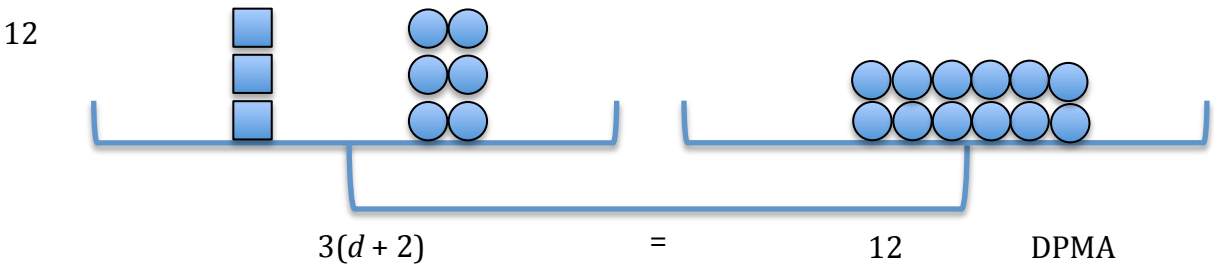
   We can represent $d + 2$ as 1 square and 2 circles. Draw 1 square and 2 circles.

   In your Student Booklet, draw a representation of $3(d + 2) = 12$ on the scale. Write "$3(d + 2)$," "=," and "12" below the appropriate section of the scale.

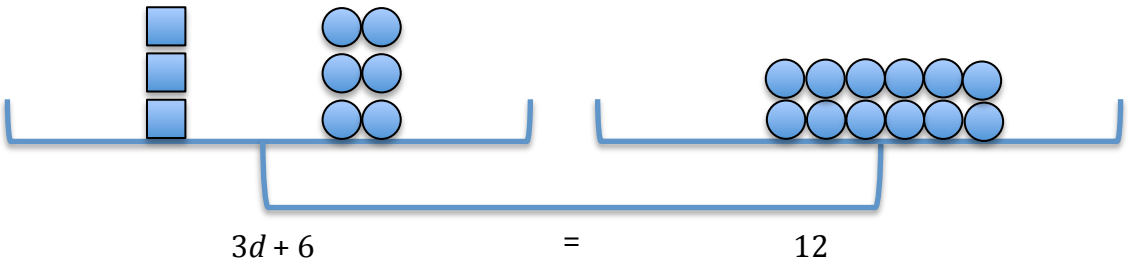
Ask a student to show their representation on the board. Ask if anyone has a different representation and have him or her draw their representation on the board.

A possible representation is:

To represent $3(d + 2) = 12$, we can use 3 sets of the $d + 2$ model.



What is another expression that you could use to describe what the representation of $3(d + 2)$ is? Write this expression below your model.

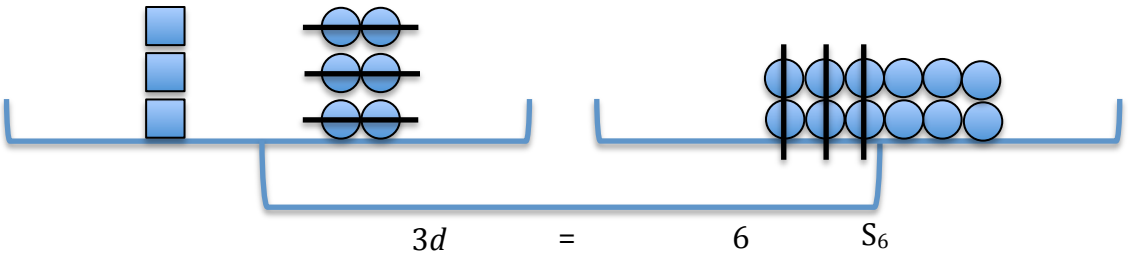


They should see that the expression $3d + 6$ is the same as $3(d + 2)$ as it represents the distributive property of multiplication over addition. Ask students to use their diagram to find the value of d that will make $3d + 6 = 12$ a true statement.

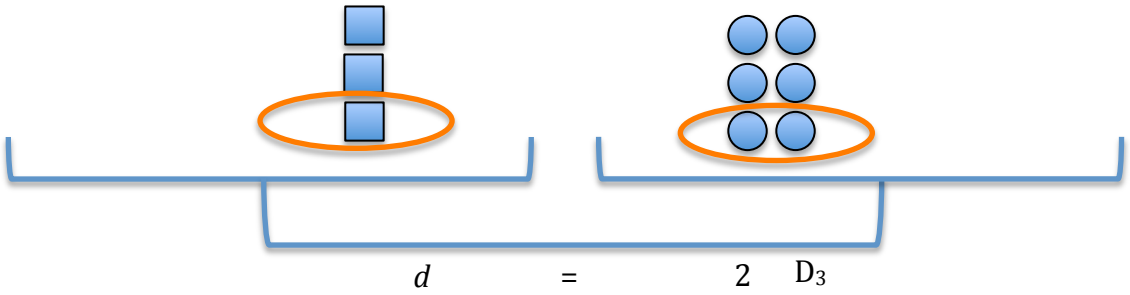
Use the diagram you have drawn in your Student Booklet and find the value of d that makes the equation $3d + 6 = 12$ a true statement.

A possible solution is:

Remove 6 from each side.



1 *d* is equal to 2.



Ask a student to show the steps they used to find the value of *d*. If students have a different method, ask them to show their process. Watch for different applications of the distributive property.

2. Students will solve an equation that involves the distributive property by using inverse operations.

   Complete the second problem. As you go through these steps, the students should be writing their answers in the Student Booklet. Have students say their answer to each step of the process.

   Let's solve another problem using a different method. The problem is 8 = 8(*h* – 4).

   What is the first step? *(use the distributive property to simplify 8(h – 4))*

   What is 8(*h* – 4) simplified? *(8h – 32)* **What property did you use to simplify the expression?** *(distributive property of multiplication over subtraction)*

What is the equation now? *(8 = 8h – 32)* **Write it.**

Now that we have simplified the equation, what do you think is the next step? *(add 32)*

The inverse of subtracting 32 is adding 32. How do we write this? Show this step. *(8 = 8h – 32 A_{32})* **Write it.**

In your Student Booklet, write, "the inverse of subtracting 32 is adding 32, A_{32}" next to Step 2.

Ask a student to come to the board and show this step.

What is the equation now? *(40 = 8h)*

The equation has been simplified to 40 = 8*h*. What is the next step? *(Divide by 8)* **Write it.**

Write, "the inverse of multiplying by 8 is dividing by 8, D_8."

Have students perform this division in the Student Booklet.

Ask another student to come to the board and show this step.

What is the value of *h*? *(5)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Working with your partner, solve each problem by using any method you prefer.

Ask student pairs to show their solution process on the board.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

After applying the distributive property, an expression is $8x + 36$. What might the expression have been before applying the distributive property? *(possible solutions include: $2(4x + 18)$ or $4(2x + 9)$)*

Expressions and Equations 2
Lesson 14

Lesson 14: Formulas

Lesson Objectives	Students solve problems involving formulas. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Formula**: a statement that expresses the relationship between two or more quantities; a generalization of an important relationship such as area of a rectangle.	
Requisite Vocabulary	Area, perimeter, volume, distributive property	
Misconception(s)	Students are sometimes confused when working with formulas because finding the missing value requires them to analyze the relationships expressed in the formula.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet. Read then solve the problem.

Ask students to give their answer for the problem and explain how they found it.

Learning to Solve

TEACHER NOTES

As students work with formulas, they become adept at finding different quantities that are missing. Presenting problems that have missing quantities in different positions in the formulas creates more flexible thinking. A variety of situations are presented that include the use of different formulas.

1. Students will solve problems involving formulas.

In the Warming Up section, you wrote an expression for finding the rental fee at the roller skating party. What was the expression? *(4r + 150)* The roller-skating rink managers can use this expression to calculate the rental fee for a skating party. You have done this before when you evaluated the expression for a certain value of the variable. This is also called "substituting" a value for the variable.

Cost of party = $4r + 150$ is called a formula because it gives a way to always find the cost for renting this skating rink.

Display Problem 1 found on the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **In Learning to Solve, you will work with the same formula to solve problems.**
>
> **Shannon wants to invite 10 people to her skating party. What will be the fee for the party?**
>
> **The formula is cost of party = 4*r* + 150. What does *r* represent?** *(the number of people invited)* **Write it.**
>
> **What does *r* equal in this problem?** *(10)* **Write it.**
>
> **How will you solve this problem to calculate the rental fee?** *4(10) + 150 = 40 + 150 = $190*

Display the next problem on the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Find the number of people invited to a roller-skating party if the total cost is $222. Use the same formula as before when you found the rental cost of a skating party at the skating rink. What formula did we use before?** *(cost of party = 4r + 150)*
>
> **What is the problem asking us to find?** *(the number of people at the party)*
>
> **How is the number of people at the party represented in the formula?** *(the variable r)*
>
> **Looking at our formula, cost of party = 4*r* + 150 what information in the problem can we substitute in the formula?** *(The total cost of the party is $222. This number can be substituted in the formula in place of "cost of party." Note that when solving equations, the dollar sign can be omitted, but the final answer should include the sign -- $)*

What is the new equation? *(222 = 4r + 150)* **Write this equation.**

We know that 150 added to 4*r* equals 222. What can we do to help us find the value of *r*?

What is the inverse operation of addition? *(subtraction)* **What is the inverse operation of multiplication?** *(division)* **We will use subtraction and division to help us solve.**

First, we will subtract, write this step in your Student Booklet, "222 = 4*r* + 150 S_{150}." What is the new equation after we subtract 150 from 150 and 222? *(72 = 4r)*

Have students think about each of these questions. Ask a student to give his or her response.

Think to yourself, 4 times a number is equal to 72. What can you do to find the number? ***(divide)***

The next step is to divide by 72 because it is the inverse of multiplication. Write 72 = 4*r* D_4.

What is the value of r? *(18 = r)*

What does this value of *r* indicate? *(there will be 18 people at a skating party if the cost of the party is $222)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

In the Practicing Together, you will use different formulas. You will use the formula to find the area of a trapezoid and the formula to solve distance problems.

Working with your partner, solve each problem.

After students have completed the problems, work through each problem with the students to find the missing values. Have student pairs provide their answers for the distance = (rate)(time) problem.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

In the Trying It On Your Own, you will work with other formulas to solve problems.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

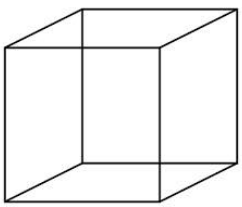
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 14A to provide extended practice before proceeding to Lesson 15.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.



In the Wrapping It Up, you will work with another formula to solve problems.

All edges of a cube are the same length. The formula for the volume of a cube is $V = lwh$, where V stands for volume, l stands for length, w stands for width, and h stands for height.

Solve this problem: The edge of a cube measures 8 centimeters. What is the volume of the cube? *(512 $cm^{3;}$ remind students that, when measuring volume, the answer must include the cubic symbol -- ^{3})*

Expressions and Equations 2
Lesson 15

Lesson 15: Using Geometric Formulas

Lesson Objectives	Students solve problems involving formulas. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Formula	
Misconception(s)	Students may think that if a quantity is missing from the formula, they cannot use the formula.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Square pyramid model	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

You will need to use formulas to solve these problems. What is the formula for the area of a rectangle? *(area of a rectangle = (length)(width) or (base)(height))* **The area of a rectangle refers to square units, like square feet or square centimeters.**

How do you find perimeter? *(add all the sides or 2(length) + 2(width)* **What is the difference between area and perimeter?** *(Area is the amount of square units of space enclosed and perimeter is the length of the boundary of the shape. Perimeter is a length measure so it is reported as foot, for example.)*

The last formula is volume. Volume is how many cubes can fit inside a shape. The formula of volume of a rectangular prism is length times width times height. The volume of a shape is recorded as cubic units, such as cubic feet or cubic inches.

Solve the problems, remembering the formulas and how the results are reported.

Ask students to give their answers for the problems and explain how they found them.

Learning to Solve

Teacher Notes

Students apply their understanding of solving equations to continue their work with formulas.

1. Students will solve problems involving geometric formulas.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

The image is of a square pyramid found in Egypt. It is called a "square" pyramid because the base of the pyramid is in the shape of a square.

If you have a model of a square pyramid available, you may want to show it so that students can see the base.

We are interested in finding the total surface area of this pyramid. What do you think "total surface area" means? *(the sum of the areas of all surfaces or faces)* **Write that in your booklet.**

How many surfaces make up this square pyramid? *(5 surfaces: 4 triangular surfaces and 1 square surface)* **Write that in your booklet.**

What is the formula for the area of a square? *($A = s^2$ or $A = lw$ or $A = bh$)* **Write that in your booklet.**

What is the formula for the area of a triangle? *($A = \frac{1}{2}bh$)* **Write that in your booklet.**

Have students review the answers to the questions.

> **For problem 5, in order to determine the total surface area of this square pyramid, we need to have some measurements. What is the height of one of the triangular faces?** *(610 feet)* **What is the length of the side?** *(755 feet)*
>
> **How do these measurements help us find the surface area?** *(give us the information needed for find the area of each part of the square pyramid)*
>
> **As a reminder, how many triangles make up the surface of the pyramid?** *(4)*
>
> **As a reminder, how many squares are on the surface of the pyramid?** *(1)*
>
> **Work with a partner to identify how can we find the total surface area if we know the parts of the pyramid?**

Allow students time to talk with a partner or group about the process. Ask them to report their ideas. As they share, have them use the model to explain why their process would work.

> **Use your method to find the total surface area of this square pyramid.** *($4((\frac{1}{2})(755)(610)) + (755)^2 = 4(230,275) + 570,025 = 1,491,125$ square feet)*

Have a student from each group show their solution process on the board. There may be multiple solution methods:

- One group might calculate the area of each triangle, not realizing that the area of each triangle will be the same, and will add each triangular area plus the area of the square base.
- Another group may see this but will still add each triangle area separately and then add the area of the square base.

- A third group may discover that they can multiply the area of the triangle by 4 because there are 4 triangular surfaces and then add the area of the square base.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. Review the formulas on the first page with the class if needed.

Working with your partner, solve each problem. Round your answer to the hundredths place.

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Revisit the concept map that was made in Lesson 1. Have students review their responses and determine what revisions or additions they would make. Discuss any of their changes.

AFTER this lesson: Administer Progress Monitoring
FORM C

Appendices

Expressions and Equations 2

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Expressions and Equations 2
Lesson 3A

Lesson 3A: Evaluating Expressions

Lesson Objectives	Students evaluate expressions with a range of values. Students use order of operations to simplify expressions. Students look for and make use of structure. (SMP 7) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Equivalent expressions:** 2 algebraic expressions are equivalent when the value of 1 expression equals the value of the other expression for *all* replacements of the variables.	
Requisite Vocabulary	Expression, equivalent, substitute	
Misconception(s)	Students sometimes believe that substituting 1 value into an expression is sufficient to determine equivalency.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the problems.

Pause for students to complete the problems.

What are the answers? How did you solve? Did anyone solve them differently?

Learning to Solve

Students will evaluate expressions.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Write the numbers 6, 7, 8, and 9 on the board.

What is the relationship among these numbers: 6, 7, 8, and 9?

Ask a student to describe their thinking about the relationship. Students should see that the numbers are in consecutive order or that each number increases by 1.

Using these 4 numbers, we will evaluate the expression $7 + 2e$. We can organize our process and solutions by using a table.

Discuss each step, including rewriting the expression.

First, rewrite the expression, substituting 6 for the variable e. Using order of operations, what do we do first? *(multiply)* **What is 2 times 6?** *(12)* **What is 7 + 12?** *(19)*

We now will evaluate the same expression, but with 7 as the value of e. What is 2(7)? *(14)* **What is 7 + 14?** *(21)*

Complete the next 2, using 8 and 9 has the value for *e*.

Pause for students to complete.

Earlier, we found that each value of *e* (6, 7, 8, and 9) increased by 1. Look at the values of 7 + 2*e* you found when you substituted. Talk to your neighbor about what you notice about the results of the substitutions you made.

Students should notice that the value of 7 + 2e increases by 2 as the value of *e* increases by 1.

On the left side of the table, write "Value of *e*: Increases by 1" and on the right side, write, "Value of 7 + 2*e*: Increases by 2."

Value of *e*: Increases by 1

$7 + 2e$	
Value of e	Value of $7 + 2e$
6	*7 + 2(6) = 7 + 12 = 19*
7	*7 + 2(7) = 7 + 14 = 21*
8	*7 + 2(8) = 7 + 16 = 23*
9	*7 + 2(9) = 7 + 18 = 25*

Value of 7 + 2*e*: Increases by 2

Why do you think the value of 7 + 2*e* increases by 2 as *e* increases by 1? *(because the value of e is being multiplied by 2)*

Have students complete the second table on the Learning to Solve sheet.

In the same way as you did with the first table, complete the next table by evaluating the expression 7 + 2*e*, using the numbers 5, 10, 15, and 20.

Ask a student to put the answers on the board.

Talk quietly with the person next to you. Determine the pattern between the values for *e* and the results of the evaluation of the expression. How would you describe the relationship?

Allow students to discuss.

What patterns did you notice? Write the pattern on the sides of the table like before.

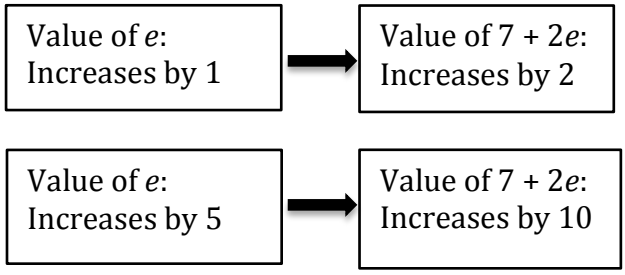
Value of *e*: Increases by 5

$7 + 2e$	
Value of *e*	**Value of $7 + 2e$**
5	*7 + 2(5) = 7 + 10 = 17*
10	*7 + 2(10) = 7 + 20 = 27*
15	*7 + 2(15) = 7 + 30 = 37*
20	*7 + 2(20) = 7 + 40 = 47*

Value of $7 + 2e$: Increases by 10

Students should notice the following:

- As the value of the variable increased, the result of the evaluation of the expression increased.
- As the value of the variable increased by 5, the value of the expression increased by 10.
- The increase of 10 in the value of the expression was due to the $2e$ term in the expression and that *e* increases by 5.



This pattern is called a generalization. A generalization is formed from patterns and relationships. The generalization we have modeled states that as the value of the variable increases, the result of the evaluation of the expression increases.

Turn to your Generalization sheet in your Student Booklet. Write the generalization: When the value of the variable increases, the result of the evaluation of the expression increases. When the value of the variable increases by the same amount each time, the value of the expression will increase by 2 times the value of the increment.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Divide the students into groups. Allow students to use a calculator if they wish.

> **Working with your group, evaluate the expression in each table, using 1 of the sets of variable values provided, and describe the pattern.** *(Make sure students understand that they can substitute values found in Set A or B – it's their choice.)*

Check for understanding by having pairs share their work.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Find an expression whose value is 7 if the value of the variable x is 1. Write the expression in your booklet. Be able to explain how you determined the expression.

Equations and Expressions 2
Lesson 5A

Lesson 5A: Translate Sentences into Equations and Inequalities

Lesson Objectives	Students translate sentences into equations and inequalities. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent expressions, Distributive property of multiplication over addition (subtraction)	
Misconception(s)	Students may think that in the acronym PEMDAS for Order of Operations, they must do all multiplication before they do any division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Translate the phrases into algebraic expressions.

Ask students to give their answer for one of the problems and explain how they found the answer.

Learning to Solve

1. Students will translate sentences into equations.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

An equation is a statement that two algebraic expressions are equal. For example, $2x + 3 = 15$ means that there is a value for x that will make $2x + 3$ equivalent to or equal to 15.

Word problems usually do not include the equation $2x + 3 = 15$.

Instead, the problem could be "Twice some unknown increased by 3 is equal to 15" or "The number of miles Jerry ran in a week is doubled and then increased by 3 to equal 15 miles in the week."

We can represent these sentences or situations with the algebraic equation $2x + 3 = 15$ where x represents the unknown amount.

Working with your partner, write the algebraic equation for each sentence in the table for Problem 1.

Have student pairs write their equations on the board and check for agreement with other students.

2. Students will translate sentences into inequalities.

> **When two quantities are not equal to each other, they are called inequalities. What symbols can we use to show that two quantities are not equal? In your booklet for Problem 2, write as many different symbols that show inequality, and then write out what each symbol means.**

Ask students to write one of the inequality symbols on the board until all the symbols are shown.

Students should come up with:

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\neq$	not equal to

On the board, write "*s* increased by 8 is greater than 10"

> **What does the phrase "*s* increased by 8 is greater than 10" mean?** *(when we take a number and add 8 to it, the result is a number that is larger than 10)*

> **We can write this relationship using symbols. On your whiteboard, write the inequality using symbols.** *($s + 8 > 10$ or $10 < s + 8$)*

Have students work together to complete the table under Problem 3.

Working with your partner, write the algebraic inequality for each sentence in the table under Problem 3.

Practicing Together

This lesson does not have a Practicing Together section. Students work together to develop an understanding of translating sentences to algebraic equations and inequalities during the Learning to Solve portion.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Write a sentence(s) in words that might be represented by the algebraic equation, $2b - 7 = 13$.

Ask students to read their sentences.

Expressions and Equations 2
Lesson 14A

Lesson 14A: Using Formulas to Solve Problems

Lesson Objectives	Students use formulas to solve problems. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Interest:** the amount of money you earn or owe when investing or spending money **principal:** the initial amount of money invested or spent **rate:** the percentage of the principal that results in the interest **time:** a measured or measurable period, such as days, weeks, months, years, etc.	
Requisite Vocabulary	Formulas, radius, pi, exponent, area, perimeter, volume	
Misconception(s)	Students are sometimes confused when working with formulas because finding the missing value requires them to analyze the relationships expressed in the formula.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up problem in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Evaluate the expressions for $m = -5$.

Ask students to give their answer for one of the problems and explain how they found the answer.

Learning to Solve

TEACHER NOTES

Formulas often pertain to situational contexts. As students apply formulas in the problems presented in this lesson, they are using their knowledge of number relationships to find missing values. A variety of situations are presented that include the use of different formulas.

1. Students solve problems involving formulas.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

In Learning to Solve, you will work with another formula to solve problems.

The formula for the area of a circle is $A = \pi r^2$. In this formula, r is the length of the radius.

If students have not had previous experience with this formula in the core class, or if you feel the students struggle with area, show a drawing of a circle with the radius labeled and define pi and state that pi is usually stated as 3.14. Review the meaning of r^2. Apply the area formula.

Find the area of a circle with a radius of 2.4 centimeters.

Let $\pi = 3.14$

Direct students to substitute the values for the variables in the formula and write values in their booklet.

Using the information provided, substitute the value for the appropriate variable.

After you have given them time to work, have students compare what they wrote with their partner. Then, ask some students to write their equation on the board. Select students who wrote equations that are correct and some that are incorrect. Have students look at what is on the board and discuss which equations are correct and why.

Have students work on Problem 2 by substituting values for the variables in the formula.

Solve the second problem in your Student Booklet.

The radius of a circular swimming pool is 25 ft. What is the area of the pool?

After you have given them time to work, have students compare what they wrote with their partner. Then, ask some students to write their equation on the board. Select students who wrote equations that are correct and some that are incorrect. Have students look at what is on the board and discuss which equations are correct and why. Discuss any errors such as multiplying 25 by 2 instead of squaring 25.

When we use formulas, carefully read the problem to understand it and then substitute values for the variables found in the formula.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. As you work through the example problem below, fill in the blank row with the information for each heading.

In the Practicing Together, you will work with other formulas to solve problems. Before you start working on completing the table with your partner, we are going to review the vocabulary in the table.

Look at the first column, interest. Interest is the amount of money you earn or owe when investing or spending money.

The next column is principal. Principal is the initial amount of money invested or spent.

The next column, rate, is the percentage of the principal that results in the interest.

Time can be days, weeks, months, years, etc.

We will do one together. Your principal is $100. The rate of the investment is 10% each month. What is 10% of $100? *($10)*
You would make $10, this is the interest earned per month. How much interest would you earn in 10 months? *($100)* **How did you calculate this?** *(amount of money invested multiplied by rate and time)*

The last column is formula. The formula for determining the simple interest on an amount of money is
Interest = Principal × Rate × Time which is represented by
I = prt. In our example, our equation was
$100 = $100 × .10 × 10.
Working with your partner, fill in the missing values in the table.

Have student pairs provide answers for the missing values in the table. Write in these missing values for each row. Students should also explain their formula and how they calculated the missing value.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

In the Trying It On Your Own, you will work with other formulas to solve problems.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

In the Wrapping It Up, you will work with another formula to solve problems.

You are given the formula

$$F = C\left(\frac{9}{5}\right) + 32$$

This formula converts a Celsius temperature to a Fahrenheit temperature. On the Wrapping It Up sheet in your Student Booklets, write the process you would follow to convert a temperature of 16°C to Fahrenheit.

Have students read the process they would use to convert a Celsius temperature to Fahrenheit.