Fractions

Teacher Lessons

Lesson 1: Unit Fractions and the Whole

Lesson Objectives	Students identify the whole, unit fractions, equal-sized parts, and amount of iterations. Students partition area models to show fractions. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP 4) Students attend to precision. (SMP 6)
Vocabulary	**Unit fraction:** a fraction of the form $\frac{1}{b}$ **Iteration:** making copies of a smaller amount and combining them to create a larger amount **Improper fraction:** a fraction that is greater than 1. For a fraction to be greater than 1, the numerator must be greater than the denominator. **Mixed number:** a number that is composed of a whole number and a fraction that is less than 1
Requisite Vocabulary	Fraction, numerator, denominator
Misconception(s)	Students often think that fractions are 2 whole numbers, separated by a fraction bar. This idea does not help students see a fraction as a number or quantity. Fractions should be read as, for example, “one-fourth,” rather than “1 out of 4.” Reading fractions in this way will help students see fractions as numbers.
Instructional Materials	**Teacher** • Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) **Student** • Student Booklet • Red colored pencil • Square tiles (20 per pair)

	• Chart paper or Poster Board	

Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Module Check Form A** ✓	Lesson 7: Mid-assessment **Module Check Form B**	Lesson 15: Post-assessment **Module Check Form C**

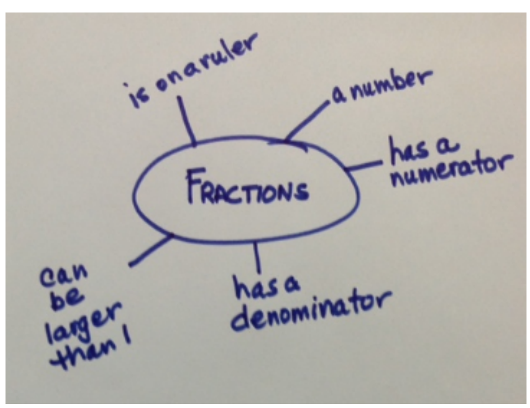
Warming Up

Create a concept map about fractions on a class poster or chart paper and display it for the duration of the module. Have students turn to the Warming Up sheet in the Student Booklet. Students will first write their own ideas on this page and then add peer responses. Draw a circle on the board with “Fractions” in the middle.

Fractions

Today, we will start by thinking about what you know about fractions. Write everything you know about fractions in your Student Booklet.

Ask for student responses. Make a concept map on the board similar to the photo below as students share ideas. Ask for students to clarify or provide examples to aid in the brainstorming of ideas.



What are some important ideas? What is a fraction? Can you add an example to any of your words?

Take a picture of the concept map when students finish sharing their ideas and/or display the map for the duration of this module. The concept map will be used again at the end of this module. Displaying during the course of the module will allow both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

TEACHER NOTES

During this lesson, you may find that some students observe that the number of iterations is equal to the number in the numerator in a fraction. This is not an accurate generalization. When the iteration results in a mixed number such as $1\frac{2}{3}$, for example, the number of iterations would be 5, not 2. Be alert during the lesson to students who make this incorrect observation.

Throughout the lessons in this module, students may find that using tracing paper to partition on the number lines or length segments helps to make the length-parts more accurate. This may aid in helping students determine the comparison of size or relationship to benchmark fractions in later lessons.

1. Students will be introduced to the terms "unit fraction" and "iteration."

The fraction $\frac{1}{2}$ has a special name; we call it a unit fraction. Why do you think we call $\frac{1}{2}$ a unit fraction?

(accept reasonable answers, such as $\frac{1}{2}$ is a unit fraction because it has a numerator of 1 or $\frac{1}{2}$ is 1 part of a whole that is partitioned or divided into 2 parts)

A fraction that has a numerator of 1 is a unit fraction.

Turn to the Notes section of your Student Booklet. Write another example of a unit fraction, but use a different denominator.

Ask students to name other unit fractions. Each student should contribute 1 unit fraction. List them on the whiteboard or ask students to write their unit fraction on the board. When all students have given a unit fraction, ask the following questions:

What do you notice about all the unit fractions? *(the numerators are all 1)*

What does the numerator of a fraction tell you? *(the number of equal-sized parts)*

What does the denominator of a fraction tell you? *(the number of equal-sized parts of the whole)*

What is the smallest number that could be a denominator in a unit fraction? *(1)* **Is $\frac{1}{1}$ a unit fraction?** *(yes, it can be; doesn't matter that it is whole unit)*

2. Students will learn the concept of iteration with fractions.

   Distribute the square tiles to each pair. Have students build areas.

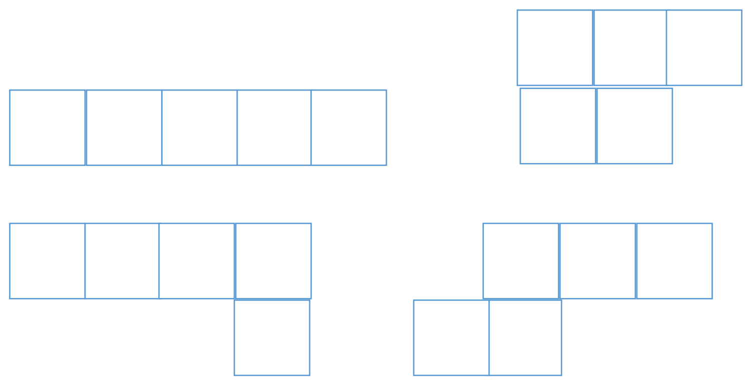
This (hold up one square tile) **represents an area of $\frac{1}{4}$. Using your square tiles, make an area that represents $\frac{3}{4}$.**

Have students share their areas. Compare the areas they make, noting that 2 areas are possible.

How many iterations of the unit fraction did you do? *(3)* **Do both areas show the same amount?** (*Yes, because the unit fraction was iterated the same number of times)*

Now let's use the area of one square tile to represent $\frac{1}{8}$. Make an area of $\frac{5}{8}$.

Have students share their areas. Compare the areas they make, noting that 12 different areas are possible. 4 examples are shown below.



How many iterations of the unit fraction did you do? *(5)* **Do both areas show the same amount?** (*Yes, because the unit fraction was iterated the same number of times)*

3. Students will determine the unit fraction and number of iterations in a fraction.

Display the Fraction Iteration Table on the Learning to Solve sheet in the Teacher Masters. Have students turn to the table in their Student Booklet. Complete the first row together. Students may then work individually, in pairs or small groups, or as a class.

Look at the table in your booklet. Let's do the first row together. What is the first fraction? *($\frac{2}{3}$)* **What is the unit fraction?** *($\frac{1}{3}$)* **Write $\frac{1}{3}$ in the Unit Fraction space. How did you determine the unit fraction?** *(because the unit fraction was iterated 2 times)* **How many iterations?** *(2)* **Write 2 in the Number of Iterations space.**

Complete the table.

Give the students time to fill in the table.

Look at the third and fifth fraction. How are these 2 fractions different from the others? *(they are greater than 1)*

How did you find the unit fraction and number of iterations for the fractions greater than 1? Was the process similar to the process you used with fractions less than 1? Why or why not? *(students should note that they followed the same procedure for all fractions)*

Have students share a different row of their table.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs. If the class is struggling with the concept, complete the sheet with the entire class.

2. Review the answers for problems 1–5. On problem 6, discuss the number of iterations needed to make 1 whole. Stress that because there are 2 wholes, the unit fraction would need to be iterated 8 times. Then, 1 more time for a total of 9 times.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

In your Notes section of the Student Booklet, write the fraction for $\frac{1}{8}$ iterated 7 times.

Discuss student responses.

Draw a representation of the unit fraction $\frac{1}{8}$ iterated 7 times.

Write the mixed number for $\frac{1}{8}$ iterated 11 times.

Draw a representation of the unit fraction $\frac{1}{8}$ iterated 11 times.

Ask students to draw their representation on the board and check for understanding.

Fractions
Lesson 2

Lesson 2: Unit Fractions: Mixed Numbers and Improper Fractions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students identify mixed numbers and improper fractions.
Students convert and explain the relationship between mixed and improper fractions.
Students make sense of problems and persevere in solving them. (SMP 1)
Students model with mathematics. (SMP 4)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">Partition: to split a whole into equal parts</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Numerator, denominator, unit fraction, improper fraction, mixed number</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">A significant misconception is that the number of partitions indicates the denominator, regardless of size. Partitions (or sub-units) do not have to be congruent, but they must be the same size.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Square tiles (20 per pair)
• Red colored pencil</td></tr>
</table>

Warming Up

Review iterations and partitioning area models.

Distribute the square tiles to each pair. Have students build areas. Hold up one square tile to represent an area of $\frac{1}{5}$.

> **This represents an area of $\frac{1}{5}$. Use your square tiles, make an area that represents $\frac{4}{5}$.**

Have students share their areas. Compare the areas they make, noting that there are multiple ways to make the area, but the sides of the square tiles must be aligned.

> **Now make an area that represents $\frac{6}{5}$.**

Have students share their areas. Compare the areas they make.

> **How many square tiles does it take to make a whole?** *(5)* **We can show a whole and 1 part.**

Separate the tiles to show the 1 whole and the $\frac{1}{5}$ part. As you say $\frac{6}{5}$ and $\frac{1}{5}$, indicate where they are in the tile arrangement. Show $\frac{5}{5}$ and the $\frac{1}{5}$.

> **Another way to represent your area of $\frac{6}{5}$ is to say it is 1 and $\frac{1}{5}$.**

Learning to Solve

Teacher Notes

When using different representations, make sure that students use the correct name. If a student does not use the name of a representation (e.g., "I showed the fraction by using rectangles."), correct the student, using the name of the representation (e.g., "You used the area model to represent the fraction.")

When students use area models and/or number lines, make sure that students make each partition, or iteration, equal sized. It is important to understand that fractional pieces have to be the same size (but not necessarily congruent) when the whole is the same.

When partitioning number lines, students often draw hash marks equal to the denominator, rather than counting spaces. Again, verify that students make the correct partitions and that the distance is equal.

Throughout the lessons in this module, students may find that using tracing paper to partition on the number lines or length segments helps to make the length-parts more accurate. This may aid in helping students determine the comparison of size or relationship to benchmark fractions in later lessons.

1. Students will symbolize a fraction by using multiple representations.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Select a student to read the problem.

Follow along as [student] reads the word problem.

Pause for the student to read.

Without drawing a picture, write on your sheet and tell me how much pie Sam ate. *($\frac{3}{4}$)* **How do you know?** *(answers may vary. For example, some students may indicate that $1 - \frac{1}{4} = \frac{3}{4}$)*

What is the unit fraction used to represent $\frac{3}{4}$? *($\frac{1}{4}$)* **How many times was $\frac{1}{4}$ iterated?** *(3 times)* **We could say that Sam ate three, $\frac{1}{4}$ pieces.**

Now, let's look at some representations and determine whether they modeled $\frac{3}{4}$ correctly.

Complete the table for each representation as the lesson progresses.

Look at the models in your Student Booklet.

The first model is a number line. What is the whole? *(the distance from 0 to 1 or the length between 0 and 1)* **If we wanted to represent fourths on the number line, how do we partition the whole, the length between 0 and 1?** *(into 4 equal-length parts)*

After you partition, how should we show $\frac{3}{4}$? *(draw a line from 0 to the third hash mark)* **Is the model correct?** *(yes)* **Why?** *(the third hash mark represents $\frac{3}{4}$ of a whole)*

Look at the next model. This is called an area model. In an area model, we use a shape, such as a rectangle, square, or circle, and partition the whole. What is the whole? *(1 rectangle)* **How should the whole be partitioned?** *(into fourths)* **How is the whole partitioned?** *(into halves)* **Is this the correct representation for $\frac{3}{4}$?** *(no)* **Why?** *(each whole is partitioned into halves, not fourths; the shapes are not combined to form a whole)*

Look at the next 2 models, numbers 3 and 4, and determine whether they correctly represent $\frac{3}{4}$.

Give students time to complete the models.

Are the representations correct? *(yes; answers will vary. For example, there are 4 equi-sized partitions and 3 of them are shaded)*

What about the space in number 4? Does it still show $\frac{3}{4}$? *(yes answers will vary. For example, there are 4 equi-sized partitions and 3 of them are shaded. The parts do not have to be continuous)*

Students will symbolize improper fractions and mixed numbers through the use of a number line.

Look at Problem 5. Can a fraction be greater than 1? *(yes)* **Think about the fraction $\frac{5}{4}$.**

What is the unit fraction? *($\frac{1}{4}$)* **How many one-fourths, or iterations, are in $\frac{5}{4}$?** *(5 one-fourths or 5 iterations)*

Look at the number line. The whole is the length from 0 to 1. Think about the pie problem we started with. Sam ate $\frac{3}{4}$ of the pie. How many pieces would he have eaten if he had eaten the whole pie? *(4 pieces, or $\frac{4}{4}$)* **In this example, the whole was 1 pie, or 4 pieces of the pie.**

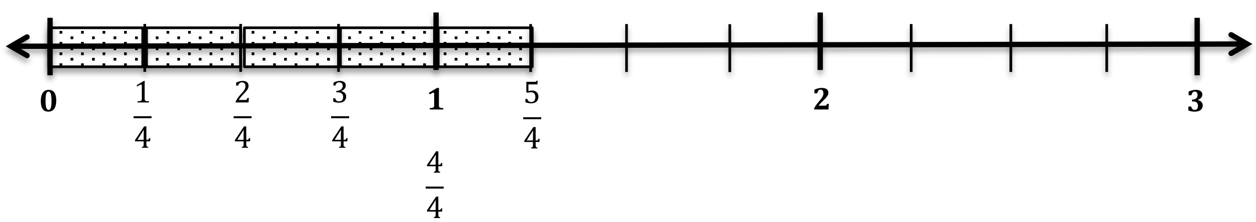
Think about the improper fraction $\frac{5}{4}$. Is it greater than 1 whole? *(yes)* **How do you know?** *(accept reasonable answers, such as 4 would be 1 whole, 1 more than 4)*

We will represent $\frac{5}{4}$ on the number line. How many partitions do we need between 0 and 1? *(4)* **How do you know?** *(the denominator)*

Do we need to make additional partitions after the 1? *(yes)* **Why?** *(greater than 1 whole)* **How many partitions do you need between 1 and 2?** *(4)*

Now we will label our hash marks, or the partitioning, up to and including $\frac{5}{4}$. Label and count.

Label the number line from $\frac{1}{4}$ to $\frac{5}{4}$, like the example below.



Draw a line, starting at 0 to show the length of $\frac{5}{4}$.

What is another way that I can represent this fraction besides $\frac{5}{4}$? *($1\frac{1}{4}$)* **Why?** *(the mixed number shows 1 whole and $\frac{1}{4}$ more)*

We can say that $\frac{5}{4} = 1\frac{1}{4}$. Write this below the number line. As we showed in the last lesson, $\frac{5}{4}$ is an improper fraction because $\frac{5}{4}$ has a numerator that is greater than the denominator. And $1\frac{1}{4}$ is a mixed number because it contains a whole number and a fraction. How is an improper fraction equal to a mixed number? *(they show the same distance or length)*

Is there a way that we can find this equivalence without drawing a model? How can we change an improper fraction to a mixed number? *(answers may vary, such as divide 5 by 4, the quotient is $1\frac{1}{4}$)*

2. Students will convert a mixed number to an improper fraction.

Have students look at number 6 on the Learning to Solve sheet. Ask students to look at the number line and decide what fraction is represented. Write student responses as the lesson progresses.

Look at the number line. The whole is the length or distance from 0 to 1. How is the whole partitioned? *(into fourths)* **Write the fraction that you think is represented.**

What fraction does this model represent? *(accept equivalent answers such as $2\frac{2}{4}$, $2\frac{1}{2}$, $\frac{10}{4}$, $\frac{5}{2}$)*

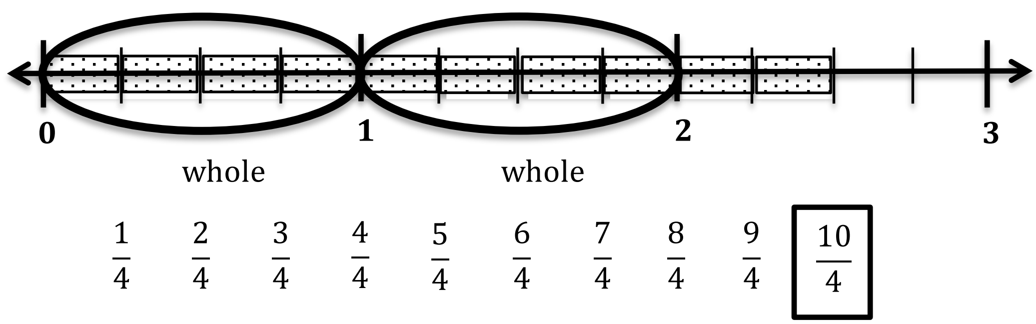
Write students' responses on the board, adding any that students did not state, including $\frac{10}{12}$.

No one said that this model represents $\frac{10}{12}$. This is what I wrote down. Is it correct? *(No)* **Why or why not?** *(accept reasonable answers, such as $\frac{10}{12}$ is less than one whole)*

What was the whole? *(0 to 1)* **I counted the whole as 0 to 3.** *(show 0 to 3 on the number line)* **If I said the whole was 0 to 3, would I be correct now?** *(yes)* **But I agreed with you when you said that the whole was 0 to 1, so my answer is incorrect.**

Be sure the erase $\frac{10}{12}$ so that students are not confused.

Circle the whole and label the hash marks, or the partitioning, up to and including $\frac{10}{4}$. What are we counting by? *(fourths)*



Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs, in small groups, or as a class to complete the Practicing Together sheet.
2. Review answers and ask students how they solved each problem.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

> **In the Notes section of your Student Booklet, write or draw 3 different representations of $\frac{9}{8}$. Circle the whole if you make a drawing.**

Have students share their work and check for understanding.

Lesson 3: Comparing Fractions, Using Benchmark Fractions

Lesson Objectives	Students compare fractions, using the benchmark fractions 0, $\frac{1}{2}$, and 1. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP 4)	
Vocabulary	**Benchmark fractions**: fractions used to judge the magnitude or size of other fractions (0, $\frac{1}{2}$, and 1)	
Requisite Vocabulary	Numerator, denominator, unit fraction, whole	
Misconception(s)	A significant misconception is that the number of partitions indicates the denominator, regardless of size. Partitions (or sub-units) do not have to be congruent, but they must be the same size.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Fraction cards (see page 137 of Teacher Masters) • Fraction mat (see page 138 of Teacher Masters) • Red colored pencil

Warming Up

Review partitioning and labeling a number line with various fractions less than 1. Students may want to use different colors as they label the number line.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Look at the number line. How is a number line different from an area model? *(accept reasonable answers, such as area models use square, rectangles, or other polygons; a number line is a length model)*

For this example, the whole will be the length from 0 to 1. First, partition the whole and label $\frac{1}{2}$. How did you determine where to place the fraction? *(accept reasonable answers, such as halfway between 0 and 1)*

Using this same number line, partition the whole into thirds. Label the hash mark $\frac{1}{3}$.

When we use number lines, we look at length. Draw a line from 0 to $\frac{1}{2}$. Now draw a line from 0 to $\frac{1}{3}$.

How are these 2 fractions similar? *(both are unit fractions, distance from 0 to $\frac{1}{2}$ is longer)*

Using this same number line, partition the whole into fourths.

Label the hash mark for $\frac{2}{4}$. What do you notice about $\frac{1}{2}$ and $\frac{2}{4}$? *(accept reasonable answers, such as same distance, same hash mark, they are equal)*

Is there another fraction we could place on this number line that is less than $\frac{1}{2}$? *(accept reasonable answers, such as any fraction that is not equal to or greater than $\frac{1}{2}$)*

Learning to Solve

Teacher Notes

When using different representations, make sure that students use the correct name. If a student does not use the name of a representation (e.g., "I showed the fraction by using rectangles."), correct the student, using the name of the representation (e.g., "You used the area model to represent the fraction.")

When students use area models and/or number lines, make sure that students make each partition, or iteration, equal sized. It is important to understand that fractional pieces have to be the same size (but not necessarily congruent) when the whole is the same.

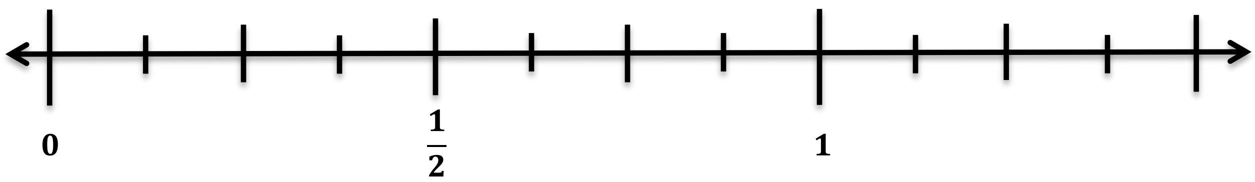
When partitioning number lines, students often draw hash marks equal to the denominator, rather than counting spaces. Again, verify that students make the correct partitions and that the distance is equal.

Throughout the lessons in this module, students may find that using tracing paper to partition on the number lines or length segments helps to make the length-parts more accurate. This may aid in helping students determine the comparison of size or relationship to benchmark fractions in later lessons.

1. Students will be introduced to benchmark fractions on a number line.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Students should be in pairs or small groups for discussion during the lesson.



We are going to use the length or distance from 0 to 1 as the whole. How is this number line partitioned? *(into eighths)* **Thinking about eighths, what fraction is equal to $\frac{1}{2}$?** *($\frac{4}{8}$)* **Write $\frac{4}{8}$ below $\frac{1}{2}$.**

Using this number line, talk to your partner and decide which fraction, $\frac{3}{8}$ or $\frac{3}{4}$, is closer to $\frac{1}{2}$. How do you know? *($\frac{3}{8}$ is closer to $\frac{1}{2}$, because the distance or length on the number line from $\frac{3}{8}$ to $\frac{1}{2}$ is shorter than the distance or length from $\frac{3}{4}$ to $\frac{1}{2}$)*

Check the distance to see which one is closer to $\frac{1}{2}$.

Is $\frac{9}{16}$ or $\frac{1}{8}$ closer to $\frac{1}{2}$? How do you know? *($\frac{9}{16}$ is closer to $\frac{1}{2}$ because the distance or length on the number line from $\frac{9}{16}$ to $\frac{1}{2}$ is shorter than the distance or length from $\frac{1}{8}$ to $\frac{1}{2}$.)*

We can use the fraction $\frac{1}{2}$ for comparing fractions. It is called a benchmark fraction. There are 3 common benchmark fractions, 0, $\frac{1}{2}$, and 1. Circle these benchmark fractions on your number line. We will use these benchmark fractions to compare fractions.

TEACHER NOTES

Some students may argue that 0 and 1 are not fractions. If a student makes this statement, show the fractions $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$. Ask, what is another way of writing $\frac{4}{4}$? Students should respond 1. Indicate that $\frac{4}{4}$ and 1 are equivalent so 1 is a benchmark fraction. In the case of $\frac{0}{4}$, ask students what the numerator would be of the fraction that comes before $\frac{1}{4}$. The numerator would be 0. 0 is equivalent to $\frac{0}{4}$ so 0 is a benchmark fraction.

Throughout the lessons in this module, students may find that using tracing paper to partition on the number lines or length segments helps to make the length-parts more accurate. This may aid in helping students determine the comparison of size or relationship to benchmark fractions in later lessons.

2. Students will compare fractions to benchmark fractions.

   Complete the sheet, modeling the fractions, as the lesson progresses.

   What if you wanted to compare the fractions $\frac{7}{9}$ and $\frac{9}{16}$? Talk with a partner about which fraction is larger. Show your answer in your student booklet by using < or > in the blank.

   Is $\frac{7}{9}$ closer to 1 or closer to $\frac{1}{2}$? *(closer to 1)* **What about $\frac{9}{16}$? Is it closer to 1 or $\frac{1}{2}$?** (*closer to $\frac{1}{2}$*) **Which fraction is larger?** *($\frac{7}{9}$)*

Because $\frac{7}{9}$ is closer to 1 and 1 is greater than $\frac{1}{2}$, we know that $\frac{7}{9}$ is greater than $\frac{9}{16}$. What comparison symbol, greater than or less than, can we write between the fractions? *(greater than)* **We can say that $\frac{7}{9} > \frac{9}{16}$ without having to find common denominators.**

Use the number line and benchmark fractions, 0, $\frac{1}{2}$, and 1, to compare the rest of the fractions.

Look at the first comparison. Is $\frac{2}{3}$ closer to 0, or $\frac{1}{2}$ or 1? *(closer to $\frac{1}{2}$ or 1)* **Is $\frac{1}{16}$ closer to 0, $\frac{1}{2}$, or 1?** *(closer to 0)* **Because $\frac{2}{3}$ is closer to $\frac{1}{2}$ or 1, this fraction is greater than $\frac{1}{16}$. What symbol do we write between these fractions?** *(greater than)* **What if I wrote $\frac{1}{16} < \frac{2}{3}$? Is this still correct?** *(yes, it shows the same relationship)*

Talk with your partner and determine which fraction is greater for the next 2 comparisons. First, determine whether each fraction is closest to 0, $\frac{1}{2}$, or 1. Write a greater than or less than sign between the 2 fractions.

Ask a student pair to write answers on the board.

Write a nonexample on the board: $\frac{9}{7} < \frac{3}{4}$.

Look my example on the board. Did I compare these fractions correctly? *(no)* **How would you explain to someone why my comparison is incorrect?** *(accept reasonable answers, such as using benchmark fractions, you can determine that $\frac{9}{7} > \frac{3}{4}$.)*

Have students share their ideas, then you can share the following questioning process if needed.

Using benchmark fractions, is $\frac{9}{7}$ closer to 0, $\frac{1}{2}$, or 1? *(1)* **Is $\frac{3}{4}$ closer to 0, $\frac{1}{2}$, 1?** *(1)* **Because they are both closer to 1, how do we decide which is greater?** *($\frac{9}{7}$ is closer to 1, but it is greater than 1)* **Thinking of sevenths, what is 1 whole?** *($\frac{7}{7}$)* **What sign should be written between these fractions?** *(greater than)*

Practicing Together

1. Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. Distribute a set of Fraction Cards and a Fraction Mat (see pages 137 – 138 of Teacher Masters) to each pair or small group.

   You are going to sort the fractions cards into 1 of the 3 categories. You will need to decide with your partner(s) if the fraction on the card is closest to 0, $\frac{1}{2}$, or 1. You will put that card on the mat under that benchmark fraction.

   When you have sorted all of the fractions, talk to your partner(s) about how you would describe the fractions you placed in each category. Write any patterns you notice at the bottom of each column.

Select some pairs to share their sorts. You may want to have a different pair share their fractions for each column. After writing the fractions in the columns for the class to see, discuss. Spend more time on the improper fractions and how the decisions were made to sort these types of fractions.

Do you agree with the way the fractions were sorted? Did anyone sort the fractions in a different way?

Discuss any discrepancies. As students share patterns, write the patterns on the displayed Practicing Together sheet from the Teacher Masters.

What patterns did you notice about the fractions in each of the categories? *(may include the size of the unit fractions, the amount of iterations, or partitioning)*

In the 0 column, what do you notice about the relationship between the numerator and the denominator? *(the numerator is less than half of the denominator)*

In the $\frac{1}{2}$ column, what do you notice about the numerator compared to the denominator? *(the numerator is about half of the denominator or, if you multiply the numerator by 2, you get a product close to the denominator)*

In the 1 column, are there any fractions greater than 1? *(yes)* **How do you know? What do you notice about the numerator and denominator?** *(the numerator is greater than the denominator)*

2. Students write generalization statements about the relationship between benchmark fractions and a given fraction in the Notes section of their Student Booklets.

   We have created generalizations about how fractions relate to the benchmark fractions of 0, $\frac{1}{2}$, and 1. Turn to the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships. Record the patterns for each benchmark fraction. Then, give 3 examples of fractions that fit with the generalization.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other colored) pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

> **Turn to the Notes section of your Student Booklets. If someone asked you how to use benchmark fractions to determine the relationship between 2 fractions, what would you say? Write down what you would say.** *(accept reasonable answers, such as use benchmark fractions to determine which fraction is greater)*

Have students either share orally or in writing in the Notes section of their Student Booklet. Check for understanding.

Fractions
Lesson 4

Lesson 4: Comparing Fractions With Like Numerators

Lesson Objectives	Students compare fractions with like numerators (e.g., $\frac{4}{8}$ and $\frac{4}{5}$) by using concrete and pictorial models. Students compare fractions that equal 1 whole. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP 4)	
Vocabulary	**Generalization:** formulating and producing statements about patterns and relationships and evaluating their reasonableness	
Requisite Vocabulary	Numerator, denominator, unit fraction	
Misconception(s)	Students often think that fractions such as $\frac{5}{6}$ and $\frac{3}{4}$ are equivalent because they are "only 1 away from the whole." This idea does not take into account the size of the sub-unit and that the whole must be the same for the comparison. Students may also think that the fraction is composed of 2 whole numbers and not consider the fraction as a quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent)	• Student Booklet • Cuisenaire rods (1 set per student pair) • Red colored pencil

	• Projector (or equivalent)	

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Think about this problem. Which is larger: $\frac{1}{3}$ of a small pizza from House of Pizza or $\frac{1}{4}$ of a large pizza from House of Pizza?

What are the 2 sizes of pizza? *(small and large)* **Can we compare a small pizza to a large pizza?** *(no)* **Why?** *(the amounts cannot be compared because they refer to different wholes—a small pizza and a large pizza. They are not the same size.)* **We can say the wholes of the pizzas being compared are a small pizza and a large pizza. Can we compare fractions when the wholes are different?** *(no)* **Why?** *(The size of the wholes is different.)*

What would we have to change to compare pizza slices*? (same whole, or same size pizza)*

Learning to Solve

TEACHER NOTES

The purpose of this lesson is to focus on the whole with the understanding that greater the denominator, the smaller the piece or part of the whole. Although this is the focus, discourage students from describing fractions by using part-whole language (e.g., "3 out of 4 parts shaded"). This model does not correlate to the linear model and encourages the misconception that fractions are not numbers.

When using Cuisenaire rods, verify that students use the same whole to correctly compare fractions.

1. Students compare fractions with like numerators, using Cuisenaire rods.

   Assign students to pairs. Give each student pair a set of Cuisenaire rods. Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   We want to determine which piece is larger: $\frac{1}{5}$ of a large pizza from House of Pizza or $\frac{1}{6}$ of a large pizza from House of Pizza. What is the size of the whole in this problem? *(1 large pizza)* **The wholes are the same, so can we compare the pizza slices now?** *(yes)*

   Think about benchmark fractions. Are $\frac{1}{5}$ and $\frac{1}{6}$ closer to 0, $\frac{1}{2}$, or 1? *(both closer to 0)* **Because they are both close to 0, we cannot use benchmark fractions to compare. We need to think about the size of each pizza slice.**

   If all the slices were equal to $\frac{1}{5}$ of the pizza and everyone could have only 1 slice each, how many people could share the pizza? *(5 people)*

   How many people could share the pizza if all slices were equal to $\frac{1}{6}$ of the pizza? *(6 people)*

   Which slices would be larger, the slices shared by 5 people or the slices shared by 6 people? *(slices shared by 5 people)* **Which piece is larger: $\frac{1}{5}$ of a large pizza from House of Pizza or $\frac{1}{6}$ of a large pizza from House of Pizza?** *($\frac{1}{5}$ of a large pizza)*

   What if 8 friends shared a large pizza? What would happen to the size of the pizza slices? *(they would be smaller)*

   We can think about the size of the pieces only because we are talking about the same whole, 1 large

pizza. We cannot compare pizza slice sizes if the whole is different.

Look at the next problem. We need to compare $\frac{2}{3}$ of a medium pizza and $\frac{2}{9}$ of a medium pizza from House of Pizza. Think about it for a moment. Which do you think is the greater amount? Who thinks $\frac{2}{3}$ is greater? Who thinks $\frac{2}{9}$ is greater?

Write on the board the number of students who think $\frac{2}{3}$ is greater and the number of students who think $\frac{2}{9}$ is greater.

First, what size are the wholes, small, medium, or large pizzas? *(medium pizzas)* **Are the wholes the same?** *(yes)* **Can we compare these 2 fractions?** *(yes)*

First, think about what $\frac{2}{3}$ means. How many total slices? *(3)* **Now, think about $\frac{2}{9}$. How many total slices?** *(9)*

Which slices would be bigger? *($\frac{2}{3}$)* **How do you know?** *(only sharing among 3 people)* **If you really loved pizza, which portion would you rather have? Why?** *(accept reasonable answers, such as $\frac{2}{3}$ because the size of the denominator means there would only be 3 pieces)*

How else can we compare $\frac{2}{3}$ and $\frac{2}{9}$? *(accept reasonable answers, such as compare the size of the respective unit fractions)*

What is the unit fraction in $\frac{2}{3}$? *($\frac{1}{3}$)*

What does $\frac{2}{3}$ mean relative to its unit fraction? *(2 one-third pieces)*

What is the unit fraction in $\frac{2}{9}$? *($\frac{1}{9}$)*

What does $\frac{2}{9}$ mean relative to its unit fraction? *(2 one-ninth pieces)*

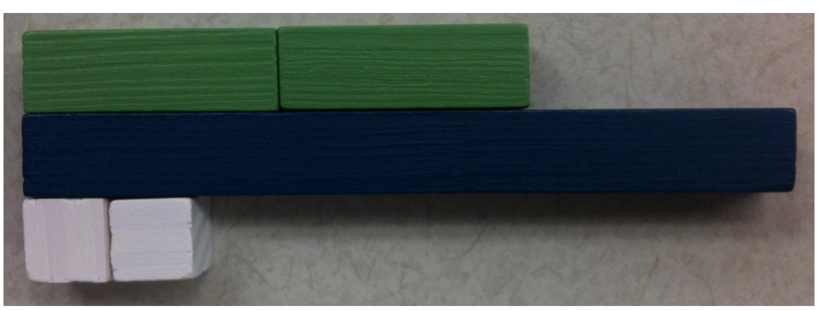
Which fraction do you think is greater, $\frac{2}{3}$ or $\frac{2}{9}$? *($\frac{2}{3}$)* **Why?** *(the one-third unit fractions are larger)*

Write your answer in your Student Booklet.

What is an important idea to remember when we are comparing fractions? *(Answers will vary, such as fractions have to refer to the same whole OR a unit fraction can be used to compare fractions.)*

Using your Cuisenaire rods with your partner, create a representation of the comparison of $\frac{2}{3}$ and $\frac{2}{9}$. The length of the blue rod will represent the whole.

Allow student pairs to create their representations. Below is the only possible representation, using Cuisenaire rods. The first photo shows the whole (blue) and the one-ninth (white) and one-third (green) parts. The second photo shows the comparison of $\frac{2}{3}$ and $\frac{2}{9}$.



Ask a few student pairs for their responses to this question: Is $\frac{2}{3}$ greater than or less than $\frac{2}{9}$?

How did you represent $\frac{2}{3}$? *(used 2 of the green pieces)*

How did you represent $\frac{2}{9}$? What color? *(used 2 of the white pieces)*

Look at your answer in your Student Booklet. Do you still think your answer is correct or do you want to change your answer? Why? *(have students share their responses)*

Ask students the following questions:

What is similar about $\frac{2}{3}$ and $\frac{2}{9}$? *(they have the same numerator)* **What does the same numerator mean?** *(the unit fraction has been iterated the same number of times)*

How many unit parts are we using? *(2)*

What is different? *(the denominator)*

What does the denominator indicate? *(the number of equal-sized parts that the whole is partitioned into)*

How is the whole for $\frac{2}{3}$ partitioned? *(into 3 parts)*

How is the whole for $\frac{2}{9}$ partitioned? *(into 9 parts)*

Although the whole is partitioned differently, we are comparing the same number of parts. Thinking about how the whole is partitioned, which fraction is greater, $\frac{2}{3}$ or $\frac{2}{9}$? *($\frac{2}{3}$)* **Why is $\frac{2}{3}$ greater?** *(one-third is greater than one-ninth because the parts are larger; therefore, 2 one-third parts is greater than 2 one-ninth parts)*

How can we symbolize that $\frac{2}{3}$ is greater than $\frac{2}{9}$? *($\frac{2}{3} > \frac{2}{9}$ or $\frac{2}{9} < \frac{2}{3}$)*

2. Students compare improper fractions that have like numerators.

Look at the next pair of fractions. Are these fractions examples of unit fractions, proper fractions, or improper fractions? *(improper fractions)*

What does $\frac{5}{3}$ mean? *(5 one-third parts)* **How was the whole partitioned?** *(into 3 parts)*

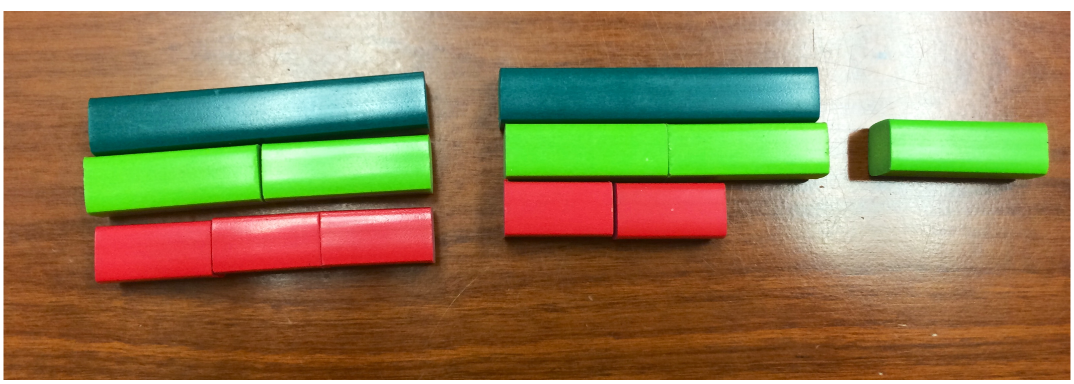
What does $\frac{5}{2}$ mean? *(5 one-half parts)* **How was the whole partitioned?** *(into 2 parts)*

Which fraction do you think is greater, $\frac{5}{3}$ or $\frac{5}{2}$? Write your answer in your Student Booklet. Who thinks $\frac{5}{3}$ is greater? Who thinks $\frac{5}{2}$ is greater?

Write on the board the number of students who think $\frac{5}{3}$ is greater and the number who think $\frac{5}{2}$ is greater.

Using your Cuisenaire rods with your partner, create a representation to show how to compare $\frac{5}{3}$ and $\frac{5}{2}$ to justify your answer. The whole is the length of the dark-green rod.

Have student pairs create their representations. Walk around and observe the models. Provide guidance as needed. Below is the only possible representation, using Cuisenaire rods. The photo shows the whole (dark-green), one-third (red) and one-half (light green).



Do you still think your answer is correct or do you want to change your answer? Why? *(have students share their responses)*

Which fraction is greater, $\frac{5}{3}$ or $\frac{5}{2}$? *($\frac{5}{2}$)* **In both fractions, we were comparing the same amount of parts, 5. What does the denominator tell us?** *(number of equal-sized parts that the whole is partitioned into)*

What about the size of each part? What do you notice when the denominator is larger? *(the parts are smaller)*

Think about the denominator as the amount of people equally sharing a candy bar. If you really like candy, would you rather share with 3 people or 2 people? Why? *(with fewer people, you get a larger piece of the candy bar)*

Write the comparison relationship between $\frac{5}{3}$ and $\frac{5}{2}$ in your Student Booklet. *($\frac{5}{3} < \frac{5}{2}$)*

Verify that students have written the relationship correctly.

From these 2 examples, how can you determine which fraction is greater if both fractions have the same numerator but different denominators? *(if both fractions have the same numerator, the smaller the denominator, the greater the fraction)*

Have students turn to the Notes section in their Student Booklets and complete on board so students can copy the fraction generalization.

Turn to the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships.

A smaller denominator indicates that there are less partitions of the whole, and a larger denominator indicates that there are more partitions of the whole.

Therefore, the area of each fractional part of the whole that is partitioned into less parts, or that has a smaller denominator, is larger than the fractional part of the whole that is partitioned into more parts, or has a larger denominator.

Write this generalization on the Generalizations page: The <u>area</u> of each fractional part of the whole

that is partitioned into fewer parts, or that has a smaller denominator, is larger than the fractional part of the whole that is partitioned into more parts, or has a larger denominator.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet. Allow students to use their Cuisenaire rods or an area model.
2. Review the ways that the fractions were categorized and the generalizations hat students made for each category.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Think of ways to generalize how to compare fractions using the example on the sheet. For example, which fraction is greater, $\frac{6}{25}$ or $\frac{6}{30}$? Write your answer. *($\frac{6}{25}$)* **How could you compare without making a model? Write your answer.** *(accept reasonable answers, but should make mention of unit fractions and parts)*

Have students share their work and check for understanding.

Fractions
Lesson 5

Lesson 5: Comparing and Ordering Fractions

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students compare and order fractions, including improper fractions.
Students reason abstractly and quantitatively. (SMP 2)
Students use mathematical models. (SMP 4)
Students attend to precision. (SMP 6)
Students look for and express regularity in repeated reasoning. (SMP 8)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Numerator, denominator, unit fraction, improper fraction</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may think that $\frac{1}{2}$ of something is always equivalent to $\frac{2}{4}$ of something else, without considering that the whole must be the same to compare fractions.
Students often compare fractions by simply comparing the numbers in the denominators and/or numerators, rather than referring to the size of the fractional part of the whole.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>Teacher Masters
Whiteboard (or equivalent)
Projector (or equivalent)
Symbol and Fraction Cards (see pages 139 -</td><td>Student Booklet
Whiteboard
Dry erase marker
Red colored pencil</td></tr>
</table>

	147 of Teacher Masters)	

Warming Up

Review benchmark fractions.

Have students turn to the Warming Up in their Student Booklet. Read the problem with the students.

> **Kari said, "$\frac{7}{6}$ is closer to 2 than the benchmark number 1." "I agree," said Rob. "You cannot have a fraction more than 1 close to the benchmark number 1." Do you agree with Kari and Rob? Why or why not?**
>
> *(Disagree. $\frac{7}{6}$ is closer to 1 than to 2.)*

Allow time for them to respond to the problem. Before they share their responses, have them share with a partner.

As students share their reponses, discuss as needed.

Learning to Solve

TEACHER NOTES

In this lesson, students will use both benchmark fractions and previous experience with number lines. It is vital that students understand that the whole needs to be the same to compare fractions and that when modeling (either concrete or pictorial), the representation needs to be the same—meaning that the number line or area model need to be the same size. Additionally, students often compare fractions by simply comparing the number in the denominator or numerator, rather than referring to the size of the fractional part of the whole. To address this misconception, students should make the generalization that the greater the denominator, the smaller the part. The use of models will support the development of this generalization.

Throughout the lessons in this module, students may find that using tracing paper to partition on the number lines or length segments helps to make the length-parts more accurate. This may aid in helping students determine the comparison of size or relationship to benchmark fractions in later lessons.

1. Students order fractions, using benchmark fractions.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Read the first problem and complete the sheet as the lesson progresses.

   Look at the first problem. Brad, Denise, and Sharla have been practicing their running every day after school. Today during gym class, Brad ran $\frac{7}{6}$ miles, Denise ran $\frac{5}{3}$ miles, and Sharla ran $\frac{4}{5}$ miles. Their gym teacher wants to place the students in order from the

longest distance ran to the shortest distance ran. How can you help the teacher organize these distances?

We will use benchmark fractions to help us compare the distances. What are the 3 benchmark fractions we have used previously? *(0, $\frac{1}{2}$, 1)* **We can also use these benchmark fractions with the next whole number, meaning 1$\frac{1}{2}$ and 2. Let's work through this problem together.**

What is the question we are answering? *(how can you organize the distances from longest to shortest)* **What information do we need to answer our question?** *(the distances ran by the 3 students)*

What is our whole for this problem? *(1 mile)*

Let's organize the information. How many miles did Brad run? *($\frac{7}{6}$ miles)* **What does $\frac{7}{6}$ mean?** *(7 one-sixths or 1 one-sixth, which is equal to 1, and an additional $\frac{1}{6}$)* **Which benchmark fraction is $\frac{7}{6}$ closest to?** *(1)* **How do you know?** *(accept reasonable answers, such as he ran more than a mile but not much farther, 1 whole is $\frac{6}{6}$)*

How many miles did Denise run? *($\frac{5}{3}$ miles)* **What does $\frac{5}{3}$ mean?** *(5 one-thirds or 3 one-thirds, which is equal to 1, and an additional $\frac{2}{3}$)* **Which benchmark fraction is $\frac{5}{3}$ closest to?** *(2)* **Why?** *($\frac{5}{3}$ is equal to 1 and $\frac{2}{3}$. $\frac{2}{3}$ is close to 1 whole; therefore, $\frac{5}{3}$ is almost 2.)*

How many miles did Sharla run? *($\frac{4}{5}$ mile)* **What does $\frac{4}{5}$ mean?** *(4 one-fifths)* **Which benchmark fraction is $\frac{4}{5}$ closest to?** *(1, but less than 1)*

Two fractions, $\frac{7}{6}$ and $\frac{4}{5}$ were close to 1. Which is smaller? *($\frac{4}{5}$)* **Why?** *(it is less than 1)* **What is the order of the fractions from the longest distance to the**

shortest distance ran? *($\frac{5}{3}$, $\frac{7}{6}$, $\frac{4}{5}$)* **What are the names of the students in order from the longest distance ran to the shortest distance ran?** *(Denise, Brad, Sharla)*

2. Students compare fractions with like numerators.

   For this activity, us the Symbol and Fraction Cards (see pages 139 - 147 of Teacher Masters). Review how to compare fractions with like numerators, using a number line as needed.

   Call 2 students to come to the front of the room. Give them 2 fractions that have the same numerator from the Fraction Card set They should hold up and show their fractions to the class. Have another student come to the front and stand between the students with the correct relational Symbol (<, >, or =). Ask the class to determine if the relationship is correct. Then, have someone explain how they know it is correct.

   Look at the statement that (students' names) have made. Do you agree with the statement? Why or why not?

   Do as many pairs of fractions as time allows. You may want to add in other fractions as appropriate.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the activity sheet in groups or with the whole class if more appropriate.

2. Discuss the answers and check for understanding.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Help the teacher grade the assignment. The students were told to order the fractions from greatest to least. For each set of fractions, explain if the order is correct and why.**

If time permits, have students share their work and check for understanding.

Fractions
Lesson 6

Lesson 6: Compare Fractions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students compare fractions.
Reason abstractly and quantitatively. (SMP 2)
Look for and make use of structure. (SMP 7)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may think that $\frac{1}{2}$ of something is always equivalent to $\frac{2}{4}$ of something else, without considering that the whole must be the same to compare fractions.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)
• 1 Deck of cards (number cards and Ace cards only)</td><td>• Student Booklet
• Whiteboard
• Dry erase markers (2 for every pair of students
• Cuisenaire rods (1 set per student pair)
• Find A Place Game Sheet (2 per student pair and 1 each to take home, see page 148 of Teacher Masters)
• Red colored pencil</td></tr>
</table>

Warming Up

Review benchmark fractions and fractions between set benchmarks. Give each student a whiteboard. Provide a number line as needed.

Using a number line, as needed, write 3 fractions that are between $\frac{1}{4}$ and $\frac{1}{2}$. You have 1 minute. *(examples include: $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{3}{7}$)*

Ask students to share 1 fraction. Write all fractions on the board.

Which fractions are closest to $\frac{1}{2}$? Let's circle the fractions closest to $\frac{1}{2}$. How do you know that the fraction is closest to $\frac{1}{2}$? What method do you use to decide?

Learning to Solve

Teacher Notes

Refrain from providing strategies too quickly for students to find equivalent fractions. Allow students time to explore and find their strategies, using Cuisenaire rods and/or number lines.

1. Students will use Cuisenaire rods to identify equivalent fractions.

   Give each student pair a set of Cuisenaire rods.

   Take out 1 brown rod. This will be our whole. Now use your white rods. How many white rods are needed to match the length of the brown rod? *(8)*

   Using the white rods, show $\frac{2}{8}$ of the length of the brown rod.

Now find a fraction that is equivalent to $\frac{2}{8}$. How will you know whether it is equivalent? *(if it is the same length)* **Do we have to compare the 2 fractions to the brown rod?** *(yes)* **Why?** *(you must use the same whole to compare fractions)*

What fraction is equivalent to $\frac{2}{8}$? ($\frac{1}{4}$) **Using one rod, how would you represent $\frac{1}{4}$ of the length of the brown rod?** *(1 red rod)*

Using the same whole, the brown rod, show $\frac{2}{4}$ and $\frac{4}{8}$ of the length of the brown rod.

Now find another fraction that is equivalent to these 2 fractions.

What fraction? ($\frac{1}{2}$) **Using one rod, how would you represent this fraction?** *(1 pink rod)*

Write "$\frac{2}{4} = \frac{4}{8} = \frac{1}{2}$" on the board.

Is this statement true? (*yes*) **How do you know?** (*accept reasonable answers*)

2. Students will build and compare fractions while playing Find-a-Place.

   Group students into pairs. Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Display the directions for the game on the Learning to Solve sheet. The game requires a standard deck of cards, without the Jacks, Queens, and Kings.

   Distribute 4 Find A Place sheets per pair (see page 148 of Teacher Masters), one for each game and one for each player to take home. If playing only once, distribute 3 sheets per pair and one for each player to take home.

   Read the directions and then play the game.

Game Directions:

1. You will play in pairs. The person on the left is Player A. The person on the right is Player B.

2. The goal of the game is to create a fraction using the digits 0 through 9 that is as close as possible to the fraction in the center of the page.

3. To create the fractions, I will draw a card from this deck and show it to you. The deck contains the 2 through 9 cards. The ten card represents 0 and the ace represents 1. There are 4 of each number in the deck.

4. The first card I draw will be for Player A. Player A may put the number on the card in any numerator or denominator that is blank on his or her side of the game sheet. Once I draw the card, it will not be drawn again.

5. Player B will get the next card. That player will place the number in any numerator or denominator on his or her side of the game sheet.

6. I will keep drawing cards until all blanks are filled.

7. Once you place a number in a box, you may not change the number or move it.

8. You must play the number on your turn. In other words, you cannot save the number and play it later.

Scoring Directions:

1. To score, you and your partner will decide who created the fraction that is closer to the number in the middle.

2. The player who created the fraction closest to the number in the middle receives one point.

3. If you both created a fraction that is equally close to the number in the middle, you both receive a point.

4. If you created a fraction that has a 0 as the denominator, the other person automatically gets a point.

5. **The person with the most points wins.**

Tell students that they will play the game again, this time using the strategies they discovered while playing the game the first time.

We will play the game again using the second sheet. Talk with your partner about strategies you used to place your number.

What strategies did you use to place your numbers? Think about the strategies you discovered in the first game that were helpful and use them in this game.

Play the game a second time. Allow students to take a sheet home with them to play outside of class. Continue the discussion regarding strategies as time permits in later classes. This game can be used at any point in the unit to review the comparison of fractions.

Practicing Together

There is no Practicing Together sheet. Instead, have students play Find-a-Place for the majority of the class period. Review strategies if the class needs it.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 6A to provide extended practice before proceeding to Lesson 7.

Wrapping It Up

Ask students to share an idea about fractions that they used to play Find-a-Place.

Think for a minute about something you did or could do to help you play the Find A Place game more successfully. Some of you may be asked to share the strategies or ideas with the class.

Have students share their work and check for understanding.

Fractions
Lesson 7

Lesson 7:
Equivalent Fractions

Lesson Objectives	Students find equivalent fractions, using multiplication and division. Students create generalizations about fraction relationships. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent, equivalent fractions, generalization, partition	
Misconception(s)	When using multiplication to create equivalent fractions, students think they multiply by a whole number. For example, $\frac{2}{3} = \frac{4}{6}$ because $\frac{2}{3}$ was multiplied by $\frac{2}{2}$. However, many students will say they multiplied by 2.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry erase marker • Concentration Cards (1 set per pair, page 149 of Teacher Masters) • Red colored pencil

Warming Up

Give each student a whiteboard. Have students write 3 fractions between $\frac{1}{2}$ and $\frac{3}{4}$ on their whiteboard.

On your whiteboard, write 3 fractions between $\frac{1}{2}$ and $\frac{3}{4}$. *(examples include: $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{7}$)*

Have students share their whiteboard with the person sitting next to them. Write at least 1 fraction from each student on the board.

Ask students to determine which fraction is the largest.

Which fraction is the greatest in our group? How do you know?

Learning to Solve

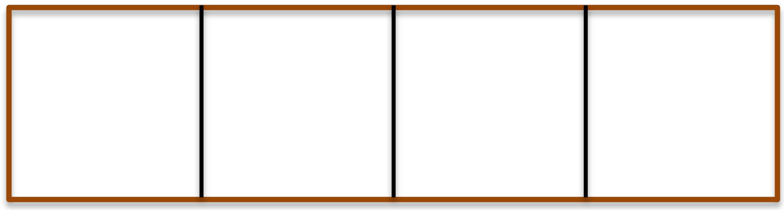
Teacher Notes

Students often view fractions as 2 separate numbers, rather than a single number with a single value. As we move from making and identifying equivalent fractions with manipulatives and diagrams into the symbolic representations in this lesson, be sure to reinforce that a fraction represents a number or quantity.

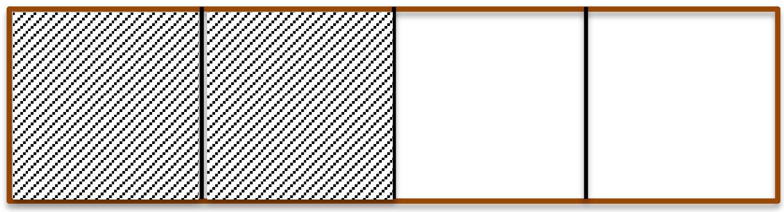
1. Students partition the same whole in order to identify equivalent fractions.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Display the rectangle, which represents a rectangular garden.

Under problem 1 of the Learning to Solve sheet, there is a rectangular model that represents a garden.



Heather has been given $\frac{2}{4}$ of the garden to plant her vegetables. Shade $\frac{2}{4}$ of the garden to indicate Heather's area.



Heather wants to share her $\frac{2}{4}$ of the garden with 5 friends.

Get together in small groups.

Talk to your group members about how to partition the garden area.

What do we have to do first? *(partition, or break apart, $\frac{2}{4}$ of the whole garden)* **Yes, we need to partition the whole to help us figure out how much of the garden each friend will receive. How many total people are sharing the garden?** *(6, Heather and 5 friends)*

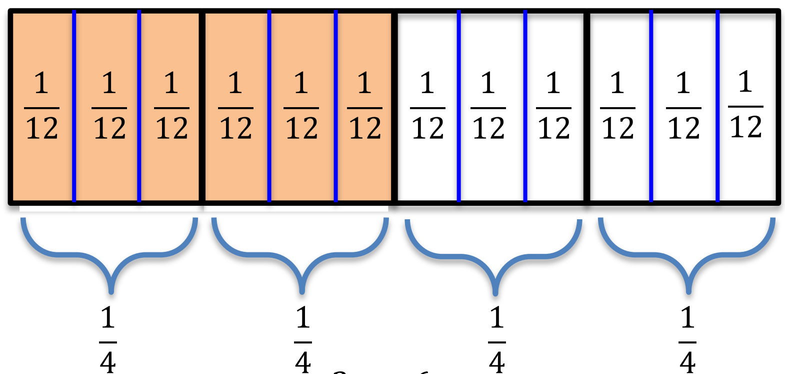
How many parts does she need to split each of the 2 one-fourth parts into so that all 6 of them get the same amount of the garden? *(into 3 parts each)* **How do you know?** *(partitioning each 4^{th} into 3 parts will make 12ths, which can be shared by 6 people)*

Note that 12 is a multiple of 6. This may come up in the discussion and, if so, should be highlighted.

Partition the entire garden into twelfths, including the shaded portion. Does it have to be equal sized

parts? *(yes)* **Why?** *(accept reasonable answers, such as partitioning creates equal parts)*

Allow students to partition. Remind students to try and make each part as equal as possible. The following picture shows the completed model.



$$\frac{2}{4} = \frac{6}{12}$$

Based on your model, what fraction is equivalent to $\frac{2}{4}$ of Heather's garden? *($\frac{6}{12}$)*

How much of the garden will Heather and each of her friends get? *($\frac{1}{12}$)*

Below your model in your booklet, symbolize the comparison between $\frac{2}{4}$ and $\frac{6}{12}$. *($\frac{2}{4} = \frac{6}{12}$)*

In the model, how did we create twelfths from the fourths? *(partition each fourth into 3 parts)*

There were 4 fourths and we partitioned each into 3 parts, so that means we had 4 equal-sized parts of 3. Why did we partition the 4 fourths into 3 parts? *(6 friends wanted to plant $\frac{2}{4}$ of the garden; each one-fourth section had to be partitioned into 3 parts so that each friend had an equal-sized area; 12 is a multiple of 6)*

What operation do we use to find the total if we have 4 equal-sized groups of 3? *(multiplication)* **What did we multiply the numerator and denominator by?** *(3)* **How did we partition each fourth?** *(into thirds)*

Look again at problem 2. Think to yourself for a minute about this question. We can see from our model that $\frac{2}{4} = \frac{6}{12}$. If we do not want to draw a model, how can we determine that $\frac{2}{4} = \frac{6}{12}$?

Ask a student for an idea. Focus on $\frac{2 \times 3}{4 \times 3} = \frac{6}{12}$. Write this on the board under the rectangle partitioned into twelfths.

Why were both the numerator and the denominator multiplied by 3? *(the denominator was multiplied by 3 because each $\frac{1}{4}$ unit was partitioned into 3 parts; the numerator was multiplied by 3 because there are 2 units of size $\frac{1}{4}$, so there will be 6 units)*

If we multiply the numerator and the denominator by the same number, it is the same as multiplying the whole fraction by 1. Multiplying by 1 does not change the value of the fraction.

Multiplying the numerator and the denominator by the same number creates equivalent fractions without using manipulatives or Cuisenaire rods.

Look at problem 3. In your Student Booklet, write a fraction that is equivalent to $\frac{2}{3}$. Explain how you found your equivalent fraction.

Answers may vary. Check that students multiply the numerator and denominator by the same number. Students may choose any number, as any number will result in an equivalent fraction—except 0.

Have students write the equations on the whiteboard to demonstrate that a fraction can have many different equivalent fractions. Examples may include:

$\frac{2 \text{ x } 2}{3 \text{ x } 2} = \frac{4}{6}$, $\frac{2 \text{ x } 3}{3 \text{ x } 3} = \frac{6}{9}$, $\frac{2 \text{ x } 4}{3 \text{ x } 4} = \frac{8}{12}$, $\frac{2 \text{ x } 5}{3 \text{ x } 5} = \frac{10}{15}$

If we think about the examples on the board, what does the number that we multiply the numerator and the denominator by represent? *(how many parts we partition each of the original parts into)*

Would it make sense to multiply the numerator and the denominator by different numbers? Why or why not? *(no, the number we multiply by is how many parts we partition each original part into and that applies to both the numerator and the denominator) If the numerator and denominator are multiplied by different numbers, it is not the same as multiplying by 1. When we multiply by a number other than 1, you do not get an equivalent fraction.*

2. Students write generalization statements about finding equivalent fractions in the Notes section of their Student Booklets.

   Write the generalization on the whiteboard:

   Turn to the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships.

   If we multiply the numerator and the denominator by the same number, it is the same as multiplying the whole fraction by 1. Multiplying by 1 does not change the value of the fraction.

   We created a generalization about how to find equivalent fractions today. We found that if we multiply the numerator and the denominator by the same number, it is the same as multiplying the whole fraction by 1. Multiplying by 1 does not change the value of the fraction.

   Record this generalization in your Student Booklet. Then, using $\frac{3}{8}$ as your fraction, use this generalization to create 3 fractions that are equivalent to $\frac{3}{8}$.

Practicing Together

Distribute a set of Concentration Cards to each pair of students (see page 149 of Teacher Masters). Have them play as many rounds of Concentration as time allows.

> **Each pair has a set of cards. Shuffle your cards, then keeping them face down, lay them out on the table. Decide who will go first. That person will turn over 2 cards. If they are equivalent fractions, the player will remove them from the board and keep them. If they are not equivalent fractions, turn them back over. Keep going until all the cards are removed from the board. Whoever has the most cards at the end of the game is the winner.**

Have students share their work and check for understanding.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to Wrapping It Up in their Student Booklet. Ask students to work in a pair to solve the problems.

As time permits have students share their solution.

Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Module Check Form A**	Lesson 7: Mid-assessment **Module Check Form B** ✓	Lesson 15: Post-assessment **Module Check Form C**

Fractions
Lesson 8

Lesson 8: Simplify Fractions

Lesson Objectives	Students simplify fractions. Students reason abstractly and quantitatively. (SMP 2) Students construct viable arguments and critique the reasoning of others. (SMP 3) Students model with mathematics. (SMP 4)	
Vocabulary	**Fraction in simplest form**: a fraction where the numerator and denominator have no common factors	
Requisite Vocabulary	Equivalent fractions, mixed number, improper fraction, factor, multiple	
Misconception(s)	Students often think that simplifying a fraction, such as $\frac{2}{4}$, means dividing the fraction by 2, rather than dividing by $\frac{2}{2}$, or 1.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry erase marker (2 different colors for each pair) • Pattern blocks (1 hexagon, 6 triangles, 3 blue rhombi, 2 trapezoids) per pair • Red colored pencil

Warming Up

Review multiples and factors.

Give each student a whiteboard.

We will review some mathematics vocabulary, "multiples" and "factors." What is a factor? *(a number that is multiplied by another number in a multiplication problem)*

Write 2 factors that give a product of 24. What are 2 factors that equal a product of 24? *(2 and 12, 3 and 8, 4 and 6, 1 and 24)*

Write at least 3 pairs of factors for the product 48 *(answers may vary, such as 16 and 3, 12 and 4)*

We also will use multiples in this lesson. A multiple is the product of 2 factors. What other factors are BOTH 24 and 48 multiples of? *(Answers will vary, such as 3, 4, 6, 8, 12, 24)*

Learning to Solve

Teacher Notes

Make sure to use the word "simplify," rather than "reduce." "Reduce" is often interpreted to mean "make smaller," but fractions written in simplest form are equivalent.

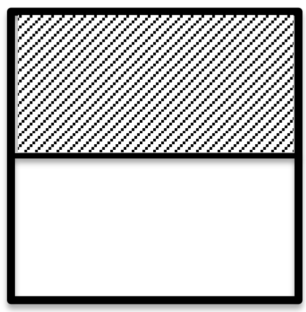
Students may say that "simplest form" means that there are no more numbers that both the numerator and denominator can be evenly divided by. This is correct, but using mathematical language, restate this as the simplest form of a fraction means that the numerator and denominator have no more factors in common.

Students may mention that both the numerator and denominator have a common factor of 1. This is true; however, every number has 1 as a factor and therefore 1 is not used when determining the simplest form of a fraction.

1. Students identify equivalent fractions, using an area model.

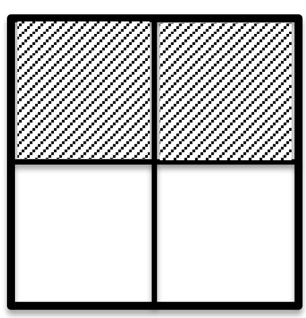
Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the whole. Our whole is the area of the square. First partition the square into 2 parts. Shade to show $\frac{1}{2}$ of the area of the square.

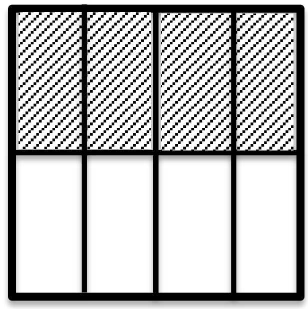


Using this same model, is there a way to show fourths?

Cut the halves in half, then partition the model into 4 equal parts. Now what fractional part is shaded? *($\frac{2}{4}$)* **Is the same area of the whole shaded as when we were showing one-half?** *(yes)*



Is there a way to not change the area shaded, but partition more? *(yes, there are multiple ways. For example, into 8ths, 12ths, and so on)* **Let's partition the whole into eighths.**



What is the shaded fractional part? *($\frac{4}{8}$)* **What can we say about $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$?** *(they are equivalent)*

All 3 fractions cover the same area of the whole, so they are equivalent. We can also say that $\frac{1}{2}$ is the simplest form of $\frac{2}{4}$ and $\frac{4}{8}$.

"Simplest form" means that the numerator and the denominator have no common factors.

Let's look at problem 2 on your sheet. Using any method you choose, find 4 fractions that are equivalent to $\frac{1}{2}$. Write these 4 fractions in the $\frac{1}{2}$ column.

First, what does "equivalent" mean? *(equal to or the same as)*

Have pairs report the fractions that they found. Record them in the $\frac{1}{2}$ column. When the list exhausts the possibilities that students found, ask whether anyone disagrees with any of the fractions listed as being equivalent to $\frac{1}{2}$. Make changes if needed.

The fractions that we listed in this column are all equivalent to $\frac{1}{2}$. How did you determine which fractions were equivalent? *(accept reasonable answers, such as using multiplication, number line)* **They represent the same amount as $\frac{1}{2}$. In this case, $\frac{1}{2}$ is the simplest form because 1 and 2 have no common factors other than 1.**

If students are not convinced that there are no other common factors, have students list the factors of 1 and 2 and provide corrective feedback as needed.

Have students work with a partner or in a small group to complete the other column of the table.

> **For problem 3, find 4 fractions that are equivalent to $\frac{3}{4}$. Write these 4 fractions in the second column of the table for Problem 2 in your Student Booklet.**

Record the fractions students share in the $\frac{3}{4}$ column. When the list exhausts the possibilities that students found, ask whether anyone disagrees with any of the fractions listed as being equivalent to $\frac{3}{4}$. Make changes if needed.

> **The fractions we listed in this column are all equivalent to $\frac{3}{4}$. In your own words, what does that mean?** *(accept reasonable answers, such as represent the same amount as $\frac{3}{4}$)* **How did you find a fraction that is equivalent to $\frac{3}{4}$?** *(accept reasonable answers, such as multiply both the numerator and denominator by the same factor)*
>
> **What would be the simplest form of these fractions?** *($\frac{3}{4}$ is in simplest form)* **Why?** *(because 3 and 4 have no factors in common except 1)*
>
> **Let's look at problem 4. Using the area of the rectangle as the whole, first partition and shade to show $\frac{3}{4}$. Then partition to show another fraction equivalent to $\frac{3}{4}$.**

Have students share the models created.

2. Students will determine how to find a fraction in simplest form.

> **Suppose we started with the fraction $\frac{9}{12}$ from your list of fractions equivalent to $\frac{3}{4}$. Let's think about how you found equivalent fractions: You multiplied.**

What operation do you think we would use to simplify? *(division)*

When we multiplied earlier, did we have to multiply by the same or different numerator and denominator? *(the same)* **What common factor can both 9 and 12 be divided by?** *(3)* **What is 9 divided by 3?** *(3)* **What is 12 divided by 3?** *(4)* **Is there a multiple, other than 1, that 3 and 4 can be divided by?** *(no)* **When we cannot divide the numerator and denominator by a common factor, we are done and the fraction is in simplest form.**

Let's try another fraction. On your whiteboard, write the fraction $\frac{24}{48}$. Are these numbers even or odd? *(even)* **All even numbers can be divided by what factor?** *(2)* **Divide 24 and 48 by 2. What fraction did you write?** *($\frac{12}{24}$)* **Is this in simplest form?** *(no)* **How do you know?** *(can be divided by 2 or 4 or 6)* **Because we can divide by another common factor, this fraction is not in simplest form.**

What are common factors of 12 and 24? *(3, 4, 6, 12)* **Select one of the common factors and divide 12 and 24 by it. Keep dividing until the fraction is in simplest form.**

What is the simplest form of $\frac{12}{24}$? *($\frac{1}{2}$)* **Why?** *(12 and 24 have no common factors?*

Have students practice writing fractions in simplest form. Say and write each fraction, 1 at a time. Ask students to write the fraction on their whiteboard, show how they found the simplest form, and write the simplest form.

Problem 5 in your Student Booklet has a table. For each fraction listed in the first column, you will show how you changed the fraction to the simplest form. Write that in the second column. In the third column, write the fraction in simplest form.

After students finish, ask for the simplest form and write them on table displayed.

How did you know when the fraction was in simplest form? *(the numerator and denominator have no common factors)*

If students disagree, ask for their simplest form, and provide error correction as needed.

Practicing Together

Distribute the pattern blocks to each pair of students. Review the fractional amount each block represents, holding up each block as you describe it. Ask them to find as many combinations of patterns that they can that show 1. They should write the equations in their Student Booklet.

Today, you will work with partner to find combinations of the pattern blocks. We are going to use the area of the pattern blocks to model fractions. The area of the hexagon (hold up the hexagon) is 1. You will find combinations of the other blocks that will cover the area. For example, (model as you talk), if you use 2 trapezoids, you cover the area. The area of each trapezoid represents $\frac{1}{2}$. The equation you will write in your Student Booklet is $\frac{1}{2}+\frac{1}{2}=1$. You will find as many combinations as you can that equal 1 whole. Write the equations of the combinations in your Student Booklet.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 8A to provide extended practice before proceeding to Lesson 9.

Wrapping It Up

Have students turn to Wrapping It Up in their Student Booklet. Select a student to read the problem. Have them work independently or with a partner.

> Miguel said, "I made a fraction equivalent to $\frac{2}{3}$ by adding 3 to the numerator and denominator. $\frac{2}{3} = \frac{5}{6}$." Do you agree with Miguel? Why or why not?

Students should disagree. Their justification may include logical reasoning ($\frac{5}{6}$ is closer to 1 than $\frac{2}{3}$ is) or indicating that finding an equivalent fraction is a multiplicative process, not an additive one.

Lesson 9: Addition and Subtraction of Fractions With Like Denominators

Lesson Objectives	Students add and subtract fractions (including mixed numbers and improper fractions) with like denominators. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students create mathematical models. (SMP 4)	
Vocabulary	**None**	
Requisite Vocabulary	**None**	
Misconception(s)	Students may try to add fractions by adding the numerators and the denominators.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry erase marker • Factor Game Board (1 per pair, see page 150 of Teacher Masters) • Factor Game Record Sheet (1 per pair, see page 151 of Teacher Masters)

		• Red colored pencil

Warming Up

Play the Factor Game (see pages 150-152 of Teacher Masters). Distribute two Factor Game Boards and Factor Game Record Sheets to each pair.

1. You are going to play the Factor Game. This is a two-person game, Player A and Player B. Decide now who will be Player A and who will be Player B. The object of the game is to get as many points as you can to win the game. To show how the game is played, to start, the teacher is Player A and the class is Player B for this first game.

2. Look at the numbers 1 to 30 on the Factor Game Board. Each number will be used only one time during the game, but some numbers may not be used at all.

3. Player A first selects one number from 1 to 30 from the Factor Game Board and circles it using a colored marker. In this case, Player A selects the number 6, which is circled on the game board and also is written in the Player A Score column of the Factor Game Record Sheet.

4. Now it's Player B's turn, who identifies factors of 6 and circles the factors using his or her marker and also writes them in the Factor column of the record sheet. Because any number times 1 equals that number, 1 is circled and written. Are there any other factors of 6? Yes, 3 and 2 are also factors of 6; so Player B circles 3 and 2 on the game board and writes the numbers next to the 1 in the Factor column of the record sheet. Because 1 + 3 + 2 = 6, a 6 is written in the Player B Score column.

5. Play continues until no two numbers remain on the game board that can be multiplied together to form a product that matches the number chosen.

6. Once there are no more factors available, each person determines the point total by adding the numbers in his or her column of the record sheet. The winner is the person with the most points. If more than one game is played, players alternate who

goes first, while maintaining their role as Player A or B.

Play a sample game against all of the students to show how to keep score. Then set a time limit and have the pairs play as many games as they can in the time allowed. If time permits, consider the following debriefing questions.

> **What is your best first move**? (Answers will vary; for example, 4 because the sum of the remaining factors is only 3, 1 and 2; 4 x 1, 2 x 2, so 2 + 1 = 3.) **What number on the first turn that will give your opponent the largest score?** *(30, the score will be 42; 1, 15, 2, 6, 5, 10, 3)* **What is a number you could select on your first turn that would give your opponent the least score?** *(1; the opponent gets 0 points)* **If 20 is picked on the first turn, what will your opponent's score be?** *(22; 10, 2, 5, 4, 1)*
>
> **What is not a good first move?** (Answers will vary; for example, 30 because the sum of the remaining factors is larger than 30.) **What is a number that will give your opponent a score of 21 on the first move?** *(18 with factors of 1, 2, 9, 3, 6)* **What is a number you could select on your first turn that would give your opponent a score of 28?** *(28; 1, 14, 2, 7, 4)*

When finished, have students share their work and check for understanding.

Learning to Solve

TEACHER NOTES

This lesson focuses on unit fractions because the parts are from the same whole, or have like denominators. This is an important concept that students need to understand, yet a nuance that students might miss. If a student wants to add denominators, point out the common whole and/or unit fraction.

If students consider the numerator and denominator to be 2 whole numbers rather than seeing the fraction as one quantity, you may notice that they will add the numerator AND denominator.

1. Students will model subtraction of fractions with like denominators.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the subtraction equation in problem 1. What is the whole for both fractions? *(sixths)*

First, we will draw a model to represent the problem, similar to how we used the number line for addition. We will use the area model to represent the whole. How should we partition the area? *(into sixths)*

Partition and shade the model to represent $\frac{5}{6}$ of the area. What is the unit fraction? *($\frac{1}{6}$)* **What does $\frac{5}{6}$ mean in terms of the unit fraction?** *(5 one-sixths)* **What does $\frac{3}{6}$ mean in terms of the unit fraction?** *(3 one-sixths)*

What does the equation $\frac{5}{6} - \frac{3}{6}$ mean in terms of the unit fraction? *(5 one-sixths minus 3 one-sixths)* **Cross out 3 one-sixth pieces on the model. What is the solution to the difference of $\frac{5}{6} - \frac{3}{6}$?** *(2 one-sixths or $\frac{2}{6}$)*

2. Students will add and subtract improper fractions and mixed numbers with like denominators.

Look at the next problem on your Learning to Solve sheet. What type of fraction is $\frac{7}{6}$? *(improper fraction)*

What is the unit fraction for both fractions? *($\frac{1}{6}$)* How many times was the unit fraction iterated to create $\frac{7}{6}$? *(7 times)* **How many times was the unit fraction iterated to create $\frac{3}{6}$?** *(3 times)*

How many total one-sixth pieces? *(10)* **Write the sum of $\frac{7}{6}$ and $\frac{3}{6}$.** *($\frac{10}{6}$)*

Have students share their answer. If someone has a different answer, ask for the equation and have the student explain how the sum was obtained. Provide error correction as needed.

Select a student to read problem 3.

Follow along as [student] reads problem 3.

Pause for the student to read and for the students to do the problem.

What is the unit fraction? *($\frac{1}{12}$)* **Thinking about just the fraction part of the mixed number, what does $\frac{5}{12}$ mean in terms of unit fraction?** *(5 one-twelfths)* **If you subtract $\frac{1}{12}$ from the 5 one-twelfths, what is the difference?** *(4 one-twelfths)*

Think about the whole numbers. You had 3 feet and need to remove 1 foot. What is the difference? *(2 feet)*

What is $3\frac{5}{12} - 1\frac{1}{12}$? *($2\frac{4}{12}$)*

Is $\frac{4}{12}$ in simplest form? *(no)* **How do you know?** *(the numerator and denominator can both be divided by 4; they share a common factor of 4)* **If we simplify the fraction, does it change how much of the board needs to be removed for the fence?** *(no, it does not change the amount of board to be removed.)*

What is the simplest form of $\frac{4}{12}$? *($\frac{1}{3}$)*

Write the equation in simplest terms *($3\frac{5}{12} - 1\frac{1}{12} = 2\frac{1}{3}$ feet)*

Practicing Together

1. Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.
2. Have students work in pairs or groups to complete the activity sheet.
3. Have pairs or groups share their answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to Wrapping It Up in their Student Booklet.

> Think about two fractions that have a sum of $\frac{1}{2}$. Write an addition equation that has a sum of $\frac{1}{2}$.

As time permits have students share the equations they wrote. Discuss as needed.

Fractions
Lesson 10

Lesson 10: Add and Subtract Fractions With Unlike Denominators

Lesson Objectives	Students add and subtract fractions (including mixed numbers and improper fractions) with unlike denominators. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Multiple, common denominator	
Misconception(s)	Students may try to add fractions by adding the numerators and the denominators.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Close to 1 Game Sheet (1 per pair, see page 154 of Teacher Masters) • Close to 1 Game Cards (1 set per pair, see page 155 of Teacher Masters) • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Answer the questions on your Warming Up sheet in your Student Booklet.**

There are multiple answers. Ask students to share a pair of fractions. Record their answers, using an equation—for example, $\frac{3}{4} + \frac{1}{4} = 1$ or $\frac{7}{4} - \frac{3}{4} = 1$.

If there is any disagreement, discuss the equations with students. Draw a model if necessary to provide a rationale for the correct answer.

Learning to Solve

Teacher Notes

This lesson focuses on finding like denominators, but rather than first finding a common denominator by listing multiples or factors, students draw a model. This shows that the area of the whole has not changed, just the way it is partitioned. Emphasize that when we add or subtract fractions with unlike denominators, we find equivalent fractions, not a new fraction.

> **We have been finding equivalent fractions such as 1/3 = 3/9. We found 3/9 by multiplying 1/3 x 3/3** (or 1). **We have also added and subtracted fractions with like denominators such as 1/8 + 3/8 = 4/8. In order to add or subtract fractions, we have to change the problem so that the fractions have the same denominator.**

> **Let's try 1/8 + 1/4. It's difficult to know what the sum is without changing at least one fraction to have the**

same denominator. Talk with your partner. What would you change in the problem? *(Students may suggest to change 1/4 to 2/8. Other answers are possible such as changing both fractions to have a denominator of 16.)*

Using 8 as a denominator would be great! We can rewrite the equation 1/8 + 2/8 = 3/8.

You are going to play a game that may require you to create equivalent fractions in order to be able to add or subtract.

Play Close to 1. Distribute a Close to 1 Game Sheet and a set of Close to 1 Game Cards (see pages 154 - 155 in Teacher Masters) to each pair.

We are going to play Close to 1. Your pair has a game sheet and a set of cards. Each set of cards has 4 of each number: 1, 2, 3, 4, and 5. You will shuffle the cards, then each of you will draw 4 number cards. The object of the game is to use the numbers to create a sum that is close to 1. You will write your numbers of the cards in the boxes *(one in each box to the left of the = sign)*. **Add the two fractions together, and the sum should be close to 1** *(write the sum in the boxes after the = sign)*. **Whoever gets a sum closer to 1 gets a point. If your sums are the same distance from 1, you both earn a point. You will play 2 rounds, shuffle the cards again and play 2 more rounds. Whoever has the most points at the end of the 4 rounds is the winner.**

Let's try a practice round.

Draw 4 cards. Have students work with a partner to create 2 fractions that they believe will have a sum close to 1. Have pairs share their work.

Find the sums by making equivalent fractions with a common denominator. Decide if the sum is close to 1.

After the first draw, have a few students share their work and check for understanding. When the game is completed, have students share their equations they made. Watch for incorrect equivalent fractions or inaccurate determinations of who was closer to 1.

As time permits, the game can be changed to be subtraction, with a goal of getting a difference close to 0.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs or as a whole class to complete the activity sheet.
2. Have pairs share their answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 10A to provide extended practice before proceeding to Lesson 11.

Wrapping It Up

As time permits replay the Close to 1 game. You may also change the game to Close to 0 if you feel students need more practice with subtraction.

Fractions
Lesson 11

Lesson 11: Multiplication of Fractions

Lesson Objectives	Students use models to show and explain how to multiply fractions. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Factor, product	
Misconception(s)	Students may think that when "of" is used in a word problem, it means to multiply.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 1 set of number cards	• Student Booklet • Red colored pencil • Product Game Sheet (1 per student; see page 156 of Teacher Masters) • Deck of playing cards, minus face cards

Warming Up

Review the partitioning of area models and number lines.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

We have used area models, number lines, and Cuisenaire rods to represent fractions.

Look at the list of fractions on your Warming Up sheet. You will use a model to represent each fraction.

How do you determine how to partition the area of a rectangle or a number line? *(the denominator gives the number of partitions in the whole)*

Once you partition, you can shade if using an area model or draw the length if using a number line. How do you know how much to shade, or the distance you need to show? *(the numerator tells the number of parts)*

When the denominator is large, what can be said about the area or the length of each part? *(it is smaller)* **What about when the denominator is small?** *(the area or length of the partition is larger)*

After you model each fraction, write the benchmark fraction it is closest to.

Have students work and then show their models and share their benchmark fractions. Ask whether anyone disagrees and why.

Learning to Solve

Teacher Notes

A fraction multiplication problem, for example $\frac{1}{2}$ x $\frac{3}{4}$ is often read as $\frac{1}{2}$ of $\frac{3}{4}$. The use of the word "of" may cause students to overgeneralize that "of"' means to multiply. As you go through the lesson, watch for signs of students making this overgeneralization.

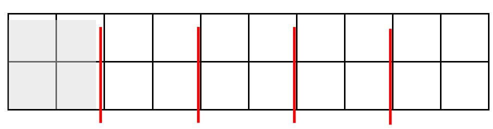
1. Students will multiply fractions, using models.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the first model on the sheet. We have used an area model before, but this one looks a little different. In this model, the whole is area of the rectangle that is made up of 20 area units.

Let's show $\frac{1}{5}$ of the whole, or $\frac{1}{5}$ of the 20 area units. How do you think we can show $\frac{1}{5}$ of the whole? *(shading)*

Let's partition the 20 area units into equal-sized groups. How many groups do we need? *(5)* **How do you know?** *(the denominator)* **Partition the whole into 5 equal groups. Now shade to show $\frac{1}{5}$ of the area.**



What is the whole? How many area units? *(20 area units)* **Write it.**

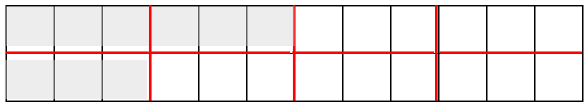
How many area units are in 1 one-fifth piece? *(4 area units)* **What is $\frac{1}{5}$ of the whole or the area?** *(4 area units)*

Look at the next model. What is the whole? How many area units? *(24 area units)* **Write it. For this model, we need to show $\frac{3}{8}$ of the whole.**

How do we partition the whole? *(into eighths)* **How do you know?** *(the denominator)*

Partition the whole into eighths. How will you determine how to partition? *(accept reasonable answers, such as divide 24 by 8)* **How many area units in each eighth?** *(3 area units)*

Shade 3 one-eighth parts.



What is $\frac{3}{8}$ of the whole or the area? *(9 area units)*

Look at problem 3.

Select a student to read the problem.

Follow along as [student] reads.

Pause for the student to read.

This problem is similar to the previous models, but it now contains a situation we haven't worked with yet. What is it asking us to find? *(how many cars are blue)* **What is our whole?** *(15 model cars)*

First, estimate: Out of 15 cars, about how many are blue? How did you estimate? *(accept reasonable answers, such as 11 because I kind of pictured in my mind where $\frac{2}{3}$ of 15 might be on a number line)*

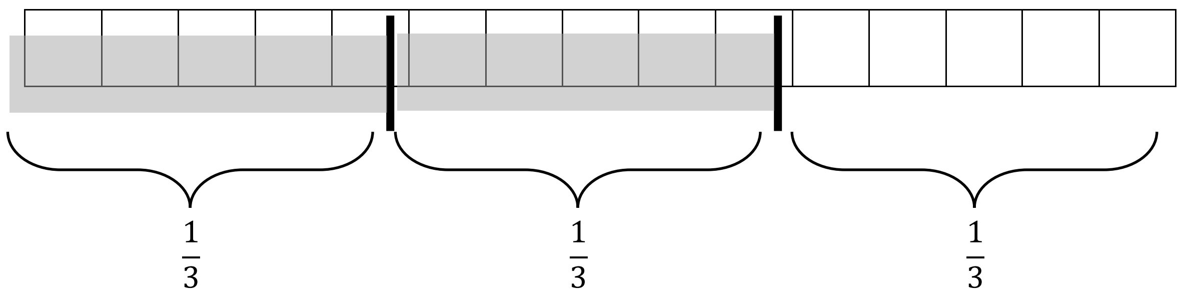
We will create a model of this problem. What should we do first? *(draw a model with 15 partitions)*

Make a rectangle using the gridded area in your Student Booklet. How many area units should be in your model? *(15)* **Make this model in your booklet.**

What we should do to find out what $\frac{2}{3}$ of the model looks like? *(partition the 15 area units into thirds)*

How did you know to partition into thirds? *(the denominator)* **Partition the 15 area units into thirds.**

How many area units in 1 third? *(5)* **Shade to show what $\frac{2}{3}$ of the model of 15 cars looks like.**



How many blue cars does James have in his model car collection? *(10)* **What is $\frac{2}{3}$ of 15?** *(10)* **Is this reasonable? Does it match your estimate? Is it more than or less than half the cars**?

Practicing Together

Play the Product Game. Use a regular deck of cards, without the face cards. An Ace will be 1. Distribute a Product Game Sheet (see page 156 of Teacher Masters) to pairs or individuals.

We are going to play the Product Game. I will draw 4 number cards from my deck of cards. You will write the numbers in the blanks to make 2 fractions. Then, you will find the product. You get points depending on the product. Write the number of points in the space to the right. If your product is greater than or equal to 1, you will score 5 points. If your product is greater than $\frac{1}{2}$ but less than 1, you will score 3 points. If your product is less than or equal to $\frac{1}{2}$, you will score 1 point. We will play as many rounds as we can in the time we have. At the end of time, whoever has the highest score is the winner.

Shuffle the deck; draw and say the numbers. Do a practice round. You may want to do additional practice rounds. Then, allow time for students to play the game. On the first two draws, have a few students share their work and or monitor pairs to check for any inaccuracies. Do so periodically throughout the games. Reshuffle the deck as needed.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Replay the Product Game as time allows.

Fractions
Lesson 12

Lesson 12: Multiplication of Fractions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students multiply fractions.
Students make sense of problems and persevere in solving them. (SMP 1)
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Product</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may think that when “of” is used in a word problem, it always means to multiply.
Students may also think that multiplication should always result in a product larger than the factors.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)
• Deck of playing cards, minus face cards</td><td>• Student Booklet
• 2 different colored pens, pencils, or markers
• Red colored pencil
• Product Game Sheet (1 per student; see page 156 of Teacher Masters)</td></tr>
</table>

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Match each model by drawing a line to the expression that it represents.

Ask a student to display his or her answers.

Learning to Solve

Teacher Notes

Students may want to find common denominators to multiply because that was the process when they were adding and subtrcting. While it is not an efficient method, this does work but it may create additional steps for students. If students try to do this, you may want to model so that they generalize that numerators are multiplied and denominators are multiplied.

1. Students will multiply fractions, using models.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the first problem. Follow along as [student] reads.

Read the problem.

What is the whole in this problem? *(a gallon of paint)*

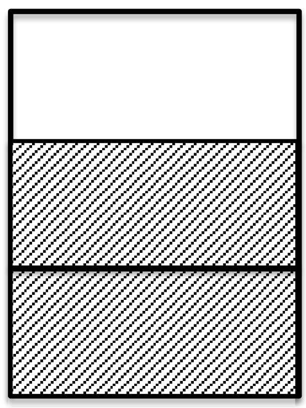
If she will use $\frac{3}{4}$ of $\frac{2}{3}$ of a gallon, is that amount closer to using the entire $\frac{2}{3}$ of the gallon or is it closer to $\frac{1}{2}$ of

the $\frac{2}{3}$ gallon of paint? *($\frac{1}{2}$)* **Estimate how much of a gallon Marcia will use.**

As students share their estimates, write them on the board.

Let's draw a model to represent this problem. The area of the square on your sheet will represent a gallon of paint.

Marcia has $\frac{2}{3}$ of a gallon of paint. Represent this on your model. How do we partition it? *(into thirds)* **How do you know?** *(denominator tells the number of partitions of the whole)*

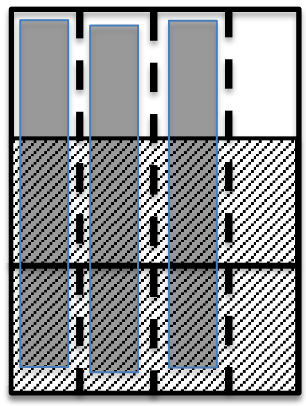


To finish painting her room, Marcia needs $\frac{3}{4}$ of the $\frac{2}{3}$ gallon of paint represented in our model.

Are we using the whole model to think about the $\frac{3}{4}$ or just the shaded portion, the $\frac{2}{3}$? *(just the shaded portion)*
Although we want to know how much $\frac{3}{4}$ of the $\frac{2}{3}$ is, we always need to compare it to the whole.

We partitioned $\frac{2}{3}$ horizontally. To help us see the model, using a different colored pen, partition the whole vertically into fourths.

Now shade to represent $\frac{3}{4}$.



$\frac{3}{4}\ of\ \frac{2}{3}$

How many equal-sized parts did we make in the whole? *(12)* **What part is shaded twice, for both $\frac{3}{4}$ and $\frac{2}{3}$?** ($\frac{6}{12}$) **Is there another way to think about the $\frac{6}{12}$?** *(yes)* **What is another way to describe $\frac{6}{12}$?** ($\frac{1}{2}$) **Are $\frac{6}{12}$ and $\frac{1}{2}$ equivalent?** *(yes)* **We can say $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent because they represent the same area of the rectangle.**

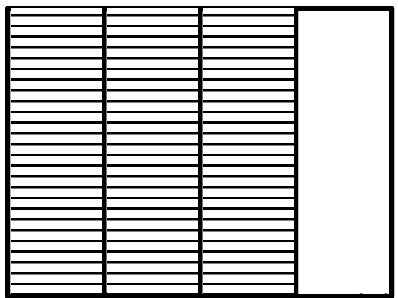
2. Students model the algorithm for multiplication of fractions.

Look at the expression in problem 2. How would you model the expression? *(answers will vary, such as shown in the Teacher Masters)*

Solicit students' responses and write them on the board or document camera. As you go through the next portion of the lesson, compare students' responses to this process.

To model this expression, would we draw a representation of the $\frac{3}{5}$ first or of the $\frac{3}{4}$? ($\frac{3}{4}$) **Why?** *(because we want to find what $\frac{3}{5}$ of $\frac{3}{4}$ is, so we have to draw the $\frac{3}{4}$ first)*

In your Student Booklet, draw an area model of $\frac{3}{4}$, using the rectangle provided.

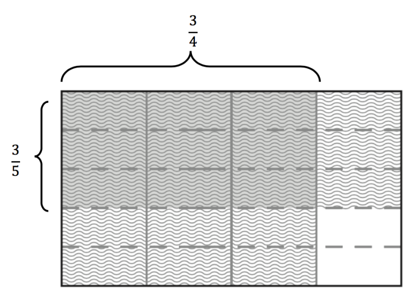


What is the whole? *(the area model, rectangle)*

We need to find what $\frac{3}{5}$ of $\frac{3}{4}$ is. Using our model of $\frac{3}{4}$, what should we do to show $\frac{3}{5}$ of the $\frac{3}{4}$? *(model $\frac{3}{5}$ on top of the $\frac{3}{4}$)*

Represent $\frac{3}{5}$ of $\frac{3}{4}$ on your model, extending the partitioning lines into the empty space.

Model the partitioning on the board.



Now the whole is partitioned into equal-sized parts. How many total equal-sized parts? *(20)*

What area of the rectangle is represented by $\frac{3}{5}$ of $\frac{3}{4}$? *($\frac{9}{20}$)*

$\frac{3}{5}$ of $\frac{3}{4}$ can be solved by the multiplication problem $\frac{3}{5} \times \frac{3}{4}$

$$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$$

Our model shows us that $\frac{3}{5} \times \frac{3}{4}$ is equal to $\frac{9}{20}$. Can $\frac{9}{20}$ be simplified? *(no)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Complete the first problem using a number line to model. If you feel students are more success with an area model, you may want them to complete using an area model, like in Learning to Solve, in groups or with the whole class.

   Similar to the area model, we can also model multiplication of fractions on a number line. What is the whole on this number line? *(0 to 1)* **Look at the first fraction, $\frac{2}{5}$, how do we partition the length between 0 and 1?** *(partition it into 5 equal-sized parts)*

   Now draw a line or small rectangle to show the length. Next, we have to partition each fifth part into thirds, because the problem is trying to determine how many two-thirds of two-fifths. How many total partitions from 0 to 1? *(15)* **What is the length, or how many parts did we shade twice?** *(4)* **What is $\frac{2}{5} \times \frac{2}{3}$?** *($\frac{4}{15}$)*

2. Discuss the answers.

3. Have the students do the second problem on their own and share their answers. Check for understanding and provide error correction as needed.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Continue to play the Product Game from Lesson 11. Use a regular deck of cards, without the face cards. An Ace will be 1. Distribute a Product Game Sheet (see page 156 of Teacher Masters) to pairs or individuals. Play as many rounds as you can in the remaining time.

> **We are going to play the Product Game. I will draw 4 number cards from my deck of cards. You will write the numbers in the blanks to make 2 fractions. Then, you will find the product. You get points depending on the product, and write the number of points in the space to the right. If your product is greater than or equal to 1, you will score 5 points. If your product is greater than $\frac{1}{2}$ but less than 1, you will score 3 points. If your product is less than or equal to $\frac{1}{2}$, you will score 1 point. We will play as many rounds as we can in the time we have remaining. At the end of time, whoever has the highest score is the winner.**

Shuffle the deck; draw and say the numbers and allow time for students to work. On the first two draws, have a few students share their work and check for understanding. Do so periodically throughout the games. Reshuffle the deck as needed.

Fractions
Lesson 13

Lesson 13: Division of Fractions

Lesson Objectives	Students use models to show and explain division of fractions. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Dividend**: the number being divided, the whole amount **Divisor:** the number or quantity that divides the dividend **Quotient**: the answer to a division problem	
Requisite Vocabulary	**None**	
Misconception(s)	Students do not understand that a fraction can be a representation of division. Students overgeneralize the rule and will “flip” the second fraction without understanding the reason to multiply by the reciprocal.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Deck of playing cards, minus face cards	• Student Booklet • Red colored pencil • Product Game Sheet (1 per student; see page 156 of Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet. Solve each problem.

Have students work and then give their answers and explain their reasoning.

Look at the first problem from the Warming Up sheet. A daily serving of fruit is 2 apples. How many servings can you make from a group of 14 apples?

What is the problem asking? *(how many groups of 2 were in 14)* What was the answer? *(7)* **How did you solve?** *(division problem: 14 ÷ 2 = 7)*

In division, the answer is called a quotient. What is the quotient? *(7)* **The dividend is the whole, the amount you started with. In this problem, what is the dividend?** *(14)* **The divisor is the number being divided into the dividend. What is the divisor?** *(2)*

Look at the second problem. A daily serving of cookies is $\frac{1}{2}$ of a cookie. How many servings of $\frac{1}{2}$ cookies are there in 6 cookies?

What did you have to find? *(how many groups of $\frac{1}{2}$ were in 6)* **What was the answer?** *(12)* **The equation for this problem is**

$$6 \div \frac{1}{2} = 12$$

Learning to Solve

TEACHER NOTES

Students have often been given acronyms (like Keep-Flip-Change) to remember how to perform the algorithm. The lessons on division focus on a conceptual understanding before an algorithm will be used.

1. Students will use models to divide fractions. Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Tell students to follow along and fill in their sheet as the lesson progresses.

Look at problem 1 on the Learning to Solve sheet.

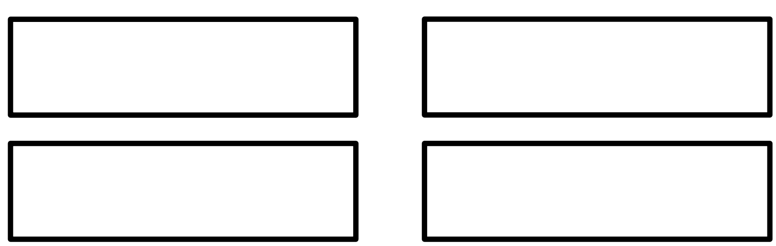
Read the problem to students or have a student read it.

Ask students for an estimate and their reasoning. Do not reply to the correctness of their answer or the logic of their reasoning.

First, estimate how many people are in the group. Let's use an area model for this problem. Before we model, write the division expression for this problem: $4 \div \frac{2}{3}$.

Now, we will draw a model to represent the 4 pizzas.

Display a model similar to the following.



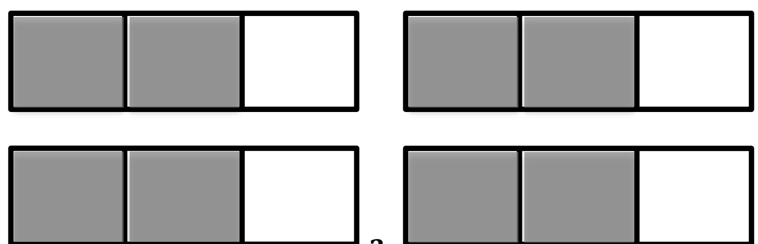
Why do you think that I chose to use the area of a rectangle to model my pizzas instead of a circle?
(sometimes circles are difficult to use when representing unit

fractions such as $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc.; creating equal-size parts can be an issue)

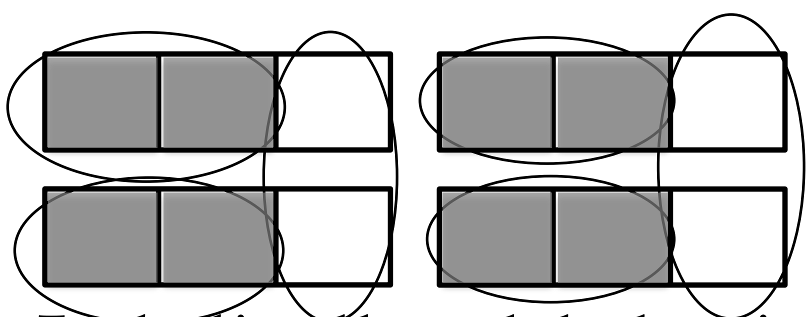
What is the unit fraction of $\frac{2}{3}$? *($\frac{1}{3}$)*

How many equal-sized parts will each rectangle be partitioned into? *(3)*

Represent $\frac{2}{3}$ of a medium pizza on each rectangle in your model.



Circle each group of $\frac{2}{3}$. How many total groups of $\frac{2}{3}$ are in your model? *(6)*



$4 \div \frac{2}{3} = 6$

To solve this problem, we had to determine how many groups of $\frac{2}{3}$ are in 4 pizzas. We took 4 medium pizzas, partitioned them into thirds, and divided them into groups of $\frac{2}{3}$.

This problem is a division problem, just like the apple problem you solved in the warm up.

In the apple problem, 14 ÷ 2 = 7, what did you notice about the quotient as compared to the dividend? *(the quotient is smaller than the dividend)*

In the pizza problem, $4 \div \frac{2}{3} = 6$, what did you notice about the quotient as compared to the dividend? *(the quotient is greater than the dividend)*

2. Have students turn to the Generalization page in the Notes section of their Student Booklets. Write the generalizations on the white board for students to copy.

Turn to the Notes section in your Student Booklet. A generalization is a statement that describes patterns and relationships.

When we create generalizations, we can add them to the Student Booklets. Now, we have two generalizations for division of fractions.

Here are 2 important generalizations about division:

1. Division does not always result in a quotient that is smaller than the dividend or divisor.

We've already shown this to be true for the dividend. If we divide $\frac{2}{3} \div \frac{2}{3} = 1$. Here, the quotient is greater than the divisor. Now we divide $\frac{1}{8} \div \frac{2}{3} = \frac{3}{15}$. Here, what do we notice when we compare the quotient to the divisor? *(the quotient is less than the divisor)*

2. **When you solve a division problem, whether the problem contains whole numbers or fractions, you are finding how many groups of a quantity are in the whole OR, the quantity in a group.**

Write these generalizations in your Student Booklet.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the activity sheet in groups or with the whole class if more appropriate.

2. Discuss the answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Continue to play the Product Game from Lessons 11 and 12. Use a regular deck of cards, without the face cards. An Ace will be 1. Distribute a Product Game Sheet (see page 156 of Teacher Masters) to pairs or individuals. Play as many rounds as you can in the remaining time.

> **We are going to play the Product Game. I will draw 4 number cards from my deck of cards. You will write the numbers in the blanks to make 2 fractions. Then, you will find the product. You get points depending on the product, and write the number of points in the space to the right. If your product is greater than or equal to 1, you will score 5 points. If your product is greater than $\frac{1}{2}$ but less than 1, you will score 3 points. If your product is less than or equal to $\frac{1}{2}$, you will score 1 point. We will play as many rounds as we can in the time we have remaining. At the end of time, whoever has the highest score is the winner.**

Shuffle the deck; draw and say the numbers and allow time for students to work. On the first two draws, have a few students share their work and check for understanding. Do so periodically throughout the games. Reshuffle the deck as needed.

Lesson 14: Division of Fractions

Lesson Objectives	Students divide fractions, using an algorithm. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Common denominator	
Misconception(s)	Students do not understand that a fraction can represent division. Students overgeneralize the rule and will “flip” the second fraction without understanding the reason to multiply by the reciprocal.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • 4 dice (optional)	• Student Booklet • 4 dice per pair (optional) • Red colored pencil • 4 to Go Game Sheet (1 per pair, see page 157 of Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet located in your Student Booklet and solve the problems.

Call upon students to share their answers and check for understanding.

Learning to Solve

TEACHER NOTES

The purpose of this lesson is for students to see the division model for fractions. Students should complete the model alongside the teacher and not "flip the second fraction and multiply across." Although students may have been taught this generalization in core instruction, it is important for them to develop the conceptual understanding through the visual model.

The common denominator method is a more conceptual approach to the "flip and multiply" method. It is important that students understand that the quotient in the denominator is 1 which results in dividing the numerator by 1.

1. Students will model division of a fraction by a fraction.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

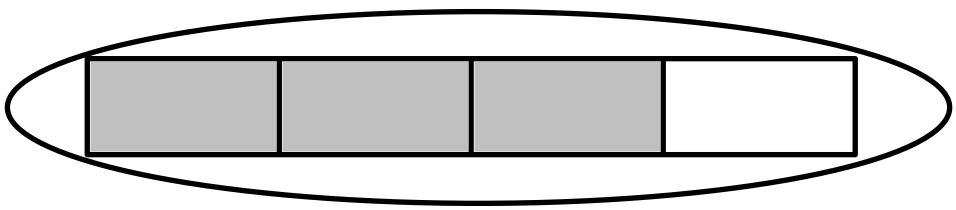
Look at problem 1. Before we begin, let's estimate. Think about benchmark fractions. What benchmark

fraction is $\frac{3}{4}$ closest to? *(1)* **And $\frac{1}{2}$ is already a benchmark fraction. What is 1 divided by $\frac{1}{2}$?** *(2)*

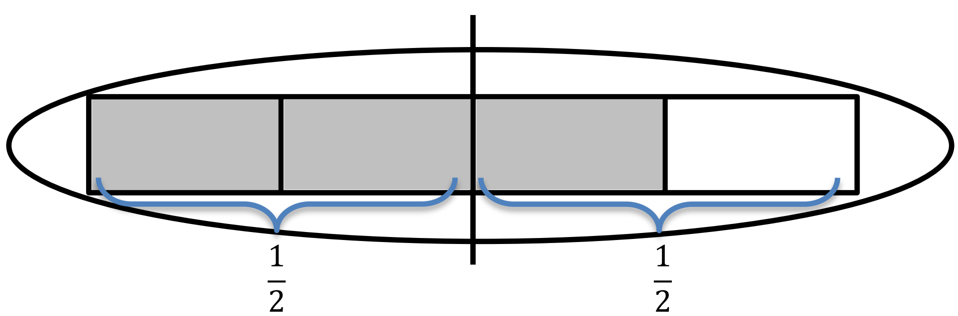
Let's model $\frac{3}{4} \div \frac{1}{2}$. In division, this means that $\frac{3}{4}$ is partitioned, or divided, into groups of how many? *($\frac{1}{2}$)* **Another way to think about division of fractions is how many $\frac{1}{2}$ units fit inside of $\frac{3}{4}$ of a whole.**

What is our first step? *(draw a model of $\frac{3}{4}$)* **Using an area model, represent $\frac{3}{4}$ and circle the whole.**

Display a model similar to the following.



To determine the size, or area, of a $\frac{1}{2}$ unit, I have to compare it to the whole. What is $\frac{1}{2}$ of the whole? *(2 one-fourth pieces)* **Label each $\frac{1}{2}$ of the whole on your model. A $\frac{1}{2}$ unit is the size of 2 one-fourth pieces.**



How many $\frac{1}{2}$ unit pieces do you see in the model? In other words, how many one-halves are there in $\frac{3}{4}$ part

of the whole? *(1 whole unit and $\frac{1}{2}$ of another)* **We have 1 whole unit, that is, $\frac{1}{2}$. There is one part left over. Because we needed 2 parts for each group, we have a half of a part remaining. Therefore, $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$. Write $1\frac{1}{2}$ next to answer.**

As you are saying this, indicate where it is found in the model.

This shows that $\frac{3}{4} \div \frac{1}{2}$ is equal to $1\frac{1}{2}$. Is this close to our estimate of 2?

2. Students will use an algorithm to divide fractions.

Now look at problem 2 on the Learning to Solve sheet.

We showed through the use of a model that $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$ units. Write the equation to the left of the area model.

As we have done previously with other operations such as addition and subtraction of fractions, we want to develop an efficient, or faster, way to divide fractions. One method that we can use is a common denominator method. It helps us to compare a part to a whole. Let's try it with the problem we just completed.

Looking at our example, what is the common denominator for $\frac{3}{4}$ and $\frac{1}{2}$? *(4)* **What are the equivalent fractions?** *($\frac{3}{4}$ and $\frac{2}{4}$)*

Substitute the equivalent fractions for the fractions in the problem, $\frac{3}{4} \div \frac{2}{4}$. Write the division equation with the equivalent fractions. Now divide the denominators, what is 4 divided by 4? *(1)* **What is the division expression for the numerators?** *(3 divided by 2)* **We can also write this as a fraction, $\frac{3}{2}$. Change this**

improper fraction to a mixed number. What is $\frac{3}{2}$ as a mixed number? *($1\frac{1}{2}$)*

$$\frac{3}{4} \div \frac{2}{4} = \frac{3 \div 2}{4 \div 4} = \frac{3 \div 2}{1} = 3 \div 2 = \frac{3}{2} = 1\frac{1}{2}$$

Is this the same answer as our model? *(yes)*

Now look at problem 3. Using the common denominator method, find the quotient of $\frac{3}{5} \div \frac{3}{10}$.

Have students show the process on the board.

$$\frac{3}{5} \div \frac{3}{10} = \frac{6}{10} \div \frac{3}{10} = \frac{6 \div 3}{10 \div 10} = \frac{6 \div 3}{1} = 6 \div 3 = 2$$

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the Practicing Together sheet.
2. Discuss the answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 14A to provide extended practice before proceeding to Lesson 15.

Wrapping It Up

Play 4 to Go in pairs. Distribute the 4 to Go sheet (see page 157 of Teacher Masters) and a set of 4 dice to each pair.

This game can be played in one of two ways. First, the teacher can roll the dice. Second, student pairs can roll the dice. Instructions should be modified depending on the way the game is played.

Have students turn to the Wrapping It Up page in their Student Booklet. You roll the dice and all students use your roll, or students can play in pairs and alternate rolling the dice.

We are going to play 4 to Go in the time we have remaining. One of you will be Player A and the other Player B; decide now. *(pause)* **You will take turns rolling your 4 dice.** *(Or, I will roll the 4 dice)* **You will both place your numbers in the boxes on the game sheet, Player A or Player B, to make 2 fractions. Perform the division. You get points depending on the quotient; write the number of points in the space to the right. If your quotient is less than or equal to $\frac{1}{2}$, you score 1 point. If your quotient is greater than $\frac{1}{2}$ but less than 1, you score 3 points. If the quotient is greater than or equal to 1, you score 5 points. The person with higher score wins.**

On the first two rolls, have a few students share their work and check for understanding. When finished, have students share their work.

Fractions
Lesson 15

Lesson 15: Application of Operations on Fractions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students apply the operations of addition, subtraction, multiplication, and division on fractions set in contextual problems.
Students make sense of problems and persevere in solving them. (SMP 1)
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Area</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may try to use key words to solve application problems.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)
• Concept Map (from Lesson 1)</td><td>• Student Booklet
• Red colored pencil</td></tr>
</table>

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve each problem.

Provide time for students to work.

How did you solve? Did anyone solve differently?

Learning to Solve

Students will apply knowledge of addition, subtraction, multiplication, and division of fractions.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at the first problem. Follow along as [student] reads.

Pause for student to read.

What is the whole? *(the playground, 1)*

First, estimate the area of the playground that Francis and Gabe shoveled. *(about $\frac{1}{2}$ of the playground)*

What is the actual area of the playground that Francis and Gabe shoveled? *($\frac{11}{15}$)*

How did you solve? Did you draw a model? Did anyone solve differently?

If Francis, Gabe, and Pam shovel the snow from the area of the playground that they are responsible for, what area remains for Elizabeth to shovel? First, estimate. What area remains, about how much of the playground? *(less than half)*

What area is left for Elizabeth to shovel? *($\frac{1}{15}$ of the playground)*

How did you solve? Did you draw a model? Did anyone solve differently?

Now look at problem 2. What values will make each of the 3 equations true? Work with your partner to solve.

What missing values did you use?

Discuss students' answers.

Now look at the last problem.

Read the problem.

What is the whole? *(12 miles)* **How can you solve?**

Discuss students' answers.

Is there another way? Does anyone disagree?

Solve the problem now.

Provide time for students to work in pairs or groups.

How many miles will you have biked when you stop to rest? *(8)*

Have students show their models and explain how they solved.

Practicing Together

There is no Practicing Together section in this lesson to allow time to complete the Learning to Solve sheet.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the fraction concept map completed in Lesson 1. Ask the following questions and update, change, and add to the concept map.

1. **Can we add anything to our concept map?**
2. **What do we know about adding and subtracting fractions?**
3. **What do we know about multiplying and dividing fractions?**
4. **What could we add to describe comparing fractions, mixed numbers, improper fractions, and simplifying fractions?**
5. **Can we add examples to help explain or highlight the ideas on our map?**

Have students share their work and check for understanding.

Progress Monitoring Schedule

BEFORE	**AFTER**	**AFTER**
Lesson 1: Pre-assessment **Module Check Form A**	Lesson 7: Mid-assessment **Module Check Form B**	Lesson 15: Post-assessment **Module Check Form C** ✓

Appendices

FRACTIONS

Fractions
Lesson 6A

Lesson 6A: Equivalent Fractions

Lesson Objectives	Students identify and create equivalent fractions, using concrete and pictorial models. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Equivalent:** same portion or quantity	
Requisite Vocabulary	Mixed number, improper fraction, area model	
Misconception(s)	Students may think that $\frac{1}{2}$ of something is always equivalent to $\frac{2}{4}$ of something else, without considering that the whole must be the same to compare fractions.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Cuisenaire rods	• Student Booklet • Cuisenaire rods (1 set per student pair) • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Brian, Donna, and Sally finished an after-school run. They ran the following distances in the same amount of time: Brian ran $\frac{12}{8}$ of a mile, Donna ran $\frac{6}{4}$ of a mile, and Sally ran $\frac{3}{2}$ of a mile.

What is our whole in this problem? *(1 mile)* **How is that whole represented on the number line?** *(length between 0 and 1)*

How should we partition Brian, Donna, and Sally's number lines? *(8 equal parts, 4 equal parts, 2 equal parts, respectively)*

Partition and draw the length of each person's after-school run. Remember, that these fractions indicate a length and not just a point on a number line.

Pause for students to work in pairs or small groups.

What do you notice about the distances Brian, Donna, and Sally ran? *(they all ran the same distance)*

The grid showed 8 partitions on the number line. Look at the 3 fractions, $\frac{12}{8}$, $\frac{6}{4}$, and $\frac{3}{2}$. What relationship do you see between the fractions and the number of partitions? *(accept reasonable answers, such as the denominator for Brian's distance is 8, so the partition must be in 8 equal parts; 8 is a multiple of all the denominators)*

Learning to Solve

TEACHER NOTES

Students may begin to overgeneralize the concept of 1 whole, rather than develop the understanding of the size of the whole and the idea of how "big" the pieces are within the whole (e.g., $\frac{1}{4}$ of a watermelon is larger than $\frac{1}{4}$ of a candy bar). Be sure that students understand what is being compared.

Use precise vocabulary when discussing the attribute being used: distance, length, and/or size of the fractional part. When using Cuisenaire rods, refer to the length of the rod. If a student responds, "The white rod is $\frac{1}{9}$ of the blue rod," restate the response as "The length of the white rod is $\frac{1}{9}$ of the length of the blue rod."

1. Students review the concept of equivalency and equal shares.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Students may work in pairs or as entire class if appropriate. Select a student to read the problem.

   Look at the problem. Follow along as [student] reads.

   Work with your partner and draw a diagram in your Student Booklet to represent the amount of cookie each person will receive. How many people are sharing the 6 cookies? *(4)* **How do you know?** *(James and 3 friends)* **Do you think it is important to share the cookies equally, meaning that each person gets the same size? Why?** *(answers will vary, such as to be fair)*

What fraction of a cookie will each child receive? *(possible answers include $\frac{6}{4}$, $\frac{3}{2}$, $1\frac{1}{2}$)* **What did your diagram look like?** *(allow a few students to share)*

Write the answers on the board: $\frac{6}{4}$, $\frac{3}{2}$, and $1\frac{1}{2}$.

We have 3 possible answers: $\frac{6}{4}$, $\frac{3}{2}$, and $1\frac{1}{2}$.

Do the answers represent different quantities or amounts? Would each person get the same amount of cookie or different amounts? *(they represent the same quantity or amount, but they look different)*

We can say that $\frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$. Using your diagram, how can you explain that these fractions are all equal? *(answers will vary, such as all represent an equal amount of cookie)*

Give students the opportunity to justify and explain their answer. They should explain that each child would receive in total 1 whole cookie and a half of a cookie.

2. Students identify equivalent fractions.

Give each student pair a set of Cuisenaire rods. Model as the lesson progresses. Write "$\frac{3}{9}$ [ ] $\frac{1}{3}$" on the board.

Another word for "equal" is "equivalent." We will find equivalent fractions. What does "equal" mean? *(the same as)* **Equivalent fractions represent the same quantity, like our cookie example. Even though I can write 3 different fractions, each person would receive the same amount of cookie.**

Let's use Cuisenaire rods to justify, or prove, that these fractions are equivalent. Find 1 blue Cuisenaire rod from your student set. The length of this blue rod represents the length of the whole, or 1.

What is the unit fraction of $\frac{3}{9}$? *($\frac{1}{9}$)*

With your partner, determine which rod's length will represent $\frac{1}{9}$ of the length of the blue rod. Put this rod above the blue rod that represents your whole.

Which rod's length represents $\frac{1}{9}$ of the length of the blue rod? *(the white rod's length)* **How do you know?** *(it takes 9 white rods to equal the whole, or the length of the blue rod)*

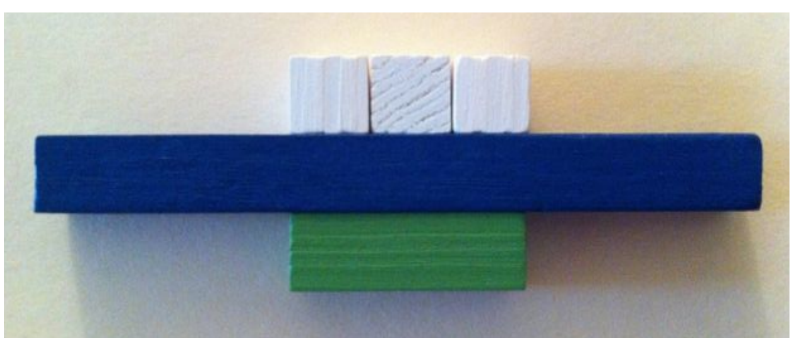
How do we represent $\frac{3}{9}$? *(3 white rods)* **Put 3 white rods above the blue rod.**

Leave the blue and white rods in place on your table. Find a rod's length that represents $\frac{1}{3}$ of the length of the blue rod and place it below the blue rod.

Which rod is $\frac{1}{3}$ of the length of the blue rod? *(the light-green rod)*

Now compare $\frac{3}{9}$ and $\frac{1}{3}$.

To compare any 2 fractions, what do they have to have in common? *(same whole)*



Do these 2 fractions refer to the same whole? *(yes)*

What is the whole? *(the length of 1 blue rod)*

Compare the length of the rods that show $\frac{3}{9}$ and $\frac{1}{3}$. Do these rods represent the same fraction of length of the blue rod? *(yes)*

Because these rods represent the same fraction of length of the blue rod, the fractions $\frac{3}{9}$ and $\frac{1}{3}$ are equivalent.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the activity sheet in groups or with the whole class if more appropriate.
2. Discuss the answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

1. Have students complete the Wrapping It Up sheet in their Student Booklet.

2. If time allows, have students share their work and check for understanding.

Fractions
Lesson 8A

Lesson 8A: Simplify Fractions

Lesson Objectives	Students simplify fractions. Students model fractions in multiple ways. (SMP 4) Students reason quantitatively with fractions. (SMP 2)	
Vocabulary	None	
Requisite Vocabulary	Equivalent fractions, factor, fraction in simplest form, greatest common factor, multiple	
Misconception(s)	Students often think that simplifying a fraction, such as $\frac{2}{4}$, means dividing the fraction by 2, rather than dividing by $\frac{2}{2}$, or 1.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry erase marker (2 different colors for each pair) • Factor Game Board (1 per pair, see page 150 of Teacher Masters) • Factor Game Record Sheet (1 per pair, see page 151 of Teacher Masters) • Red colored pencil

Warming Up

Play the Factor Game (see pages 150 – 151 of Teacher Masters). Distribute two Factor Game Boards and Factor Game Record Sheets to each pair.

1. You are going to play the Factor Game. This is a two-person game, Player A and Player B. Decide now who will be Player A and who will be Player B. The object of the game is to get as many points as you can to win the game. To show how the game is played, to start, the teacher is Player A and the class is Player B for this first game.

2. Look at the numbers 1 to 30 on the Factor Game Board. Each number will be used only one time during the game, but some numbers may not be used at all.

3. Player A first selects one number from 1 to 30 from the Factor Game Board and circles it using a colored marker. In this case, Player A selects the number 6, which is circled on the game board and also is written in the Player A Score column of the Factor Game Record Sheet.

4. Now it's Player B's turn, who identifies factors of 6 and circles the factors using his or her marker and also writes them in the Factor column of the record sheet. Because any number times 1 equals that number, 1 is circled and written. Are there any other factors of 6? Yes, 3 and 2 are also factors of 6; so Player B circles 3 and 2 on the game board and writes the numbers next to the 1 in the Factor column of the record sheet. Because 1 + 3 + 2 = 6, a 6 is written in the Player B Score column.

5. Play continues until no two numbers remain on the game board that can be multiplied together to form a product that matches the number chosen.

6. Once there are no more factors available, each person determines the point total by adding the numbers in his or her column of the record sheet. The winner is the person with the most points. If

more than one game is played, players alternate who goes first, while maintaining their role as Player A or B.

Play a sample game against all of the students to show how to keep score. Then set a time limit and have the pairs play as many games as they can in the time allowed. If time permits, consider the following debriefing questions.

What is your best first move? *(Answers will vary; for example, 4 because the sum of the remaining factors is only 3, 1 and 2; 4 x 1, 2 x 2, so 2 + 1 = 3.)* **What number on the first turn that will give your opponent the largest score?** *(30, the score will be 42; 1, 15, 2, 6, 5, 10, 3)* **What is a number you could select on your first turn that would give your opponent the least score?** *(1; the opponent gets 0 points)* **If 20 is picked on the first turn, what will your opponent's score be?** *(22; 10, 2, 5, 4, 1)*

What is not a good first move? *(Answers will vary; for example, 30 because the sum of the remaining factors is larger than 30.)* **What is a number that will give your opponent a score of 21 on the first move***? (18 with factors of 1, 2, 9, 3, 6)* **What is a number you could select on your first turn that would give your opponent a score of 28?** *(28; 1, 14, 2, 7, 4)*

When finished, have students share their work and check for understanding.

Learning to Solve

Teacher Notes

Use the word "simplify," rather than "reduce." "Reduce" is often interpreted to mean "make smaller," but fractions written in simplest form are equivalent.

1. Students will simplify fractions that are divisible by multiple factors.

Give each student a whiteboard.

Write $\frac{6 \div 2}{12 \div 2} = \frac{3}{6}$ on the board.

> **Is $\frac{3}{6}$ the simplest form of $\frac{6}{12}$? Write "yes" or "no" on your whiteboard.**
>
> **What does "simplest form" mean?** *(it means that the numerator and denominator do not have a common factor)* **How do you know when a fraction is in simplest form?** *(when there are no more common factors)*

Have students hold up their whiteboard to see how many students agree or disagree. Have students provide their definition of simplest form and share how they know when a fraction is in its simplest form.

> **Can 3 and 6 be divided by the same factor again?** *(yes)* **What factor can 3 and 6 be divided by?** *(3)* **What is the simplest form of $\frac{3}{6}$?** *($\frac{1}{2}$)*
>
> **What is the simplest form of $\frac{12}{16}$?** *($\frac{3}{4}$)* **Write it on your white board.**

Have students hold up their whiteboard to share what they wrote.

> **What did you divide the numerator and denominator by? What common factor? Did anyone simplify the fraction differently?** *(some students may have divided by $\frac{4}{4}$, while others may have divided by $\frac{2}{2}$ and then divided by $\frac{2}{2}$ again; either way is correct)*

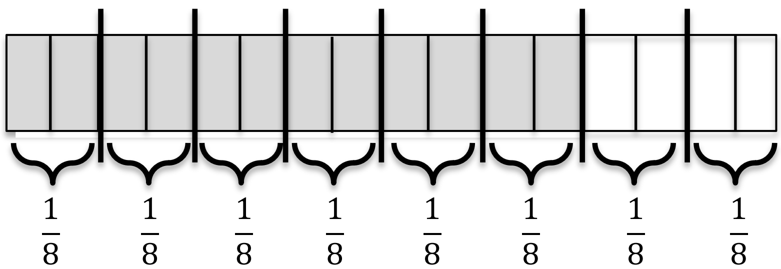
Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in the Student Booklets.

> **Let's use a model to justify, or prove, that $\frac{12}{16}$ is written as $\frac{3}{4}$ in simplest form.**
>
> **What fraction does the model represent?** *($\frac{12}{16}$)*

We want to simplify the amount of partitioning in this area model.

How many equal-sized pieces is the whole partitioned into? *(16 equal-sized pieces)*

Using your red colored pencil, add additional partitions on the model. Group 2 of the equal-sized pieces to create new equal-sized sections.



How many equal-sized sections in the whole do we have now? *(8)*

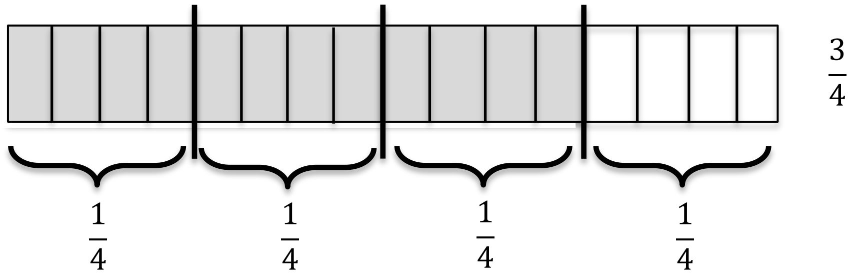
How many sections are shaded now? *(6)* **What fraction is now represented by the model?** *($\frac{6}{8}$)*

In your Student Booklet, write the relationship between the fractions represented in our first 2 models. *($\frac{12}{16} = \frac{6}{8}$)*

What did we do to our model of $\frac{12}{16}$ to show that it is equal to $\frac{6}{8}$? *(divided the model into groups of 2)*

Write this statement on the board: $\frac{12 \div 2}{16 \div 2} = \frac{6}{8}$

Can we partition our model into equal sized pieces again? *(yes, it can be grouped by 4 equal-sized pieces)* **Partition the equal-sized pieces into groups of 4 on top of the model.**



In your Student Booklet, write a mathematical expression showing how $\frac{6}{8}$ is related to $\frac{3}{4}$.

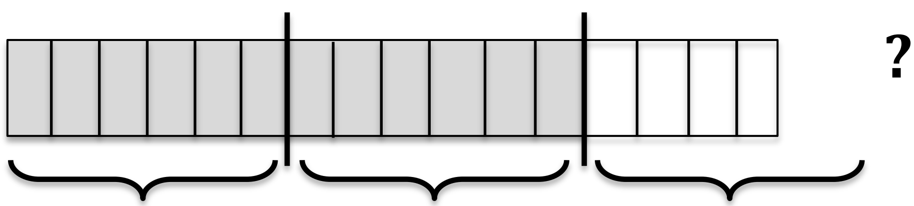
Students should write the following: $\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

If students write "$\frac{6}{8} = \frac{3}{4}$" without showing the division by 2, ask them to explain mathematically how they can support this relationship.

Is there any more partitioning that can be done that would result in equal-sized sections? *(no)*

For example, it is not possible to create an additional 2 pieces in each section because the sections would not be equal in size—there would be 6, 6, and 4 in each section.

Show the following model from the Teacher Masters.



Show the model at the top of the sheet again.

What fraction did we say that this model represents? *($\frac{12}{16}$)*

When you first wrote the simplest form of $\frac{12}{16}$ on your whiteboard, some of you might have divided the numerator and denominator by the common factor 4.

If we first partition the whole into equal-sized sections of 4 units, we can see that $\frac{12}{16}$ is equal to $\frac{3}{4}$ and no further simplification can occur.

Write the following on the board: $\frac{12 \div 4 = 3}{16 \div 4 = 4}$

What do you notice about these 2 ways of solving, or processes? *(same answer)*

One of these ways was more efficient. "Efficient" means to do something in faster way because in an accurate way. An example would be that it is more efficient to walk in a straight line from your house to your friend's house. An inefficient route would be to first stop at the store, go around the block, and then go to the house.

In our example, although dividing the numerator and the denominator 2 times by 2 is accurate, dividing them by 4 first is more efficient.

2. Students will determine how to find a fraction in simplest form.

Through the use of models, we have demonstrated how to write a fraction in simplest form. Now, let's learn a method that will allow us to find the simplest form without drawing a model.

Write the following equations on the board:

$$\frac{12 \div 2}{16 \div 2} = \frac{6}{8} \quad \text{and} \quad \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

OR

$$\frac{12 \div 4 = 3}{16 \div 4 = 4}$$

These are the equations that symbolize our representation of the simplest form of $\frac{12}{16}$.

In the first set of equations, we divide the numerator and denominator by 2 and then divide again by 2. In

the second set of equations, we divide by 4. How can we determine that we could divide by 4 instead of dividing by 2 twice? *(4 is the largest common denominator of 12 and 16)*

To help us be more efficient in simplifying fractions we can use common factors. First, for problem 1, write all the multiplication facts that you can think of that result in a product of 12. Then, for problem 2, do the same for 16.

Pause for students to work.

Now, for problems 3, write the factors that are used in the multiplication facts for 12 and 16.

Pause for students to work.

What factors do they have in common? *(1, 2, 4)* **Circle the factors they have in common. Which factor is the greatest?** *(4)* **4 is the greatest common factor of 12 and 16.**

This method allows you to find the greatest common factor of the numerator and denominator. The numerator and denominator can then be divided by the greatest common factor, which will result in the simplest form of the fraction.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work in pairs to complete the activity sheet.

2. Have pairs share their answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet from the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Jason simplified $\frac{14}{4}$ to $3\frac{1}{2}$. Explain the process that Jason might have used.

Have students read their explanation of the simplification; check for understanding.

Fractions
Lesson 10A

Lesson 10A: Addition and Subtraction of Fractions, Using Equivalent Fractions

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students add and subtract fractions with unlike denominators, using equivalent fractions.
Students reason abstractly and quantitatively. (SMP 2)
Students create mathematical models. (SMP 4)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Equivalent fraction, addend, multiple, common denominator</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may try to add or subtract fractions by adding or subtracting the numerators and the denominators.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Close to 1 Game Sheet (1 per pair, see page 154 of Teacher Masters)
• Close to 1 Game Cards (1 set per pair, see page 155 of Teacher Masters)
• Red colored pencil</td></tr>
</table>

Warming Up

Play Close to 1. Distribute a Close to 1 Game Sheet and a set of Close to 1 Game Cards (see pages 154 - 155 of Teacher Masters) to each pair.

We are going to play Close to 1. Your pair has a game sheet and a set of cards. Each set of cards has 4 of each number: 1, 2, 3, 4, and 5. You will shuffle the cards, then each of you will draw 4 number cards. The object of the game is to use the numbers to create a sum that is close to 1. You will write your numbers of the cards in the boxes *(one in each box to the left of the = sign)*. **Add the two fractions together, and the sum should be close to 1** *(write the sum in the boxes after the = sign)*. **Whoever gets a sum closer to 1 gets a point. If your sums are the same distance from 1, you both earn a point. You will play 2 rounds, shuffle the cards again and play 2 more rounds. Whoever has the most points at the end of the 4 rounds is the winner.**

On the first two draws, have a few students share their work and check for understanding. When the game is completed, have students share their work and check for understanding.

Learning to Solve

Students will add and subtract improper fractions and mixed numbers with unlike denominators.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Jake had $4\frac{1}{4}$ pizzas for his party. Before everyone came, he ate $\frac{7}{8}$ of 1 pizza. How much pizza was left for his guests?

How much pizza do you estimate will be left for Jake's guests? *(write students' estimates on the board)*

Let's model this problem. How can I represent $4\frac{1}{4}$ pizzas?

We could use many different shapes or the number line, but for this example, let's use an area model. What is the key when drawing your whole? *(have to be the same size)* **We need to make the whole and the partitions equal.**

In your Student Booklet, draw a model of $4\frac{1}{4}$.

As students are working, draw a model similar to the following on the board.

How would we solve this problem? *(accept reasonable answers; if no one states, subtract $\frac{7}{8}$ from $4\frac{1}{4}$, provide it as an option and skip the next sentence,)*

Many of you stated we should subtract $\frac{7}{8}$ from $4\frac{1}{4}$.

Write the expression $4\frac{1}{4} - \frac{7}{8}$ on the board.

How can I subtract, using our current model? I don't see any eighths and only 1 fourth.

Can I partition the model differently without changing the amount or value of the fractions? Think about common multiples. They can be used to find a common denominator. What is the least common multiple of 4 and 8? *(8)* **This least common multiple can also be used as the least common denominator. We can use the least common denominator to help us partition to find equivalent fractions.**

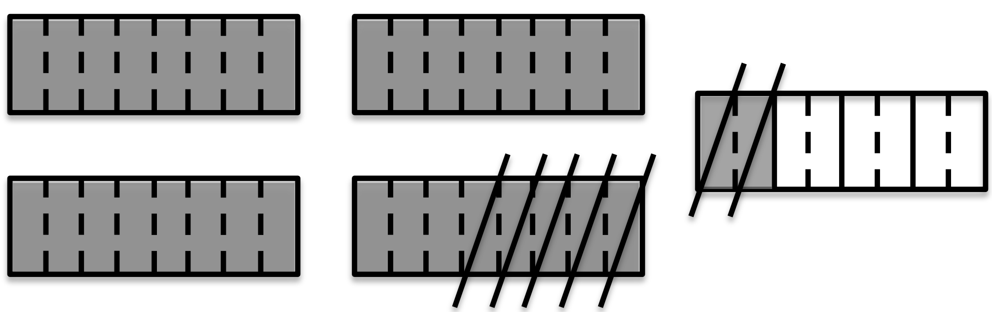
On your model, partition each whole into eighths.

As you ask the following questions, students should write their answers in the Student Booklet. Even though the Student Booklet leaves space for drawing and marking a new model, make sure that students know to use their existing model.

How many eighths represent the $4\frac{1}{4}$ pizzas in your model? *(34)*

We could rewrite our original expression of $4\frac{1}{4} - \frac{7}{8}$ as $\frac{34}{8} - \frac{7}{8}$.

Show how you will subtract $\frac{7}{8}$ from your model of $4\frac{1}{4}$ by crossing out.



How many one-eighths of the pizza remain for the guests? *(27)*

How do we write this as a fraction? *($\frac{27}{8}$)*

Read our equation with the equivalent fractions. *($\frac{34}{8} - \frac{7}{8} = \frac{27}{8}$)*

Looking at your model, what is another way to write the difference? *($3\frac{3}{8}$)*

Now rewrite our solution, turning the original expression into the equation, $4\frac{1}{4} - \frac{7}{8} = 3\frac{3}{8}$. Is the fraction $\frac{3}{8}$ in simplest form? *(yes)* **How do you know?** *(the numerator and denominator have no factors in common)*

You may have noticed that it was easier to subtract $\frac{7}{8}$ from $\frac{34}{8}$ than subtract $\frac{7}{8}$ from $4\frac{1}{4}$. To make a problem easier or more efficient, you might want to convert a mixed fraction to an improper fraction. This is not always necessary, but it is a good idea to keep in mind.

For example, add $3\frac{1}{10} + 6\frac{2}{5}$.

Provide time for students to work.

How did you solve? What equivalent fractions did you use? What is the sum? *($9\frac{1}{2}$ or $9\frac{5}{10}$)*

Did anyone have a different way to solve?

If a student solved using improper fractions, have him or her demonstrate on board; if no one points out converting to improper fractions, show how it could be done.

Did you have to covert to an improper fraction? Whether you converted to improper fractions or left them as mixed numbers, what did you have to do to the fractions to add? *(find a common denominator)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the activity sheet in groups or with the whole class if more appropriate.

2. Discuss the answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Write the following on the board:

$$\frac{\square}{\square} + \frac{3}{5} = \frac{17}{20}$$

How could you find the missing addend? What would you use to help solve? What is the answer?

If time allows, complete the problem together. Response should reflect an understanding of converting known fractions to the least common denominator and subtracting to find the missing addend, such as $\frac{17}{20}$ - $\frac{12}{20}$. Have students share their work and check for understanding.

Fractions
Lesson 14A

Lesson 14A: Multiplication and Division of Fractions

Lesson Objectives	Students multiply and divide fractions. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent, improper fraction, mixed fraction	
Misconception(s)	Students do not understand that a fraction can be a representation of division. Students overgeneralize the rule and will “flip” the second fraction without understanding the reason to multiply by the reciprocal.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • 4 dice for each pair of students • Red colored pencil • 4 to Go Game Sheet (1 per pair, see page 157 of Teacher Masters)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the problems on the Warming Up sheet by converting a mixed number to the equivalent improper fraction or converting an improper fraction to the equivalent mixed number or a whole number.

Have students give their answers and explain their reasoning.

Learning to Solve

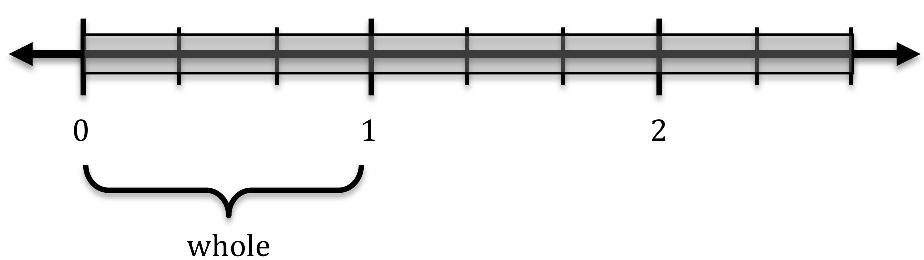
1. Students will model division of fractions.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at problem 1.

Let's model $\frac{8}{3} \div \frac{2}{3}$ on the number line. How do we model $\frac{8}{3}$? *(partition into thirds)*

On a number line, what is the whole? *(distance from 0 to 1)* **On the number line in your Student Booklet, represent $\frac{8}{3}$.**



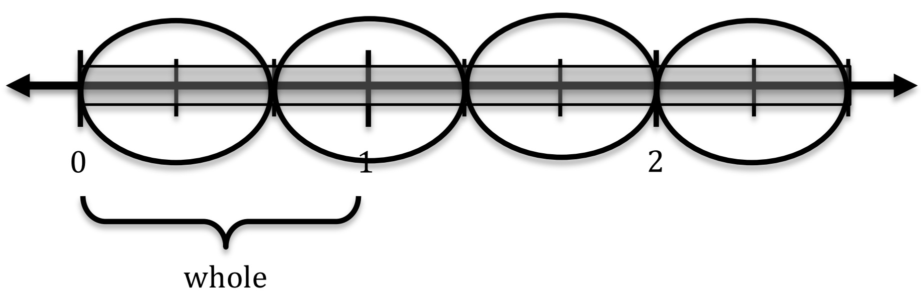
How many wholes? *(2)* **What is the mixed number that is equivalent to $\frac{8}{3}$?** *($2\frac{2}{3}$)*

Similar to other fraction problems, it is often more efficient to solve equations by using improper fractions, rather than mixed numbers.

What does $\frac{8}{3} \div \frac{2}{3}$ mean? *(the number of groups of $\frac{2}{3}$ in $\frac{8}{3}$)*

First, estimate by using benchmark fractions. What benchmark fraction is $\frac{8}{3}$ closest to? *(3)* **What benchmark fraction is $\frac{2}{3}$ closest to?** *(1)* **What is 3 ÷ 1?** *(3)* **What do you estimate that $\frac{8}{3} \div \frac{2}{3}$ is?** *(3)*

Look at your model and determine the number of groups of $\frac{2}{3}$ in $\frac{8}{3}$. Circle each group or $\frac{2}{3}$ unit.



How many $\frac{2}{3}$ units? *(4)* **What is $\frac{8}{3} \div \frac{2}{3}$?** *(4)* **Does it make sense that there are 4 groups of $\frac{2}{3}$ in $\frac{8}{3}$? Why? Is this close to our estimation?**

2. Students will use an algorithm to divide fractions.

Look at problem 2.

Recall that we divided 2 fractions by finding the common denominator. Let's do that with a problem involving a mixed number and an improper fraction.

Look at the division expression, $4\frac{2}{3} \div \frac{7}{6}$.

To be more efficient, what is a way to find the quotient? *(convert to an improper fraction)* **Convert $4\frac{2}{3}$ to an improper fraction.**

What is the equivalent fraction? *($\frac{14}{3}$)* **Write the equivalent expression.**

$$\frac{14}{3} \div \frac{7}{6}$$

Now, find the common denominator and then find the equivalent fractions for each of these fractions.

What is the common denominator? *(6)* **How did you find?** *(greatest common factor)* **What are the equivalent fractions?**
($\frac{28}{6}$ and $\frac{7}{6}$)

Write the division expression, using equivalent fractions. Now divide the numerators and the denominators and solve the problem.

$$\frac{28}{6} \div \frac{7}{6} = \frac{28 \div 7}{6 \div 6} = \frac{28 \div 7}{1} = 28 \div 7 = 4$$

What is $4\frac{2}{3} \div \frac{7}{6}$? *(4)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the Practicing Together sheet.
2. Discuss the answers.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Play 4 to Go in pairs. Distribute the 4 to Go sheet (see page 157 of Teacher Masters) and a set of 4 dice to each pair.

This game can be played in one of two ways. First, the teacher can roll the dice. Second, student pairs can roll the dice. Instructions should be modified depending on the way the game is played.

Have students turn to the Wrapping It Up page in their Student Booklet. You roll the dice and all students use your roll, or students can play in pairs and alternate rolling the dice.

> **We are going to play 4 to Go in the time we have remaining. One of you will be Player A and the other Player B; decide now.** *(pause)* **You will take turns rolling your 4 dice.** *(Or, I will roll the 4 dice)* **You will both place your numbers in the boxes on the game sheet, Player A or Player B, to make 2 fractions. Perform the division. You get points depending on the quotient; write the number of points in the space to the right. If your quotient is less than or equal to $\frac{1}{2}$, you score 1 point. If your quotient is greater than $\frac{1}{2}$ but less than 1, you score 3 points. If the quotient is greater than or equal to 1, you score 5 points. The person with higher score wins.**

On the first two rolls, have a few students share their work and check for understanding. When finished, have students share their work.