Integers

Teacher Lessons

Integers
Lesson 1

Lesson 1: Modeling Integers

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students model integers.
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">Integers: whole numbers and their opposites
Zero pair: a negative 1 matched or paired with a positive 1, resulting in a value of 0.</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Positive, negative</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students think, for example, that –3 is greater than –1 because of their extensive work with whole numbers.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)
• Chart paper or poster board
• 2-colored chips</td><td>• Student Booklet
• Whiteboard
• Dry-erase markers
• 2-colored chips (10 per student)
• Red colored pencil for each student</td></tr>
</table>

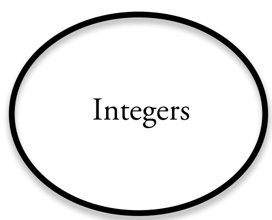
Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Module Check Form A** ✔	Lesson 7: Mid-assessment **Module Check Form B**	Lesson 15: Post-assessment **Module Check Form C**

Warming Up

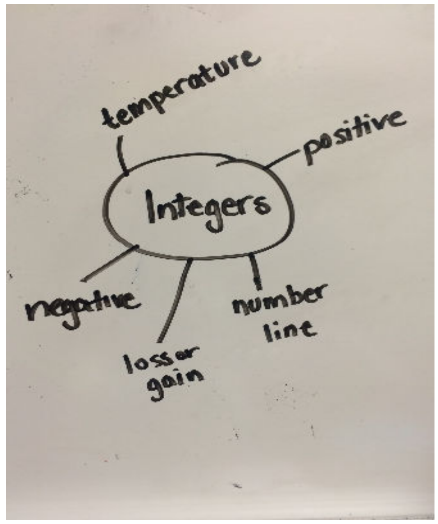
Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Draw a circle on the chart paper or poster board with “Integers” in the middle (similar to the one below). Display it for students to see.



Today, we will start by thinking about integers. Write everything you know about integers in your Student Booklets.

Ask for student responses. Ask for students to clarify or provide examples to aid in the brainstorming of ideas. Add ideas to the concept map.



What are some important ideas? What is an integer? What are examples of your ideas?

Display the concept map for the duration of this module. The concept map will be used again at the end of this module. Displaying during the course of the module will allow both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

TEACHER NOTES

A problem in working with integers is that the magnitude signs + and – symbols are also used for addition and subtraction. When reading integers, say "positive 5" and "negative 5" rather than "plus 5" or "minus 5" to differentiate between the operations and the designation of magnitude. Be sure that students use the appropriate language.

Opposites are an important concept because the addition of opposites results in 0. A zero pair is not simply "canceling out," but rather the addition of +1 and –1 to equal 0. Students should understand that any real number and its opposite always sums to 0. It is important that students understand this concept and do not think that addition is the opposite operation to subtraction.

It is an inverse operation.

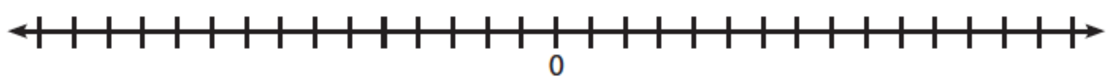
1. Students model integers using 2-colored chips.

Give each student 10 2-colored chips, a whiteboard, and a marker. Students should complete as the teacher models.

> **Today, we begin our study of integers. Temperature is a real-life example of integers. When the temperature is warm, like 80°, the value is positive because it is above 0. Sometimes the temperature gets very cold and is below 0, like negative 10°. Negative 10 is below 0, so we show this value by placing a negative sign in front of the integer. We use positive and negative symbols to show the value of integers.**
>
> **There are an infinite number of integers because an integer is any whole number, including 0, and its opposite. Integers are opposites when they are the same distance from 0.**

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **On a number line, positive numbers are to the right of 0 and negative numbers are to the left of 0. Write the integers to the right of 0.**



Students should number the hash marks 1, 2, and so on. Monitor students as they work to be sure they are numbering to the right of 0. Model how to write the numbers using the number line in the Teacher Masters.

> **When negative numbers are modeled on the number line, the first unit to the left of 0 is labeled negative 1 at the hash mark.**

What is the label for the second hash mark? *(–2)* **Label the remaining hash marks to the left of 0.**

Look at positive 3 and negative 3. What do you notice? *(answers may vary but it is important for students to recognize that these integers are the same distance from 0)*

The distance from 0 is 3 units, the same for both integers. This tells us that positive 3 and negative 3 are opposites because they are the same distance or same number of units from 0.

One way to model integers is on the number line. Another way to model integers is to use 2-colored chips. The chips are 2 colors, red and yellow. The red side represents negative 1 and the yellow side represents positive 1.

Remind students about what each color means. Write a key on the board—for example, "Red Chip = –1, Yellow Chip = +1." Model positive 5 and negative 5 using the chips. As the lesson progresses, show students how to create zero pairs after they have modeled on zero pair examples. Write the numbers modeled with the corresponding positive and negative symbols on the class whiteboard.

How can we use the chips to show positive 5? *(place 5 yellow chips on table)* **We can write positive 5 in 2 ways, 5 by itself or with the positive symbol, +, in front of 5. Write "+5" on your white board. This is a positive integer.**

What is the opposite of positive 5? *(negative 5)* **How would you model negative 5?** *(use 5 red chips)*

Using your chips, model negative 8. *(check student work with the chips)*

What is the opposite of negative 8? *(positive 8)* **Model positive 8, using the chips.** *(check student work with the chips)*

2. Students will be introduced to 0 and how to make a zero pair.

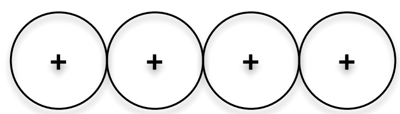
0 is a whole number but it is neither positive nor negative.

Place 1 positive chip and 1 negative chip together on your desk. What value are you modeling? *(0)* **How do you know?** *(accept reasonable answers, such as +1 plus –1 equals 0 or +1 is the opposite of –1)* **When we put a positive chip with a negative chip, or positive 1 with its opposite negative 1, the pair of chips together represent 0. We can think of it as a zero pair or as positive 1 plus negative 1 equals 0. Anytime we add a number and its opposite, the sum is 0. What is the opposite of positive 7?** *(negative 7)*

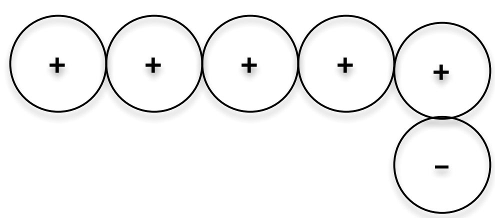
Model +7 and –7 and match each positive and negative chip to create zero pairs.

Are any chips left over or not matched? *(no)* **What is the value of positive 7 plus negative 7?** *(0)*

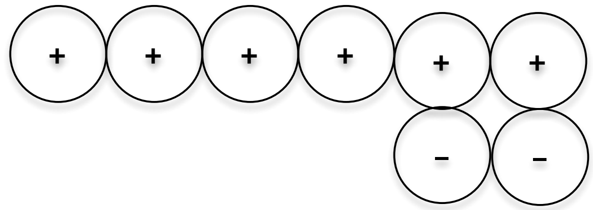
Place 4 chips on your desk. First, model positive 4; then draw this model on your whiteboard. You can make circles with the positive and negative signs inside each circle.



Take out 2 more chips. How could you model positive 4, using all 6 chips?



Now, using a total of 8 chips or circles, create another model for positive 4.



We used different amounts of chips but still modeled positive 4. What did you do with the other chips that did not change the value positive 4? *(accept reasonable answers, such as make zero pairs)*

Can we keep adding zero pairs? *(yes)* **Would the value, positive 4, change?** *(no)*

Practicing Together

1. Assign students to pairs. Have students use a certain number of chips to model positive and negative integers as you assign them to their pairs. As you assign numbers, remember that when modeling an odd number, only using an odd number of chips will work. When modeling an even number, only using an even number of chips will work. This fact should not be told to students, but rather used to guide questioning as students create their model.

Consider using the following integers: –8, +3, –6, +7, –2. Choose any integers you would like for them to model. Students may have to put their chips together to create their models or you may want to have more available.

As students work, ask them to model the assigned integer in multiple ways. For example, for –2, they may use 2 negative chips, 3 negative chips and 1 positive chip, 4 negative chips and 2 positive chips, and so on. Check student work as they model their integer assignments.

2. Have student groups show the class their models and ask the following questions.

   How did you model the integer? Did any group model it differently? What is the opposite of the integer?

When modeling the integers, what did you notice about the number of chips being used? If you want to model an odd integer, how many chips do you need? *(an odd number)* **If you want to model an even integer, how many chips do you need?** *(an even number)*

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write examples.

> **We have been modeling integers. First, write 2 examples of integers. Then, write 1 to 2 sentences that explain what an integer is. You can use examples that we did in class or are from Warming Up.**

Have students share their work and check for understanding.

Lesson 2: Modeling and Comparing Integers on a Number Line

Lesson Objectives	Students model and order integers. Students explain and identify the absolute value of an integer on a number line. Students explain that an integer and its opposite are the same distance from 0. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Absolute value:** a non-negative number that describes the distance of a number from 0 on a number line	
Requisite Vocabulary	Integer, zero pair, positive, negative	
Misconception(s)	Students often think that absolute value is always positive. This is partially true but can cause issues when they are confronted with – |7|, which equals –7.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard or equivalent • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • 20 2-colored chips per student (yellow/red, if available) • 1 red and 1 yellow colored pencil for each student

Warming Up

Distribute the chips to each student or have them work in pairs. Review the concept of opposites as it relates to operations in mathematics. Have students add notes in their Student Booklets. As the lesson progresses, write definitions and examples for students on the board for them to copy in the notes section in their Student Booklets.

> **Turn to the Notes section in your Student Booklet. We have been working with integers. Write a definition for integer. What is an integer?** *(a whole number and its opposite)*

Review the idea that positive 1 added to negative 1 equals 0. Give each student 10 2-colored chips.

> **We have used these chips before. What does the yellow side represent?** *(positive 1)* **Place positive 1 on your desk.**
>
> **What does the red side represent?** *(negative 1)* **Place negative 1 on your desk.**
>
> **One chip by itself represents a positive 1 or a negative 1. How can we show a value of 0, using our chips?** *(use 1 positive chip and 1 negative chip)* **When we pair one positive with one negative, what do we call it?** *(a zero pair)* **If we have positive 4 and add negative 4 to it, what is the value?** *(0)* **How many zero pairs do we have?** *(4)* **Recall from the previous lesson that positive 4 and negative 4 are opposites, because they are the same distance from zero.**
>
> **Place 4 yellow chips and 2 red chips in front of you. What number are we modeling with 2 negative chips and 4 positive chips?** *(positive 2)* **Why is the value of our representation positive?** *(accept reasonable answers but focus on students noting that there are 2 zero pairs and 2 positive chips not paired)*
>
> **Model positive 2 in another way, using your chips. How many chips did you use?** *(answers will vary but the number used to model positive 2 will always be an even number)* **What is the least amount of chips that could be used to model it?** *(2)* **What is the**

maximum amount of chips? *(infinite)* **I could use an infinite amount of chips, but would the total number of chips need to be even or odd?** *(even)* **Why?** *(answers will vary but you will always have 2 positive chips and an even amount of chips to make zero pairs)*

Learning to Solve

Teacher Notes

Encourage students to use the number line so that they see that an integer is a certain length or number of units away from 0. If students do not understand, allow them to use the 2-colored chips alongside the number lines.

Display the number line in Learning to Solve in the Teacher Masters. Model positive 6 with 12 2-colored chips (6 yellow chips and 3 zero pairs).

Have students turn to the Learning to Solve sheet in their Student Booklets. Students model integers on a number line.

Match my model, using your chips. How many zero pairs? *(3)* **What is the value of the chips?** *(positive 6)*

Now we will use the number line to model the same integer. Look at the first number line in your Student Booklets. What do you notice? *(0 is in the middle, extends both right and left)*

Where would positive 6 be on this number line? *(6 units to the right of 0)* **Write "positive 6" below the sixth hash mark. Any number to the right of 0 is positive.**

This number line also has hash marks to the left of 0. What are the values of the numbers to the left of 0? *(negative)* **What is the opposite of positive 6?** *(negative 6)* **Find negative 6 on the number line and write "negative 6" below the sixth hash mark to the left of 0.**

Now, using the yellow pencil, make a line from 0 to positive 6. Use your red pencil to make a line from 0 to negative 6. What do you notice about the length of both lines? *(the length of both lines is the same)* **How can we describe the length?** *(the length is 6 units in either direction from 0)*

(Point to the vertical number line.) **Look at the second number line. How is this line different?** *(it is vertical)* **Where would we show positive integers?** *(above 0)* **Where would we represent negative integers?** *(below 0)*

On this vertical number line, find and label negative 4. What is the opposite of this number? In other words, what positive number would be the same distance from 0? *(positive 4)* **Label it on the number line. Make your lines from 0 like we did with the first number line using your red and yellow pencils. What do you notice about the lengths?** *(they are the same)*

On the third number line, write these integers: positive 3, negative 5, negative 9, and positive 12 under their correct locations.

Using a number line can help us order the integers from greatest to least. Which number is the greatest of our 4 numbers? *(positive 12)*

What number is the next larger one? *(positive 3)* **Is negative 9 greater than positive 3?** *(no)* **How are negative 9 and positive 3 different?** *(one number is negative and the other number is positive)*

When we compare negative and positive numbers, which is always greater? *(positive)* **Why?** *(greater than 0, to the right of 0. Some students may note that positive numbers are always to the right of 0.)*

Now, let's compare positive 3 and negative 5. Which is farther to the left on the number line? *(negative 5)* **Because negative 5 is farther to the left, it has less value than positive 3. What is another way you can tell that positive 3 is greater than negative 5?** *(one number, positive 3, is positive and the other number is* *negative; positive numbers are always greater than negative numbers.)*

1. Students place the absolute value of integers on the fourth number line.

Write "|–7|" and "|7|" on the teacher's whiteboard. Write the absolute value of each number below on the fourth number line as the lesson progresses.

Look at my board. Make your whiteboard match mine.

Point to the absolute value symbol.

This symbol indicates absolute value. Absolute value is the distance from 0.

When we measure distance, it is positive. You are about 7 feet from me. *(point to the classroom door)* **About how many feet are we from the door?** *(point to the back wall of the room)* **About how many feet are you sitting from the back wall of the room?**

Even though we can measure behind us, we do not measure distance by using negative numbers. For example, we do not say the back wall is negative 6 feet. This is true for absolute value because it indicates the distance or number of units from 0.

Look at your number line. What is the distance, or the number of units, from 0 to negative 7? *(7)* **This distance gives the absolute value of negative 7. Write "|–7| = 7."**

What is the distance, or number of units, from 0 to positive 4 on your number line? *(4)* **What is the absolute value of positive 4?** *(4)* **Write "|4| = 4." What number would have the same absolute value as |4|?** *(negative 4; the absolute value of negative 4 is 4.)*

Write the numbers |negative 8|, negative 13, negative 4, and 2 on the board.

Place these numbers on your number line.

Have students, individually or in pairs, order the integers.

> **What is the correct order?** *(– 13, –4, 2, |–8|)* **How did you determine the order?** *(answers will vary)*
>
> **How can we describe the absolute value of negative 8? How many units away from 0?** *(8)*

Write the numbers 8, –10, –12, and |–17| on the board.

> **Listen to this situation. Jessica ordered these four numbers on a number line. She then compared them by writing them from least to greatest. She wrote: 8, –10, –12, |–17|. Did Jessica correctly order the integers? Talk with a partner to decide. You may want to use a number line to support your answer.**

Have students pair with a neighbor and discuss. After they have sufficient time to determine if they agree with Jessica, restate the question and have the pairs share.

> **Did Jessica order the integers correctly?** *(No. The correct order is –12, –10, 8, |– 17|.)* **How do you know?** *(answers will vary but should include that the absolute value of –17 is +17.)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets. Have students work in groups to act as teachers and determine errors in student samples.

1. Assign each student group a student work sample, numbered one to five. Remove any student names.

2. Have students determine if the student's response in the Student Booklets is correct. If an answer is incorrect, have students use a model or words to explain where the mistake was made. If an answer is correct, students should explain why.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 2A to provide extended practice before proceeding to Lesson 3.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write the answers to the following questions.

How is the number line helpful in understanding absolute value?

How can you use the number line to order and compare integers?

Have students share their work and check for understanding.

Integers
Lesson 3

Lesson 3: Using Models to Add Integers

Lesson Objectives	Students use models to add integers. Students state and write generalizations, evident from models, to add integers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Equation:** a mathematical number sentence in which the value of the expressions on both sides of the equal sign represents the same amount or quantity **Generalization:** formulating and producing statements about patterns and relationships and evaluating their reasonableness	
Requisite Vocabulary	Integer, zero pair, addend, sum	
Misconception(s)	Students may ignore positive and/or negative signs or revert to whole-number addition and use inappropriate generalizations to determine the sign of the sum.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard & dry-erase marker • 2-colored chips (red and yellow; 12 per student) • Red colored pencil for each student

Warming Up

Have students practice representing single-digit integers and zero pairs (positive and negative) with drawings on their whiteboards. Students may use colored-in circles to represent negative and white circles for positive or can write a negative or positive sign inside circles. Allow students to choose what works and makes sense for them.

Give each student a whiteboard for the following problems. Ask students to hold up their whiteboards after each problem is done to check their answers. After each problem is shown on the student whiteboard, have the students erase their answers and do the next problem.

What is the opposite of positive 12? *(negative 12)*

What is the value of positive 12 plus negative 12? *(0)*

What is the value of 1 negative chip added to or matched to 1 positive chip? *(0)*

A matched set, or zero pair, has a value of 0.

Draw 12 chips to show a value of positive 8. *(eight positive chips and 2 zero pairs)*

Draw four different ways to model negative 4, using different numbers of chips. For example, draw 10 chips to model negative 4. *(answers may vary by including different zero pairs; with 10 chips, four positive chips and 3 zero pairs)*

Use 6 chips to represent negative 4. Then, use a different number of chips. *(in the first instance, four negative chips and 1 zero pair; next, answers may vary by including different zero pairs)*

Allow students to discuss and share the different models.

Learning to Solve

Teacher Notes

The purpose of this lesson is to model addition and subtraction of integers, rather than simply giving rules. Some students may state that they know to “just take the sign of the bigger number” or to “switch, switch, change.” These shortcuts are often used incorrectly. Students should use the model to explain “why” the rules or generalizations exist. Developing a deeper understanding of adding integers is key to remembering the process. Students will develop these generalizations as part of the modeling once patterns and relationships are evident.

1. Students use models to add integers.

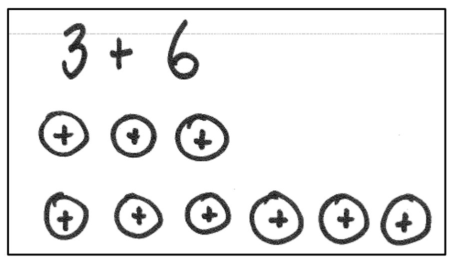
   Give each student 12 2-colored chips. Write “3 + 6” on the board. Have students continue to use their whiteboards from Warming Up.

   We will write and solve equations with integers. An equation is a mathematical number sentence in which the value of the expressions on both sides of the equal sign represents the same amount or quantity.

   Read the expression. *(3 plus 6)* **What is the sign of 3 and 6?** *(positive)* **How do you know?** *(neither number is shown with a negative symbol, all numbers are positive)*

   Model this expression with your chips on your whiteboard.

   Pause for students to model.



   What is the sign of 3? *(positive)* **What is the sign of 6?** *(positive)* **What does positive 3 plus positive 6 equal?** *(positive 9)*

   What is another way to model an expression that equals positive 9? Show the model on your board, using your chips or a drawing.

   What is the sign of the addends in your addition problem? *(positive)* **What was the sign of the sum?** *(positive)*

   What if I write “6 + 7 = 13”? What are the signs of the addends? *(positive)* **What is the sign of the sum?** *(positive)*

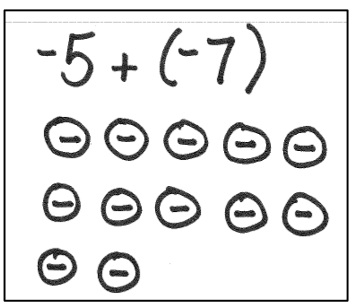
   What did you notice when we added 2 positive integers? *(the sum, or answer, was positive)*

   Think of the examples we just did as patterns. What is the pattern when we add a positive integer to a positive integer? *(the sum is positive)*

Write the generalizations on the board for students to copy in their Student Booklets. Or, write the generalizations in the Teacher Masters Generalization sheet for students to copy.

When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. The generalization we have modeled is a positive integer plus a positive integer equals a positive integer.

Turn to the Notes section in your Student Booklets. Turn to the Generalization sheet. In the first column write this generalization: "When we add a positive integer to a positive integer, the sum is a positive integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.



Clear your boards. Now write "–5 + (–7)." What is the sign of 5 and 7? *(negative)*

Model the 2 addends of the expression with your chips. How many negative chips in all? *(12)* **What is the sign of 12?** *(negative)* **So negative 5 plus negative 7 equals negative 12.**

Model another expression that equals negative12.

What expression did you model? *(ask for a variety of answers, writing them after the equal sign—for example: –5 + (–7) = –11 + (–1) or –12 = –3 + (–9))*

For the expression negative 3 plus negative 8, what do you think the sign of the sum will be? *(negative)* **Why?** *(accept reasonable answers, such as negative integer plus a negative integer equals a negative integer, or allow students to explain using a model)*

What did you notice when we added 2 negative integers? *(the sum, or answer, was negative)*

Think of the examples we just did as patterns. What is the pattern when we add a negative integer to a negative integer? *(the sum is negative)*

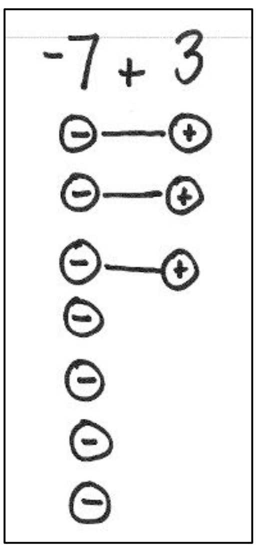
Turn to the Notes section in your Student Booklets. Turn to the Generalization sheet. In the first column write this generalization: "When we add a negative integer to a negative integer, the sum is a negative integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

2. Students model adding a positive integer and a negative integer.

Take out 10 chips. Model negative 4, like we did during the Warming Up, using these 10 chips.

How could we write this model as an equation? *(3 + (–7) = –4 or –7 + 3 = –4)*

Write "–7 + 3 = –4" on the board and have students copy the equation.



Let's try another expression. Model negative 8 plus 2. What is the sum? *(negative 6)*

How did you model it? *(for example, used 8 negative chips, 2 positive chips to make 2 zero pairs; can include additional zero pairs)*

Write the equation –8 + 2 = –6 on your board.

Do you think every answer will be negative when we add a negative integer to a positive integer? Let's try another one to help us decide.

Model 7 plus negative 3. *(allow time for students to model)* **What does 7 plus negative3 equal?** *(positive 4)* **What is the sign of 4?** *(positive)*

In the two examples we modeled, 1 addend was negative and 1 addend was positive, yet the sum was negative sometimes and positive other times.

> **Think about your models. How did you know what the sign of the sum should be?** *(depends on the sign of the greater amount of chips)*

> **Use what you noticed in the previous examples and create a model that shows a positive integer plus a negative integer that equals a positive integer.**

Allow time for students to create a model. Have students share their models. Write the equations that are shown in the models on the whiteboard. Some examples of equations might be: 8 + (–5) = +3 or –4 + 5 = + 1.

> **What do you notice about the equations and models that you created?** *(answers will vary; focus on observations that the number of positive chips in the models is greater than the number of negative chips; or, the absolute value of the positive number is greater than the absolute value of the negative number.)*

> **Now create a model that shows a positive integer plus a negative integer that equals a negative integer.**

Allow time for students to create a model. Have students share their models. Write the equations that are shown in the models on the whiteboard. Some examples of equations might be: –8 + (+ 5) = –3 or 4 + (–5) = –1.

> **What do you notice? Is there a pattern? How do you know what the sign of the sum should be?** *(answers will vary.)*

Students may notice that the greater number of positive (or negative) chips determines the sign of the sum. This also means that the absolute value of the addend with the greater number of chips is larger than the absolute value of the other addend that is modeled with the lesser number of chips. Focus on the relationship of the absolute value of the integers to determine the sign of the sum. If the absolute value of the positive number is greater than the absolute value of the negative number, the sum will be positive. If the absolute value of the negative number is greater than the absolute value of the positive number, the sum will be negative.

Write the generalizations on the board for students to copy in their Student Booklets. Or, write the generalizations in the Teacher Masters Generalization sheet for students to copy.

Turn to the Notes section in your Student Booklets. Turn to the Generalization sheet. In the first column write this generalization: "When we add a positive integer to a negative integer, the sum could be a positive integer or a negative integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Is there a pattern or a relationship to identify when the sum will be negative or positive? *(depends on the absolute value of the integer, meaning the sum will match the sign of the integer with a greater absolute value)*

Practicing Together

Activity 1: Place students in pairs or small groups. Have students model different expressions to arrive at a common integer.

Many students try to add integers without modeling. For example, Steve wrote that $-7 + 3 = -10$ and $7 + (-3) = 10$.

Use your chips to model the equations to justify your answers.

Why is his answer incorrect?

Model another equation that shows a sum of positive 10.

Activity 2: Have students turn to Practicing Together in their Student Booklets. Have students complete the two problems on the Practicing Together sheet in the same pairs or small groups. If students struggle with modeling and adding integers, complete the Practicing Together sheet as a class.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets.

> **A student wrote that –5 + (–5) = 10. Decide whether this is correct and describe how chips, a drawing, or a number line could be used to explain your reasoning or thinking.**

Have students share their work and check for understanding.

Integers
Lesson 4

Lesson 4: Using Models to Subtract Integers

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students use models to subtract integers.
Students state and write generalizations, evident from models, to subtract integers.
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Integer, equation, generalization, zero pair</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may ignore positive and/or negative signs or revert to whole-number subtraction, changing the order of the numbers to subtract from the “larger” number, rather than from the whole.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Whiteboard
• Dry-erase markers
• 2-colored chips (red and yellow; 22 per student pair)
• Red colored pencil for each student</td></tr>
</table>

Warming Up

Review modeling and adding integers by drawing a picture.

Give each student a whiteboard.

> **Instead of using the 2-colored chips, we will draw pictures of them, using circles with a positive or negative symbol inside each, to model integer values and the addition of integers.**
>
> **First, draw a representation to help you solve negative 6 + 4.**
>
> **What is the sum of negative 6 + 4?** *(–2)*

Write "–6 + 4 = –2" on the board.

> **Draw another representation using different integers to represent the sum negative 2.**

Have students share their equations.

> **How many chips did you use in your drawing?** *(answers may vary based on whether students show zero pairs)*
>
> **Draw a third way to represent the sum of negative 2.** *(answers may vary based on whether students show zero pairs)*

Have students share their equations.

> **How can we draw more chips without changing the value of your representation?** *(use zero pairs)* **There are many ways to represent the sum negative 2, but what do you notice about the number of chips used in each example**? *(the number of chips is always even)*

Learning to Solve

TEACHER NOTES

The subtraction model used in this lesson is "take away," meaning the students add zero pairs until enough positive or negative chips are available to remove. The minus sign is <u>not</u> a symbol of the number's magnitude, it represents the operation of subtraction. In Tier 1, core instruction, the symbol "–" is often read minus in both the operation and magnitude contexts. In this lesson, models will be used to develop the meaning for the definition of subtraction—for example, $a - b = a + (-b)$, read as a minus b equals positive a plus negative b. Patterns will support the development of the generalization, rather than using shortcuts or tricks. Shortcuts or tricks that are developed without meaning can cause students to apply them inappropriately.

Note that this is a long lesson and may need to be spread over 2 days. The development of the generalization for subtraction of integers requires that students have the opportunity to compare and contrast multiple examples.

1. Students model subtraction with integers.

   Write "–5 – (–2)" on the board. Pair students and give each pair a whiteboard, marker, and 22 2-colored chips.

   Look at the problem. Do you think the difference will be negative or positive? Why? *(encourage students to respond, but do not comment on their responses beyond acknowledgement)*

   We will use chips to justify our answer. Place 5 negative chips on your desk. What is being subtracted from negative 5? *(negative 2)* **Do we have 2 negative chips to remove?** *(yes)* **Remove 2 negative chips.**

What is the difference? *(negative 3)* **What does negative 5 minus negative 2 equal?** *(negative 3)*

Return to students' earlier comments to further acknowledge their responses.

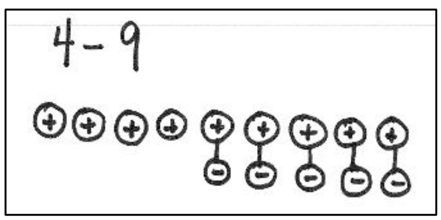
Write "–5 – (–2) = –3" on the board.
Write "–8 – (–6)" on the board.

Use your chips to model and find the difference of negative 8 minus negative 6.

What is the difference? *(negative 2)*

Repeat the process with "–13 – (–12)" and "–10 – (–6)" on the board. Students should find the differences of –1 and –4, respectively.

Write "4 – 9" on the board and have students copy the expression on their whiteboards. Students should model, using the chips.



Read the expression. *(4 minus 9)* **What are we starting with?** *(4)* **What is the sign of 4?** *(positive)*

Place 4 positive chips on your desk or whiteboard. We have to subtract 9 from the 4. What is the sign of 9? *(positive)* **Do we have 9 positive chips to take away?** *(no)*

Add 5 positive chips to the teacher model. Do not have students complete this step; just show it on the teacher's display. As you proceed with this section, constantly check for student understanding. Also, plan ahead – you will be writing many expressions and equations on your whiteboard, so plan to space accordingly.

Can we just add 5 chips to our model? What would be the value modeled if I added 5 positive chips? *(positive 9)* **So I changed the value of my starting amount, which we cannot do.**

Let's think about when we modeled numbers. Can we represent positive 4, using more chips? *(yes)* **How do we add chips without changing the value of the number represented?** *(add zero pairs)* **How many zero pairs could we add to this model, so that we can remove 9 positive chips?** *(at least 5 zero pairs)*

Add 5 zero pairs. Before we take any chips away, what number is being modeled? *(positive 4)*

How many total positive chips are on your desk now? *(9)* **How many negative chips?** *(5)*

Do we have enough positive chips to take away 9 now? *(yes)* **Remove 9 positive chips. How many chips remain?** *(5)* **What is the sign of 5?** *(negative)*

Write "4 – 9 = –5" on the whiteboard.

So positive 4 minus positive 9 equals negative 5. Is the answer negative 5 reasonable? *(yes)* **Why?** *(answers may vary, but could reflect knowledge of the number line; show students how to use the number line to answer the question)*

Write the expression "12 – (–5)" on the whiteboard.

Write this expression on your whiteboard. What do we model first? *(+12)* **Model positive 12, using chips or drawing.**

What do we subtract? *(–5)* **Do we have 5 negative chips to take away?** *(no)* **Because we do not have 5 negative chips, we cannot simply flip over 5 chips or add 5 negative chips. Why can we not just flip over 5 chips or add 5 negative chips to our model?** *(it would change the value of +12)*

What should we add to the model to give us 5 negative chips without changing the value of 12? *(zero pairs)* **How many zero pairs?** *(5)*

Add 5 zero pairs. Now take away negative 5. What is the difference? *(positive 17)*

Write "12 – (–5) = 17" on the whiteboard.

Clear your boards.

Write the expression "–9 – 12" on the whiteboard.

Now model the subtraction expression negative 9 minus positive 12, using chips or a drawing.

What is the difference? *(negative 21)* **How did you model?**

Write "–9 – 12 = –21" on the whiteboard.
Write the expression "–9 – (–11)" on the whiteboard.

Write the expression negative 9 minus negative 11. With your neighbor, model the expression to find the difference.

How did you model? What is the difference? *(positive 2)*

Write "–9 – (–11) = 2" on the whiteboard.

What if, instead of subtraction, you found the sum of negative 9 and positive 11?

Write "–9 + 11" next to –9 – (–11) on the whiteboard.

Find the sum using a model with your chips or by drawing one. What sum did you find? *(positive 2)*

Write "–9 + 11 = 2" on the whiteboard.
Write "–9 + (–12)" next to –9 – 12 on the whiteboard.

Find the sum using a model with your chips or by drawing one. What sum did you find? *(negative 21)*

What do you notice about these 2 examples? Talk with your partner about your observations.

Allow students time to discuss with a partner. Ask them to share their observations. Write their generalizations on the whiteboard. Listen for students who notice that the sums and differences are the same but the signs of the second numbers are opposites. If students do not notice this, then tell them.

When they have given all of their observations, have them continue to work with their partner to answer the following examples that are the addition partner for previous subtraction examples in Learning to Solve: 12 + 5 = , 4 + (–9) = , –13 + 12 = , –10+ 6 = .

After they complete these examples, ask them to explain their solutions. Do they notice other patterns? Ask them to share their observations. As they share, write any new observations they make and compare them with their previous observations.

> **The second addend in the addition examples has a special relationship with the number we were subtracting in our subtraction examples. What do you notice?** *(they are opposite; for example, 5 is the opposite of negative 5)*
>
> **This pattern of adding the opposite when subtracting is a way of thinking about subtraction. It is a generalization we can use whenever we are subtracting integers.**

Students write subtraction generalizations. Write the generalizations on the whiteboard or in the Teacher Masters Generalization sheet for students to copy.

> **When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed that when you are subtracting an integer, it is the same as adding the opposite of that number.**
>
> **For example, to subtract negative 9 minus 8, you can think of it as negative 9 plus the opposite of 8 or negative 8. Negative 9 plus negative 8 is negative 17. Negative 9 minus positive 8 is negative 17.**

You may want students to model this with 2-colored chips to convince themselves that these statements are correct. They would model each one separately to validate the generalization.

Turn to the Notes section in your Student Booklet. Turn to the Generalization sheet. In the first column write this generalization: "When you are subtracting an integer, it is the same as adding the opposite of that number." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students complete the Practicing Together sheet in pairs or small groups. If students struggle, complete the sheet with the whole class. Encourage them to use models for subtraction, then "check" their answer using the generalization. Have student pairs share their answers.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write their ideas.

Describe how chips or drawings can be helpful to solve subtraction problems with integers.

Have students share their work and check for understanding.

Lesson 5: Applying Generalizations to Add and Subtract Integers

<table>
<tr><td rowspan="1">Lesson Objectives</td><td colspan="2">Students apply generalizations to add and subtract integers.
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Integer, addend, equation, generalization, zero pair</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students may create generalizations about addition and subtraction of integers that are not appropriate. For example, students may say that when you add integers with different signs, the sign of the sum is the same as the largest integer.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Whiteboard
• Dry-erase markers
• 2-colored chips (as needed)
• Red colored pencil for each student</td></tr>
</table>

Warming Up

Review fact families with students.

Give each student a whiteboard.

> **When we add and subtract, the numbers used in the problem are related.**
>
> **Think about this problem: Start with 3, then add 4. What is the sum?** *(7)* **Write the addition equation on your whiteboard.**
>
> **Can we change the order of the addends and still have a sum of 7?** *(yes)* **Why?** *(accept reasonable answers)* **What do we call the property that allows us to do this?** *(commutative property of addition)* **Write the addition equation with the addends in a different order.**
>
> **Now, start with 7 and subtract 3. What is the difference?** *(4)* **Write this subtraction equation on your whiteboard. Is there another subtraction equation we can write, using the same 3 numbers?** *(yes, 7 – 4 = 3)* **Write it.**
>
> **Let's change the signs of the numbers. Write an equation to show the sum of negative 3 plus 4. What is the sum?** *(1)* **Write the other addition equation that uses these same 3 numbers.** *(4 + (–3) = 1)* **What property did you use to write the equation?** *(commutative property of addition)*
>
> **For the subtraction equations, we can only use these same 3 numbers. What subtraction equations can you write?** *(1 – 4 = –3; 1 – (–3) = 4)*

Students should create both equations. As they share the equations they wrote, if they give one of the two subtraction equations, ask for the other equation.

> **The equations you have written all belong to a fact family. Fact families can be helpful when computing with integers.**

Learning to Solve

Teacher Notes

Students may still use 2-colored chips if needed, rather than pictures. Have chips available, as needed.

Students were introduced to the generalization regarding subtraction in the previous lesson and may still be developing that idea. Students should justify their solutions for addition and subtraction problems by modeling with chips or pictures and using the generalization.

1. Students continue to develop generalizations after modeling addition and subtraction problems in the same fact family. Write the generalizations on the board or in the Teacher Masters Generalization sheet for students to copy in their Student Booklets.

   Give each student a whiteboard. Students will draw models for subtracting and adding, but they may need 2-colored chips to review or support their answers.

   Rather than using chips, we will draw representations to justify the sums and differences of integer problems. Write the expression 7 plus negative 2 on your whiteboard.

   Before you draw the representation, what do you predict the sign of the sum will be? *(positive)* **How do you know?** *(encourage students to respond, but do not comment on their responses beyond acknowledgement)*

   Draw a picture to represent 7 plus negative 2. What is the sum of 7 plus negative 2? *(5)* **Why is the sum positive?** *(accept reasonable answers, such as 7 is more units from 0 than 2, the absolute value of 7 is greater than the absolute value of –2)*

Return to students' earlier predictions to further acknowledge their responses.

Some students may notice that when the idea of "more units from 0" is discussed that this refers to absolute value. If students do not readily move to this generalization, use the following dialogue to prompt before moving to the discussion about fact families.

> **When we talk about distance from 0, to what are we referring?** *(absolute value)*
>
> **When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed that when we add numbers with different signs, the sum has the same sign as the addend that has the greater absolute value.**
>
> **Turn to the Notes section in your Student Booklet. Turn to the Generalization sheet. In the first column write this generalization: "When we add numbers with different signs, the sum has the same sign as the addend that has the greater absolute value." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.**

Write the numbers 5, 7, -2 on the board.

> **What 3 numbers are in this fact family?** (5, 7, –2) We already showed 7 + (–2) = 5. **What is the other addition equation we can write?** *(–2 + 7 = 5)*
>
> **Draw a picture on your whiteboard to find the difference of five minus negative 2. What is the difference?** *(7)* **What is the other subtraction equation we can write for this fact family?** *(5 – (7) = –2)*
>
> **Look at your drawings, what did you have to do to show the subtraction?** *(add zero pairs. Some students may say to add the opposite but this will not be represented in the drawing.)*

Write these 4 equations on the board: 7 + (–2) = 5, –2 + 7 = 5, 5 – (–2) = 7, 5 – 7 = –2.

> **Erase your board. Using the subtraction expression 5 minus 7, change the 7 to an integer that would result in a positive difference and write the equation. What integer did you use?** *(students can select any integer less than 5)*
>
> **Using your new equation, write the other fact family equations.**

Check students' equations to be sure they wrote a correct fact family. If time permits, have students share some of the fact families they wrote, especially if you have some that included a negative integer as the subtrahend.

Students develop generalizations after modeling subtraction problems.

> **Draw a representation for the expression negative 7 minus negative 12.**

Call on a student to hold up the model to display for the class.

> **What is the difference of negative 7 minus negative 12?** *(5 or positive 5)*
>
> **Did you make any zero pairs?** *(yes, 5; note that 5 is the minimal number of zero pairs that could be added but more than 5 zero pairs could be used without changing the difference)* **Why did you need to use zero pairs in the problem?** *(answers may vary; for example, we needed to subtract –12 but there were only 7 negative chips.)* **Why is the sign of the difference, 5, positive?** *(answers may vary; for example, 5 positive chips are left; some students may mention the generalization from Lesson 4 that if you add the opposite of –12, –7 + 12 = 5)*

Write the fact team on the board: –7 – (–12) = 5, –7 – 5 = –12, –12 + 5 = –7, 5 + (–12) = –7

> **When we subtract, we often have to add zero pairs to our drawing or model. What is another way to think of subtraction of integers that we discussed in Lesson 4?** *(adding the opposite)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Read the first item to help explain how the Magic Squares on the Practicing Together sheet work. Be sure to indicate the diagonals, as they have the same sum as the rows and columns. Then, have students complete the sheet in small groups or as an entire class if appropriate. Review the sheet as a class when complete.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students answer the following question.

> **Samantha subtracted 2 integers and got a difference of –7. What 2 integers could she have subtracted? Write at least 2 pairs of integers that she could have subtracted.** *(answers may vary; for example, pairs may include –7 and 0, –6 and +1, 8 and 15, and so on)*

Have students share their ideas and check for understanding.

Integers
Lesson 6

Lesson 6: Problem Solving: Addition and Subtraction With Integers

Lesson Objectives	Students use addition or subtraction to solve word problems with positive and negative integers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Variable:** a letter used to represent a quantity (e.g., $3 + x = 4$)	
Requisite Vocabulary	Integer, expression, equation	
Misconception(s)	Students read word problems for key words, not thinking about the relationships in the problem or judging the reasonableness of the answer. Students also misapply generalizations about addition and subtraction of integers.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • 2-colored chips, as needed • Red colored pencil for each student

Warming Up

Review the concept of integers.

Give students a whiteboard. Have them write the answers to each question as you read it. After each question, have students erase the previous answer so that they can easily share their answers.

What is an integer? Write your definition on your whiteboard. *(a whole number and its opposite)*

Write the opposite of each of these integers: 12, negative 36, negative 154, 48, 0. *(–12, 36, 154, –48, 0)*

What is the absolute value of an integer? *(a non-negative number that describes the distance, or the number of units, from 0)*

What is the absolute value of 78? *(78)* **What is the absolute value of –91?** *(91)* **What is the absolute value of 0?** *(0)*

Learning to Solve

Teacher Notes

Encourage students to solve problems without using concrete manipulatives, such as 2-color chips. A first scaffold would be drawing a picture as a representation that students always have available. However, if drawings are not sufficient, have 2-colored chips available as needed.

This lesson contains a word problem that refers to golf scores. In golf, the lower your score, the better. Each golf course sets a par, or standard number of strokes, for each hole. When someone scores –2, known as 2 under par, he or she did better than the standard, reaching the hole in 2 fewer strokes.

1. Students identify patterns when adding and/or subtracting integers.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve page in their Student Booklets.

We will use a table to help us solve a word problem.

Look at the first problem. This problem is about golf. In golf, there are 18 holes and you count the number of strokes it takes to get the ball into each hole. The expected score, or the expected number of strokes to get your ball into the hole, is called par. For example, let's say hole 6 has a par of 5. This means it should take you 5 strokes to make it to the hole. If you do it in 4 strokes, you are 1 under par, or –1. This means that you made fewer strokes than expected. In golf, a score that is negative is better than a score that is positive because you are under par.

Follow along as [student] reads the problem about Morgan playing golf.

First, is a negative integer less than or greater than 0? *(less than)* **You would want to finish your golf game with a negative score because this means you are under par.**

Look at the heading actual score plus negative 4 = final score on top of the table.

The table shows the days of the week Morgan played golf at the Brushy Creek golf course. The next column shows her score. We need to find her final score for the day by adding negative 4 to her actual score. We will calculate and write the final score in the last column of the table.

Using your whiteboard, calculate your final score for the day when we add negative 4 to Monday's score of negative 2? *(–6)* **Write your answer in the table of your sheet.**

What is the final score for Tuesday? *(–5)*

Find the final score for Wednesday, Thursday, and Friday and write them in the table.

Give students time to solve.

What were the final scores? *(–7, –4, and –3)*

Using the final scores, on what day did Morgan do the best? *(Wednesday).* **Why?** *(answers will vary; for example, –7 is the least of all of the scores)*

By adding negative 4 to the actual score of the day, will that always result in the final score being negative? *(no)* **Why?** *(answers will vary; for example, if Morgan scored positive 5 or greater, her score would be positive)*

If you were told that Morgan's final score of the day was positive, what would her score have to be? *(any number greater than 4).* **Why?** *(answers will vary; for example, –4 + 5 = 1. –4 + 6 = 2)*

Look at problem 2. Robert was a contestant on a student game show. He missed some questions, so his score was –200. He then correctly answered a 500-point question about cars in the 1900s. What is Robert's current score? *(300 points)*

What is the question asking? *(what Robert's new score is after he answered the car question correctly)*

How do you think Robert could have scored a negative score? Aren't most scores positive? *(yes, but he answered questions incorrectly and by the rules, lost points for an incorrect answer)*

If you start with a negative score and then answer a question correctly, how will this affect the score? *(the score will increase)*

How can we solve this problem? *(use a model, draw a picture, or use chips)* **Write an equation to show the relationships in the problem and solve it.**

How did you solve the problem? *(answers will vary; for example, some students may use a fact family, while others may use a model; some students may compute with integers without other representations)*

Record different strategies and the equations the students used.

What is Robert's current score? *(300)* **Write how you solved this problem and then write the answer in a sentence.** *(Robert's current score is 300 points.)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students complete the Practicing Together sheet in small groups or as an entire class, if appropriate. Review the sheet as a class when complete. Call on different groups to explain how they found the solutions.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write their equations.

Use a negative integer and a positive integer to write an addition equation that shows a positive sum.

Have students share their work and check for understanding.

Integers
Lesson 7

Lesson 7: Equations and Inequalities

Lesson Objectives	Students write equations and inequalities with integers and then compare sums and differences. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Inequality**: a statement that shows the relationship between 2 expressions or quantities that do not have the same value **Unequal:** the relationship between 2 expressions or quantities that do not have the same value	
Requisite Vocabulary	Absolute value, equation, simplify	
Misconception(s)	Students look at only the number, rather than both the number and its sign. For example, a student may think that –54 is greater than 9 because 54 is greater than 9.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase Markers • 2-colored chips, as needed • Red colored pencil for each student

Warming Up

Review how to compare and order integers.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

The online weather report showed this chart of temperatures around the United States. Order the temperatures from least to greatest.

Which city had the lowest temperature? *(Anchorage)*

Which city had the highest temperature? *(Orlando)*

At the North Pole, the temperature is 10 degrees lower than in Anchorage, Alaska. What expression could be used to represent the temperature? *(–10 + (–23) or –23 – 10)*

What is the temperature at the North Pole? *(–33°)*

Learning to Solve

Teacher Notes

Most students should be able to compare and solve without the use of scaffolds, such as 2-colored chips, drawings, or number lines. These scaffolds can still be made available on an as-needed basis for students who struggle with the skill, but encourage students to use what they know about integers first.

In the discussions regarding justification of which integer is greater or less than the other integer, there is reference to a number line placement of integers. For consistency in discussions, a horizontal number line is used as the frame of reference. If students bring up vertical number lines, include the up-down relationship in the discussion.

1. Students review how to compare integers.

   Give each student a whiteboard. Write the integers –95 and 63 on the board.

   We want to compare these 2 numbers. Place a symbol between the 2 integers to show how they compare. Are the integers equal? *(no)* **When quantities do not have the same value, we say they are unequal. When we write a statement to show the relationship between them, we write an inequality, which is a statement that shows the relationship between 2 expressions or quantities that do not have the same value.**

   Read the inequality statement. *(–95 is less than 63)* **How do you know that negative 95 is less than 63?** *(accept reasonable answers, such as –95 is negative and negative numbers are always less than positive numbers)*

   Erase the negative 95. Write the integer negative 103 in the place where you just erased the negative 95. Is your inequality still true? *(yes)* **How do you know?** *(–103 is farther to the left of –95 on a number line)*

   Write "|–32|" and "32" on the board.

   Erase both 63 and negative 103 and replace them with the same numbers that I just wrote. The first number, negative 32, is between 2 bars. What is this symbol called? *(absolute value)* **What is absolute value?** *(absolute value is the distance from 0)*

   The absolute value of a number is always non-negative because it is the distance from 0. We say non-negative because the absolute value of 0 is 0 and 0 is neither positive or negative. What is the absolute value of negative 32? *(32)* **What symbol can we write between the absolute value of –32 and 32?** *(equal sign)*

2. Students write comparison generalization statements in the Notes section of their Student Booklets.

We have created generalizations about adding and subtracting integers. Now, think about a generalization for comparing integers. What do you notice when we compare a negative number and a positive number? *(the positive number is always greater)*

What do you notice when you compare 2 negative integers? Think about negative 95 and negative 103. Which number is greater? *(–95)* **How can you describe negative 95 in regard to its position on a number line as compared to the position of negative 103?** *(it is farther to the right than –103 OR –103 is farther to the left than –95.)*

Turn to the Notes section of your Student Booklet. Write the inequality statement comparing negative 83 and negative 17. Which integer is greater, negative 83 or negative 17? *(–17)* **Why?** *(it is farther to the right on a number line)* **Write an inequality that shows the relationship between negative 7 and positive 2.** *(have students share their inequality with their partner; correct as needed)* **Which number is greater?** *(+2)* **Why?** *(greater than 0, positive integer is always greater when comparing a negative integer and a positive integer)*

Write an inequality that shows the relationship between positive 10 and positive 36. *(have students share their inequality with their partner; correct as needed)* **Which number is greater?** *(36)* **Why?** *(farther to the right on a number line)*

Have students turn to the Generalization page in the Notes section of their Student Booklets. Write the generalizations on the board or in the Teacher Masters Generalization sheet for students to copy in their Student Booklets.

Look at the examples you wrote in the Notes section of your Student Booklets. What pattern or relationship do you notice? *(the number farther to the right on a number line is always greater OR the number farther to the left on a number line is always less)*

When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed that the number farther to the right

on a number line is always greater OR the number farther to the left on a number line is always less.

Turn to the Generalization sheet in the Notes section in your Student Booklet. What generalization can we write to describe how we compare 2 integers? *(the number farther to the right on a number line is always greater OR the number farther to the left on a number line is always less)*

In the first column write this generalization: "The number farther to the right on a number line is always greater OR the number farther to the left on a number line is always less." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

3. Students write an equation, using 2 expressions on their whiteboards.

We have also been practicing adding and subtracting integers. Write the expression negative 5 plus 3 on your whiteboard. What is the sum of negative 5 plus 3? *(–2)*

Write this sum to make the equation, negative 5 plus positive 3 = negative 2.

Under your equation, write another addition or subtraction equation that has –2 as the sum or difference.

Allow students time to work, then, ask several students to share the equations they wrote.

The expressions that you wrote are both equal to negative 2. We can write an equation to show that two expressions represent or equal the same amount.

Write "–5 + 3 = 2 + (–4)" on the board.

Here is my example. Negative 5 plus 3 equals 2 plus negative 4. This is a true statement because the expressions, negative 5 plus 3 and 2 plus negative 4, equal negative 2. Notice how I placed parentheses on both sides of negative 4. Why did I do

this? *(answers will vary, such as it can be confusing having two symbols next to each other, so using parentheses can help avoid confusion)*

Write an equation like mine using your own expression.

Allow students time to work, then, ask several students to share the equations they wrote. Discuss any that are unique. If students do not agree with one that is shared, have them verify by using a number line that is drawn on the whiteboard to show that both expressions equal –2.

4. Students determine whether 2 expressions are equal.

Write "–8 – 9 = –8 + 9" on the board. As the lesson progresses, write "–8 – 9 < –8 + 9" and "–8 + 9 > –8 – 9."

Simplify these 2 expressions on either side of the equal sign.

What is the difference of negative 8 minus 9? *(–17)* **What is the sum of negative 8 plus 9?** *(1)* **Are these expressions equal?** *(no)* **Write a line through the equal sign to show that this is not an equal equation, like this.**

Draw a line through the equal sign (≠).

This is another type of inequality sign and we read as "not equal to."

We say this is an inequality because the 2 expressions do not equal the same value.

Another way to write this inequality is negative 8 minus 9 is less than negative 8 plus 9. Is this true? *(yes)* **Why?** *(–17 is less than 1 and 9 is a positive number OR 1 is farther to the right on the number line)*

Is this inequality true: negative 8 plus 9 is greater than negative 8 minus 9? *(yes)* **Why?** *(1 is a positive number and –17 is a negative number OR 1 is farther to the right on the number line)* **Because these 2 expressions are not equal, we can write the inequality either way.**

Write another inequality that shows an expression that is greater than –8 – 9.

Have students write the inequality on their whiteboards, using the greater than or less than sign, as in the example above. Have students write both possible inequalities (–8 – 9 > their expression and their expression < –8 – 9).

Practicing Together

Activity 1. Have students work in pairs. Assign each pair 2 integers to use as the solution to an expression. You can select the integers to use. Each pair will write on their whiteboards an inequality using the expressions they have written. Have students share with other pairs to their check work.

I will give you 2 integers. First, you will write an expression on your whiteboard that results in each integer. Then you will write an inequality comparing the two expressions, using a greater than or less than symbol.

For example, let's say my group was assigned the integers, negative 10 and 8. We could write the expression negative 10 plus 0 for negative 10 and the expression 4 + 4 for 8. Then, we could write the inequality negative 10 plus 0 is less than 4 plus 4.

What other expressions can we write that result in a value of negative 10 and 8? *(accept reasonable answers, such as 16 + (–8) > –5 – 5)*

Activity 2. Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete the Practicing Together sheet. Review the sheet as a class when complete. Call on different groups to explain how they found the solutions.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write two important ideas.

> **You are asked to help a new seventh-grader understand integers. What are the 2 most important ideas you would share with the student? Write these 2 ideas in the Notes section of your Student Booklets.**

Have students share their work and check for understanding.

Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Module Check Form A**	Lesson 7: Mid-assessment **Module Check Form B** ✔	Lesson 15: Post-assessment **Module Check Form C**

Integers
Lesson 8

Lesson 8: Practice Adding and Subtracting Integers

Lesson Objectives	Students use addition and/or subtraction to solve word problems with integers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Integer, expression, equation, variable	
Misconception(s)	Students often confuse relationships expressed in word problems, especially if students only focus on key words.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • Red colored pencil for each student

Warming Up

Review how to add and subtract integers.

Give each student a whiteboard and have students turn to the Generalization page in the notes section of their Student Booklets.

> **When we add 2 positive integers, what sign will the sum have?** *(positive)*
>
> **When we add a negative integer and a positive integer, will the sum be positive or negative?** *(could be either)* **How do you know what the sign of the sum will be?** *(accept reasonable answers relating to earlier generalizations that focused on absolute value of the addends or model representations)*
>
> **You are a number detective. I'm going to give you clues and you have to find the addends that match the clues. Write the addends on your whiteboard.**
>
> **Your sum is –29. One addend is positive and the other is negative. What are the addends? In other words, what 2 integers added together equal –29?** *(accept all expressions, writing them on the whiteboard, such as –58 + 29)*

Give time for students to work, then allow them to share. Write their responses as they share on the whiteboard. Have students verify that the addends meet the criteria. Ask for multiple solutions. Continue with additional clues if students struggled (as time permits).

Learning to Solve

TEACHER NOTES

When solving word problems, students may search for key words. The key-word approach is problematic because 1) key words are not necessarily specific to the mathematical process, 2) key words are not applicable to multi-step problems, and 3) key words do not motivate students to consider the relationships expressed in the problem.

1. Students solve word problems with negative and positive integers.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete each step as the lesson progresses.

   In the real world, when are integers used? *(answers may vary; for example, temperature, money, sea level, and so on)* **Think about temperature. When the temperature is above 0, is the sign negative or positive?** *(positive)* **How would we write the temperature 15 degrees below 0?** *(–15)*

   Now, think about money. If you had a savings account or checking account and your balance was $300, is the sign negative or positive? *(positive)* **How much money would you have to spend to have a balance of $0?** *($300)*

   A positive amount in your checking account means that you have that much money you can spend. What do you think a negative amount would mean? *(answers may vary, such as students may say that they owe the bank that much money, the checking account is overdrawn)*

   Look at your sheet. Let's read the first problem together. Debbie has $213 in her checking account. She wrote checks for $23, $126, and $67. The bank called her and said she had

overdrawn her account, meaning she spent more money than she had in the bank. What was her new balance?

Before you answer the question, what is the question asking? *(what is the current balance of her checking account)*

Solve the problem with a partner.

What was your solution? *(–$3 or overdrawn $3)* How did you solve it? (*answers may vary, such as some students may add the amount of the checks and then subtract from $213; other students may subtract, for instance, $213 – 23, then successively subtract the remaining amounts, one at a time)*

As students share their solutions, have them either display their work or you can record the steps as they describe the process. Ask for other solution methods. The two methods listed above are the most common methods. Students should note that both processes lead to a common solution.

How much money would Debbie need to deposit, or put into her account, for the total to no longer be negative? *(at least $3)*

Look at problem 2. A scuba diver was 430 feet below sea level. He rose at a rate of 12 feet per minute. What was the depth of the diver after 9 minutes? Work with your partner to solve the problem.

What was your solution? *(322 feet below sea level or –322 feet)* **How did you solve it?** (*answers may vary, such as some students may multiply 12 x 9, then add to –430; other students may add 12 nine times to –430)*

As students share their solutions, have them either display their work or you can record the steps as they describe the process. Ask for other solution methods. The two methods listed above are the most common methods. Students should note that both processes lead to a common solution.

Look at problem 3. The price of a share of stock started the day at $24. During the day, it went up $3, down $4, down $7,

and up $6. What was the price of a share of the stock at the end of the day? Work with your partner to solve the problem.

What was your solution? *($22)* **How did you solve it?** (*answers may vary, such as, some students may add the increases and the decreases ($3 + 6 = $9; –4 + (–7) = –11), then add to $24 ($24 + 9 + (–11) = $22; others may perform each calculation in the order of the problem ($24 + 3 – 4 – 7 + 6 = $22)*)

As students share their solutions, have them either display their work or you can record the steps as they describe the process. Ask for other solution methods. The two methods listed above are the most common methods. Students should note that both processes lead to a common solution.

Practicing Together

There is no Practicing Together for this lesson.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Turn to the Wrapping It Up sheet in your Student Booklet. Complete the Magic Square with a partner.**

Have students share their work and check for understanding.

Integers
Lesson 9

Lesson 9: Multiplication of Integers

Lesson Objectives	Students identify patterns in multiplication problems and use the commutative property to solve. Students state and write generalizations related to multiplying integers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Factors**: numbers being multiplied together **Product**: the answer to a multiplication problem **Commutative property:** factors can be multiplied in any order, resulting in the same product	
Requisite Vocabulary	Expression, equation, generalization	
Misconception(s)	Students may think of multiplication as repeated addition. This does not work with multiplying, for example, -4×3.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • 2 different colored pens • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • 2 different colored pencil • 2-colored chips if needed • Red colored pencil for each student

Warming Up

Review generalizations about subtraction with integers.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Give each student a whiteboard.

> Carmen said, "15 – (–18) = –3 because you add a negative number to find the difference." Do you agree with Carmen? Why or why not? *(No, Carmen is not correct. To subtract, you add the opposite of the subtrahend. The opposite of –18 is 18. 15 – (–18) = 15 + 18 = 33)*

Give time for students to work, then allow them to share. If needed, students can model the subtraction problem with pictures or 2-colored chips.

Learning to Solve

TEACHER NOTES

Similar to addition and subtraction, the goal is not for students merely to recite the rules for multiplication of integers, but rather to identify the patterns and form generalizations from the examples. Students may try to use the notion of repeated addition but this strategy does not work with integers.

Write generalizations in words only, not in symbols and words.

1. Students define vocabulary in multiplication.

Display the Learning to Solve sheet located in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Write the equation 4 × 2 = 8 on the board.

> Read the equation. (4 × 2 = 8) We will label each part of this equation with the vocabulary we use with multiplication.

Look at the numbers being multiplied; these are factors. Name the 2 factors in this expression. *(4 and 2)* Below 4 and 2, label these numbers as factors.

The answer to a multiplication problem is the product. What does 4 × 2 equal? *(8)* What is the product? *(8)* Below 8, label this number as product.

2. Students identify patterns when multiplying positive numbers by positive numbers and when multiplying positive numbers by negative numbers.

Have each student use 2 different colored pencils to label the integers—one color for the factors and the other color for the product.

Complete the Learning to Solve sheet in the Teacher Masters as the lesson progresses using 2 different colors.

First, let's review basic multiplication. Look at problem 1 with the problems, 6 × 6, 6 × 5, and 6 × 4. Write the 3 products.

What pattern do you notice about the factors? *(one factor stays the same, the other factor decreases by 1.)* Write this pattern below the question.

What pattern do you notice about the products? *(the products decrease by 6 each time)* Write this pattern below the question.

What is the sign of both factors? *(Both factors are positive)* What is the sign of the product? *(positive)* Write that.

Will the product always be positive when the factors are positive? *(yes)* Explain your reasoning. *(answers may vary, such as multiplying positive integers results in a positive product)*

Look at problem 2 with the next set of multiplication problems. 6 × 3 and 6 × 2 are already done. Continue the pattern, decreasing the second factor by 1. What are the next 4 factors? *(1, 0, –1, –2)*

Now continue the pattern for the products, decreasing by 6. What are the products? *(6, 0, –6, –12)*

What pattern do you notice about the products? *(the products decrease by 6 each time)* Write it below the question.

When both factors are positive, what is the sign of the product? *(positive)* Write it below the question.

When one factor is positive and the other factor is negative, what is the sign of the product? *(negative)* Write it below the question.

Multiplication is similar to addition because the commutative property can be applied to the factors. The commutative property states that we can change the order of the factors or addends and not change the answer. When you multiply a negative integer and a positive integer, such as 6 × –2, the product is negative. In the same way, when you multiply a positive integer and a negative integer, such as 2 × –6, the product is still negative. We can also think of this in another way. For example, when you have 2 groups of negative 6, you get the product negative 12.

Work with a partner and identify two examples of factors that, when two positive and negative groups are multiplied together, give you a product of negative 20. *(answers may vary, such as 10 × -2 = –20; when you have 10 groups of negative 2, you get the product negative 20; 4 × -5 = 20)*

2. Students identify patterns when multiplying negative numbers by negative numbers.

   Now turn to problem 3 in your Student Booklets. Using the generalization that a positive integer times a negative integer equals a negative integer, find the products.

   What are the products? *(–18, –9, 0)*

As you ask the following questions, write the answers on the whiteboard and have students write the answers in their Student Booklets.

What pattern do you notice about the products? *(they increase by 9 each time)* Write it below the question.

When one factor is negative and the other factor is positive, what do you notice about the product? *(the product is negative)* Write it below the question.

When one factor is negative and the other factor is 0, what do you notice about the product? *(the product is 0)* Write it below the question.

Continue the pattern for both the second factor and the products for the next 4 equations. Be sure to write your answer under each question.

Complete the Learning to Solve sheet while asking the following questions.

What are the values of the second factor? *(–1, –2, –3, –4)* What are the products? *(9, 18, 27, 36)*

Look at the first factor. What is the sign of the integer? *(negative)*

Look at the second factor. What is the sign of the integer? *(negative)*

Look at the products. What is the sign of the integer? *(positive)*

What do you notice about the sign of the product when both factors are negative? *(the product is positive)* What do you notice about the sign of the product when 1 factor is negative and 1 factor is positive? *(it is negative)*

Practicing Together

Have the students turn to the Generalization sheet in the Notes section of their Student Booklets. Have students write the four generalizations about the patterns on the Learning to Solve sheet. Write the generalizations on the whiteboard or in the Teacher Masters Generalization sheet for students to copy.

> When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed patterns when multiplying negative and positive integers.
>
> Turn to the Notes section in your Student Booklet. Turn to the Generalization sheet. We will write 4 generalizations.
>
> First, in the first column write this first generalization: "A negative integer times a positive integer equals a negative integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.
>
> Second, in the first column write this second generalization: "A positive integer times a negative integer equals a negative integer. In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.
>
> Third, in the first column write this third generalization: "A negative integer times a negative integer equals a positive integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.
>
> Fourth, in the first column write this fourth generalization: "A positive integer times a positive integer equals a positive integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It On Your Own, branch to Lesson 9A to provide extended practice before proceeding to Lesson 10.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Ask students to decide whether each equation is true or false. Have students raise their hands, give a thumbs-up, or stand if they think an equation is true or put their hands on their desk, give a thumbs-down, or stay seated if an equation is false.

> Previously, we used fact families for addition and subtraction. Multiplication and division are inverse operations, meaning they have a relationship that let us use fact families with the factors and products. We worked on multiplication, but using our knowledge of fact families, think for a minute about these division problems and determine whether each equation is true or false. Be able to explain your thinking of why you chose true or false.

Go through the equations on the activity sheet with students.

Lesson 10: Division of Integers

Lesson Objectives	Students identify patterns of division to solve. Students state generalizations of dividing with integers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Dividend**: the number or quantity being divided, the whole amount **Divisor**: the number or quantity to be divided into the dividend **Quotient**: the answer to a division problem	
Requisite Vocabulary	Expression, equation, inverse operations	
Misconception(s)	Students do not see division as a related operation to multiplication and thus do not apply the generalizations from multiplication to division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • 2 different colored pencils • Red colored pencil for each student

Warming Up

Review fact families using different operations to find unknowns.

Give each student a whiteboard.

Write the equation 12 + 17 = ______.

What is the sum of 12 + 17? *(29)* Using your knowledge of fact families, write 2 subtraction equations, using these 3 numbers.

How are addition and subtraction related? *(they are inverse operations)*

Review adding and subtracting integers.

Find 2 integers whose sum is –8. *(answers may vary, such as –6 + –2, –15 + 7)*

Find 2 integers whose difference is –8. *(answers may vary, such as –6 –2, –15 – (–7))*

Learning to Solve

Teacher Notes

Similar to addition and subtraction, the goal is not for students to merely recite the rules for division of integers, but rather to identify the patterns and form generalizations from the examples. They will use generalizations from the multiplication lesson to form similar generalizations for division of integers.

Write generalizations in words only, not in symbols and words.

1. Students review vocabulary of division.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Read the equation in problem 1. *(12 ÷ 3 = 4)* We will label each part of this equation with vocabulary we use with division.

The first number is the dividend. Write "dividend" on the line below 12. The next number is the divisor, the number or quantity of groups the dividend will be divided by. Write "divisor" on the line. The solution in a division problem is the quotient. Write "quotient" on the line.

You have also seen division written in another way. Look at problem 2. Which number is the dividend? *(12)* Draw a line to connect the word "dividend" to the 12. Where is the divisor? *(to the left)* Draw a line from the word "divisor" to the number 3. What is the mathematical name for the solution, 4? *(quotient)* Draw a line from the word "quotient" to the number 4.

2. Students identify patterns when dividing a positive integer by a positive integer and a positive integer by a negative integer.

Have students turn to the next sheet in their booklets. Have each student use 2 different colored pencils— one color for the quotient and the other color for the divisor.

Display the Learning to Solve sheet in the Teacher Masters and complete as the lesson progresses, using 2 different colored markers. Have students write answers on their sheets.

On the next page, look at problem 3. Find the quotients of the first 3 division problems in your book. What are the quotients? *(6, 5, 4)*

Complete the Learning to Solve sheet while asking the following questions.

What number is the divisor for each problem? *(6)* What number changes? *(the dividend)* What pattern do you notice about the dividend? *(they decrease by 6 each time)* Write the pattern below the question. The dividend decreases by 6 each time.

What pattern do you notice about the quotients? *(they decrease by 1)* Write the pattern below the question.

We can say that when the dividend decreases by 6, the quotient decreases by 1.

What pattern do you notice about the signs of the dividends and the divisors? *(they are both positive)* Write the pattern below the question.

What do you notice about the relationship between the sign of the quotient and the signs of the dividends and the divisors? *(the signs of the quotients are positive and the signs of the dividends and divisors are positive)* Write the pattern below the question. The signs of the quotients are positive and the signs of the dividends and divisors are positive.

Let's continue our pattern for the remaining equations, decreasing the dividend by 6 and the quotient by 1. The first two are done for you.

What are the next 4 dividends? *(0, –6, –12, –18)* What are the quotients for these problems? *(0, –1, –2, –3)*

0 is neither positive or negative. Other than the 0, what is the sign of our new dividends? *(negative)* What is the sign of the quotients? *(negative)* In thinking about the signs, what pattern do you notice? *(when we divide a positive integer by a negative integer, the quotient is negative)* Write the pattern below the question. When we divide a positive integer by a negative integer, the quotient is negative.

Think about fact families and our multiplication generalizations. Using the last equation, $-18 \div 6 = -3$, what multiplication equations can we write that would be in the same fact family? *($6 \times -3 = -18$ and $-3 \times 6 = -18$)*

Do these equations match the multiplication generalization that a positive integer times a negative integer equals a negative integer? *(yes)* Is it reasonable that –18 divided by +6 would equal a negative integer? *(yes)* Why? *(accept reasonable answers with a focus on the inverse relationship with multiplication)*

3. Students identify patterns when dividing a negative integer by a negative integer.

Now solve the 3 division problems in problem 5.

What is the sign of each dividend? *(positive)* Write positive below the question. What is the sign of each divisor and quotient? *(negative)* Write negative below the question. What are the quotients? *(–2, –1, –0)* Is it reasonable that a positive integer divided by a negative integer equals a negative integer? *(yes)* Why? *(accept reasonable answers; students should focus on the inverse relationship between multiplication and division)*

What pattern do you notice about the dividend? *(decreases by 5)* Write the pattern below the question.

Does the divisor change? *(no)* What about the quotient? *(increases by 1)*

Continue the pattern—the dividend decreasing by 5 and the quotient increasing by 1—for the next 4 equations.

Have students write the next 4 equations for problem 6.

Look at problem 6. we left off with the divisors being 10, 5, and 0. Continue the pattern with the 4 equations in number 6.

What are the dividends? *(–5)* What are the divisors? *(–5, –10, –15)* What are the quotients? *(1, 2, 3, 4)* What are the signs of the dividends? *(negative)* What are the sign of the divisors? *(negative)* What are the sign of the quotients? *(positive)* Thinking about multiplication generalizations, is it reasonable that a negative integer divided by a negative integer equals a positive integer? *(yes)* Why? *(students should focus on the inverse relationship between multiplication and division)*

Practicing Together

Have students turn to the Generalization page of their Notes section of the Student Booklets. They will write the generalizations they learned in the patterns on the Learning to Solve sheet. Write the generalizations on the whiteboard or in the Teacher Masters Generalization sheet for students to copy.

When we note patterns or similarities, we call them a generalization. A generalization is formed from patterns and relationships. We noticed patterns when dividing negative and positive integers.

Turn to the Notes section in your Student Booklet. Turn to the Generalization sheet. We will write 4 generalizations.

In the first column write this first generalization: "A negative integer divided by a positive integer equals a negative integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization. How is this generalization linked to multiplication? *(a negative factor times a positive factor equals a negative product, and there is an inverse relationship between multiplication and division)*

In the second column write this second generalization: "A positive integer divided by a negative integer equals a negative integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

In the third column write this third generalization: "A negative integer divided by a negative integer equals a positive integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

In the fourth column write this fourth generalization: "A positive integer divided by a positive integer equals a positive integer." In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers. Students mark the total number correct at the top of their page.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It On Your Own, branch to Lesson 10A to provide extended practice before proceeding to Lesson 11.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write two statements.

> Write two statements that describe something you learned today or surprised you about dividing with integers.

Have students share their work and check for understanding.

Integers
Lesson 11

Lesson 11: Solving Multiplication and Division Word Problems

Lesson Objectives	Students solve multiplication and division word problems containing integers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Equation, integers, generalizations	
Misconception(s)	Students read word problems for key words, not thinking about the relationships expressed in the problem.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil for each student

Warming Up

Review multiplication and division of integers.

Display the Generalization page from the Notes section of the Teacher Masters. Have students turn to the Generalization page in the Notes section of their Student Booklets.

As students add examples for the generalizations, write a few examples on the whiteboard or in the Teacher Masters Generalization sheet. Have students share their responses.

We have been practicing multiplying and dividing integers.

I will ask you some questions about the generalizations we found, and then you will write equations to illustrate the generalizations.

1. **If you multiply 2 negative integers, what can you say about the sign of the product?** *(the product will be positive)* **Write an example.**
2. **If you divide 2 negative integers, what can you say about the sign of the quotient?** *(the quotient will be positive)* **Write an example.**
3. **If you multiply a negative integer times a positive integer, what can you say about the sign of the product?** *(negative)* **Write an example.**
4. **If you multiply a positive integer times a negative integer, what can you say about the sign of the product?** *(negative)* **Write an example.**
5. **If you multiply a positive integer times a positive integer, what can you say about the sign of the product?** *(positive)* **Write an example.**
6. **If you divide a positive integer by a negative integer, what can you say about the sign of the quotient?** *(negative)* **Write an example.**

7. **If you divide a negative integer by a positive integer, what sign can you say about the sign of the quotient?** *(negative)* **Write an example.**

8. **If you divide a positive integer by a positive integer, what can you say about the sign of the quotient?** *(positive)* **Write an example.**

Learning to Solve

TEACHER NOTES

When solving word problems, students may search for key words. The key-word approach is problematic because 1) key words are not necessarily specific to mathematical process; 2) key words are not applicable to multi-step problems; and 3) using key words does not motivate students to consider the relationships expressed in the problem.

1. Have students read and determine the operation needed to solve word problems.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete each problem as the lesson progresses. Students may work in pairs or work as an entire class if more appropriate.

   Select a student to read problem 1.

   Follow along as [student] reads problem 1 about Sherman.

   Sherman's football team lost 8 yards on each of 3 consecutive plays. How many total yards did they lose?

   Work with your partner and solve the problem. Be ready to explain how you solved it.

Give students time to solve the problem. Have students share the solution and the process they used. Ask that they show their work so that others can see how they solved the problem as they explain.

> **What was your solution?** *(–24 yards)* **How did you solve it?** *(answers may vary, such as multiply 3 x –8; students may need help understanding that in football losing yards is negative gains; drawing a football field on the board to illustrate this idea may help some students better understand this problem)* **Did anyone solve it differently?** *(if any student used repeated addition, encourage him or her to use multiplication)*

> **Look at the problem 2.**

Select a student to read problem 2.

> Follow along as [student] reads problem 2 about the length of the island.

> **The length of an island is shrinking at a rate of 19 inches per year. How long will it take for the island to shrink 209 inches?**

> **Work with your partner and solve the problem. Be ready to explain how you solved it.**

Give students time to solve the problem. Have students share the solution and the process they used. Ask that they show their work so that others can see how they solved the problem as they explain.

> **What was your solution?** *(11 years)* **How did you solve it?** *(answers may vary, such as divide 209 by 19)* **Did anyone solve it differently?** *(if any student used repeated subtraction, encourage him or her to use division)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, small groups, or the entire class, depending on student needs.

2. If done in groups, review the answers, asking students to model how the problems were solved.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write 3 solutions.

Write 3 solutions: Find 2 integers such that their product is –36. *(answers may vary, such as 12 and –3, –6 and 6, 9 and –4)*

Have students share their work and check for understanding.

Integers
Lesson 12

Lesson 12: Order of Operations With Integers

Lesson Objectives	Students simplify numerical expressions with integers, using order of operations. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Associative property of addition:** if *a*, *b*, and *c* are real numbers, then $a + (b + c) = (a + b) + c$ **Distributive property of multiplication over addition:** if *a*, *b*, and *c* are real numbers, then $a \times (b + c) = (a \times b) + (a \times c)$ **Order of operations:** the rules of how to simplify expressions containing different operations **Simplify an expression**: writing an equivalent expression in the most compact or efficient manner	
Requisite Vocabulary	Commutative property, expressions, equations	
Misconception(s)	Students sometimes misinterpret the order of operations, especially with the acronym PEMDAS. They think that multiplication, for example, is always done first, before division. Similarly, they think that addition is always performed before subtraction.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • Sticky notes • Red colored pencil for each student

Warming Up

Review the associative, commutative, and distributive properties of addition and multiplication.

Give each student 3 sticky notes and a whiteboard.

Write the number 2 on one sticky note, 3 on the 2nd sticky note, and 5 on the 3rd sticky note. We will use these 3 numbers to review properties of multiplication and addition.

Place the sticky notes on your whiteboard and write an addition symbol, on your whiteboard, between each sticky note, 2 + 3 + 5. What is 2 + 3 + 5? *(10)* **Does it matter what order we add the numbers?** *(no)* **Why?** *(changing the order of the addends does not change the sum)*

What property refers to being able to change the order of addends in addition and getting the same solution? (*the commutative property of addition*)

Change the addition symbols to multiplication. What is the answer? *(30)* **Does it matter what order we multiply numbers?** *(no)* **Why?** *(changing the order of the factors does not change the product)*

What property refers to being able to change the order of factors in multiplication? *(the commutative property of multiplication)* **Notice how similar the commutative property of addition is to the commutative property of multiplication.**

Does the commutative property work with subtraction? *(no)* **Can we subtract in any order?** *(no)* **Why?** *(answers may vary, such as when the order is changed, it changes the difference, for example, 5 – 3 ≠ 3 – 5.)*

Can we change the order in division? *(no)* **Why?** *(answers may vary, such as when the order of the dividend and the divisor are changed, this changes the quotient, for example, 20 ÷ 5 ≠ 5 ÷ 20)*

Now move the 5 next to 2 and place parentheses around the 2 + 5. Leave the +3 outside of the parentheses. The parentheses

group the addition of 2 and 5. If we change the grouping, we can use the associative property. We solve the problem in the parentheses first and then perform the next operation. What is the sum? *(10)* **Erase your parentheses and regroup the 5 with the 3. Place parentheses around 3 + 5 and leave the +2 outside of the parentheses. What is the sum?** *(10)*

Change the addition symbols to multiplication. What is the answer? *(30)* **Does it matter how we group or regroup the 3 numbers?** *(no)* **Why?** *(answers may vary, such as the associative property of multiplication allows us to group the factors in different ways without changing the product)*

Write "2(3 + 5)" on the whiteboard.

Now show 2(3 + 5). What operation is it when a number is outside of but next to parentheses, the 2 in this case? *(multiplication)* **First, perform the operation in the parentheses. What is 3 + 5?** *(8)* **What is 2 × 8?** *(16)*

Let's try it another way. This time we will multiply both addends of the expression in the parentheses by 2, resulting in the expression, 2(3) + 2(5). What is 2(3) equal to? *(6)* **What is 2(5) equal to?** *(10)* **What does 6 + 10 equal?** *(16)*

This problem models what is called the distributive property of multiplication over addition. We will be using these properties in this lesson.

Learning to Solve

Teacher Notes

Even though the order of operations provides a structure for performing computations, there is still some flexibility. For example, in the expression 3(4 – 3) + 8 ÷ 4, the division can be performed first without changing the outcome.

1. Students learn the order of operations.

   Write the expression 5 + 3 • 2 ÷ 1 on the board. Do not talk about order of operations or write any grouping symbols. Have students simplify the problem on their whiteboard as it is written, based on their best judgment.

   > **Write this expression on your whiteboard. Simplify the expression by performing the computations. Simplifying means writing an equivalent expression in the most compact or efficient manner.**

   Allow students time to simplify the expression. Have them work independently. When they are finished, have them share their answers. Record the answers on the board.

   > **There can be different answers for this problem, depending on how you chose to solve. To avoid confusion, mathematicians have agreed to an order of operations. This guarantees that when we simplify an expression, we will all get the same answer.**
   >
   > **The order of operations is a certain order for performing computations in math. We can use the acronym PEMDAS to remember the order of operations.**

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Fill in the Teacher Masters sheet for students to copy.

   > **Turn to the Learning to Solve page in your Student Booklet.**
   >
   > **Write "Order of Operations" on the top line. Now write the letters "PEMDAS," with 1 letter at the beginning of each line.**
   >
   > **The first letter is "P." The P stands for parentheses, the first step or order. If parentheses or any other grouping symbol, such as brackets or braces, are part of an expression or equation, you complete any operation in parentheses first. Write "parentheses and other grouping symbols" next to the "P."**

The next letter is "E." The E stands for exponents; the next step is to calculate any exponents, if they are part of the problem. Write "exponents" next to the "E."

The third and fourth letters are "M" and "D," which stand for multiplication and division. You perform either of these operations next, depending on the left to right order that they appear in. Write "multiplication" next to the "M" and "division" next to the "D."

"A" and "S" are the last 2 steps. What operations do you think those letters stand for? *(addition and subtraction)* **You perform either of these operations next, depending on the order they appear. Write "addition" next to the "A" and "subtraction" next to the "S."**

Point to the expression 5 + 3 • 2 ÷ 1 on the board.

Simplify the expression you wrote on your board again, using the order of operations. What is the answer? *(11)* **What operation did you calculate first?** *(multiplication, 3 • 2)* **Then what did you do?** *(divided, 6 ÷ 1)* **The last step?** *(added, 5 + 6)*

Who completed it differently but still got the result of 11?

Simplify expressions, using order of operations. Display the expressions on the board as the lesson progresses or have a student complete them after discussion. Fill in the Teacher Masters sheet for students to copy.

Let's try simplifying an expression, using the order of operations. Look at the first expression on your Learning to Solve sheet.

What is the first step in order of operations? *(parentheses)* **There are no parentheses in this expression.**

There also are no exponents, so what do we do first? *(multiplication)* **What is 5 times 1?** *(5)*

What is the next step? *(division)* **What are we dividing 10 by?** *(5)* **What is the quotient?** *(2)*

After you divide and multiply, the expression is now 2 + 2 + (–4). Now what do we do? *(add)* **What is the sum?** *(0)*

Look at the next expression on your sheet. Use the order of operations to simplify the expression.

What is the answer? *(–24)* **Explain how you simplified, making sure to use the mathematically correct vocabulary like "factor," "product," "quotient," and "sum."**

Have students explain how they simplified, identifying mistakes in applying order of operations.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, in small groups, or with the entire class, depending on student needs.
2. Review the answers, asking students to model how the problems were solved.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write in the Notes section of their Student Booklets when answering the following question.

> **Rate your understanding of order of operations on a scale of 1 to 5. A rating of 1 means that you have a lot of questions. A rating of 5 means that you could easily explain it to a new student.**

Have students share their ratings and discuss any unanswered questions.

Integers
Lesson 13

Lesson 13: Writing and Solving Equations With Integers

<table>
<tr><td>Lesson Objectives</td><td colspan="2">Students use order of operations to simplify expressions containing integers.
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr><td>Vocabulary</td><td colspan="2">None</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Order of operations, integer, expression, equation, variable</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students think that there are specific key words that will provide clues about which operation to use to solve a problem.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Red colored pencil for each student</td></tr>
</table>

Warming Up

Students complete "always, sometimes, never" statements.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets and complete the items with a partner.

> **Look at the table. The first column is a generalization. In the next column, you will write whether the generalization is always true, sometimes true, or never true. In the last column, write an example to justify the "always true" statements. For generalizations that are "sometimes true," write a true example and an example that is not true.**

Have students complete and then share their examples, writing them for the class to see.

Learning to Solve

1. Review order of operations.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **We will review the order of operations. Write the order of operations by filling in the blanks. Write what each letter means.** *(parentheses and other groupings, exponents, multiplication, division, addition, subtraction)*

> **Look at the expression in number 1. Simplify the expression.**

Have students evaluate and then share their answer. Ask them to explain the order of operations they used.

> **Look at the word problem in number 2.**

Select a student to read the problem.

> **Follow along as [student] reads.**

A store sells ribbons for $3 each and pins for $2. Cora buys 3 ribbons and a pin. Jack buys 2 ribbons and 5 pins. Find the total amount spent.

Write an expression to use to find the total amount they spent. Then, simplify your expression.

Provide time for students to work. Have them share the expressions they used to solve the problem. If they do not suggest a second expression, write the expression on the board.

How is the expression 3(3 + 2) + 6(2) related to the expression 3(3) + 2 + 2(3) + 5(2)? *(answers will vary, but students should notice that the distributive property of multiplication over addition allows them to write the second expression in a simpler way)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, in small groups, or with the entire class, depending on student needs.
2. Review the answers, asking students to model how the problems were solved.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write 2 – 3 sentences.

> **A friend is having trouble simplifying expressions, using order of operations. Write 2 to 3 sentences that could help your friend better understand how to use order of operations to simplify an expression.**

Have students share their work and check for understanding.

Integers
Lesson 14

Lesson 14: Solving Word Problems With Integers

Lesson Objectives	Students solve word problems containing integers. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Integer, expression, equation, generalizations	
Misconception(s)	Students think that there are specific key words that will provide clues about which operation to use to solve a problem.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil for each student

Warming Up

Review vocabulary related to integers and models of integers.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Have students complete the Warming Up sheet in their Student Booklets with a partner (or in small groups or as a class if appropriate).

> **With a partner, complete the Warming Up sheet, defining the vocabulary, drawing the models, and solving the problems.**

Have students complete and then review the answers.

TEACHER NOTES

Many students rely on key words to identify the operation needed to solve. Rather than focus on the key words, have the students analyze what the question is asking and whether or not the solution found is reasonable.

Learning to Solve

1. Students are introduced to multiplicative comparisons.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Select a student to read word problem 1 about Sean.

> **Follow along as [student] reads word problem 1 about Sean.**
>
> **Sean had $115 in the bank. He spent $35 on jeans and $63 on 3 shirts. He deposited $45 in the bank. How much does he have in the bank now?**
>
> **Work with a partner to find the amount that Sean spent.**

Give students time to solve the problem. Have students share their solution and the process they used. After one pair shares, ask if anyone solved it a different way.

> **How did you solve the problem? Who solved it a different way?** *(answers may vary—one method is to add the amounts spent, then subtract; finally, add the deposit; another method is to perform each computation as it appears in the problem)*

Select a student to read word problem 2.

> **Listen carefully to the next word problem as [student] reads word problem 2 about Margo.**
>
> **Margo was playing a trivia game. Each correct answer is worth 200 points, but if she answers incorrectly, she loses 300 points. She ended the game with –300 points. If she answered 6 questions, how many did she get correct? How many did she get incorrect?**
>
> **What are we trying to find?** *(the number of questions answered correctly and the number of questions answered incorrectly)*
>
> **Work with a partner to find the number of questions Margo answered correctly and incorrectly.**

Give students time to solve the problem. Have students share their solution and the process they used. After one pair shares, ask if anyone solved it a different way.

> **How did you solve the problem? Did anyone solve it a different way?** *(answers may vary; for example, some students may use guess-and-test; other students may use logical reasoning and their understanding of multiples)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, in small groups, or as a class if appropriate.

2. As a class, discuss how each pair or group solved the problems. Check for understanding and ensure that students use the correct mathematical vocabulary—for example, "factors," "products," "addends," and "quotient." Below are example questions:

    How did you solve?

    How did you know what operation to use to solve?

    Was there another way to solve?

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write 2 to 3 sentences.

Word problems can be solved using a variety of strategies including making a model, writing an equation, or using a number line. Write 2 to 3 sentences that explain a strategy you use most often when you solve a word problem.

Have students share their work and check for understanding.

Integers
Lesson 15

Lesson 15: Solving Multistep Word Problems

Lesson Objectives	Students solve multistep word problems containing integers by using order of operations. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Expression, inverse operation	
Misconception(s)	Students think that there are specific key words that will provide clues about which operation to use to solve a problem.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil for each student

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Apply concepts and skills of integer computations.

> **In the first box on your sheet, each block in the box has an operation symbol. Write 4 expressions, one in each block, using addition, subtraction, multiplication, and division that result in an answer of –36. In the second box, write 4 expressions that give a result of 28.**

Have students complete and then review.

> **Below the first box, write an expression with more than 1 operation that results in an answer of –36. Below the second box, write an expression with more than 1 operation that results in an answer of 28.**

Allow students time to complete. Have them share their expressions as time allows. Discuss the similarities and differences among the expressions students share. Have students share their expressions while other students check the computations.

Learning to Solve

Teacher Notes

Many students rely on key words to identify the operation needed to solve. Rather than focusing on key words, have students analyze what the question asks, the relationships given in the problem, and whether the solution makes sense.

1. Students solve word problems containing integers.

Display the Learning to Solve sheet located in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as the lesson progresses, making notes as needed for students to solve the problems.

Select a student to read the word problem 1 about Hunter.

> **Follow along as [student] reads word problem 1 about Hunter.**
>
> **Hunter read 15 books in October. He read 3 times as many books as Paul. How many books did Paul read?**
>
> **What are we trying to find?** *(how many books Paul read)* **What information do we know?** *(Hunter read 15, which is 3 times as many as Paul.)*
>
> **Work with your partner to solve the problem.**

Give students time to solve the problem. Have students share their solution and the process they used. After one pair shares, ask if anyone solved it a different way.

> **How did you solve the problem? Did anyone solve it a different way?** *(answers may vary, such as some students may use division; other students might use multiplication.)*

Select a student to read word problem 2 about the football team.

> **Follow along as [student] reads word problem 2 about the football team.**
>
> **The middle school football team scored 4 field goals worth 3 points each and 3 touchdowns with extra points worth 7 points each. What was the team's total points?**
>
> **Work with a partner to solve the problem.**

Give students time to solve the problem. Have students share their solution and the process they used. After one pair shares, ask if anyone solved it a different way.

How did you solve the problem? Did anyone solve it a different way? *(answers may vary, such as some students may multiply the number of scoring events by the number of points, and then add)*

With your partner, solve problems 3 and 4.

Give students time to solve the problems. Have students share their solutions and the processes they used. After one pair shares, ask if anyone solved it a different way.

Practicing Together

There is no Practicing Together in this lesson.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Return to displayed integer concept map completed in Lesson 1. Ask the following questions and update, change, and add to the concept map.

1. **What can we add to our concept map about integers?**
2. **What do we know about adding and subtracting integers?**
3. **What do we know about multiplying and dividing integers?**

4. What can we add to describe comparing integers and absolute value?
5. What examples can we add to help explain or highlight the ideas on our map?

Progress Monitoring Schedule

BEFORE	AFTER	AFTER
Lesson 1: Pre-assessment **Module Check Form A**	Lesson 7: Mid-assessment **Module Check Form B**	Lesson 15: Post-assessment **Module Check Form C** ✔

Appendices

Integers

Integers
Lesson 2A

Lesson 2A: Modeling and Comparing Integers on a Number Line

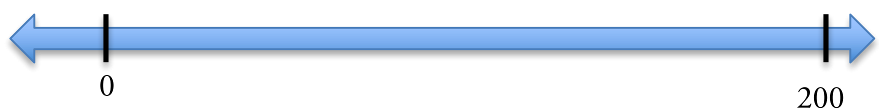
<table>
<tr><td rowspan="2">Lesson Objectives</td><td colspan="2">Students model and order integers.
Students explain and identify the absolute value of an integer on a number line.
Students explain that an integer and its opposite are the same distance from 0.
Students reason abstractly and quantitatively. (SMP 2)
Students attend to precision. (SMP 6)</td></tr>
<tr></tr>
<tr><td>Vocabulary</td><td colspan="2">Absolute value: a non-negative number that describes the distance of a number from 0 on a number line</td></tr>
<tr><td>Requisite Vocabulary</td><td colspan="2">Integer, zero pair, positive, negative</td></tr>
<tr><td>Misconception(s)</td><td colspan="2">Students often think that absolute value is always positive. This is partially true but can cause issues when they are confronted with – |7|, which equals –7.</td></tr>
<tr><td rowspan="2">Instructional Materials</td><td>Teacher</td><td>Student</td></tr>
<tr><td>• Teacher Masters
• Whiteboard (or equivalent)
• Projector (or equivalent)</td><td>• Student Booklet
• Whiteboard
• Dry-erase markers
• Red and yellow pencils for each student</td></tr>
</table>

Warming Up

Review the concept of a number line.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **This is an empty number line. A number line continues in both directions forever.**
>
> **For the first number line, place a 0 on one side and 200 on the other side.**



> **What number is halfway between, or in the middle of, 0 and 200?** *(100)* **Write "100" on the number line about where it should go.**
>
> **What type of integers could be placed on this number line?** *(positive)* **Think of 3 numbers that could be on this number line and write them where they should go.**

Have a few students add to the number line displayed.

> **Look at the second number line. We will use –25 and +25 as our end numbers. On which side do we write the –25?** *(left)* **On which side do we write the +25?** *(right)* **This time write a 0 in the middle. Think of 4 integers, 2 negative and 2 positive, that could be seen on this number line; and place them about where they should go.**

Have some students, as time allows, add their integers to the number line displayed. Ask students to determine the appropriateness of the placements of those numbers.

Learning to Solve

TEACHER NOTES

Encourage students to use the number line so they develop an understanding of the relationship between and among positive and negative integers. Using the number line will support their skill in comparing integers. Remember that plus and minus are not used when naming integers; use positive and negative. *|–4|* is read as absolute value of negative 4.

1. Students model integers on the number line.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. The additional number lines can be used or students can work from the first number line on the page.

We will use the number line to model integers. A number line can help us see the relationship of an integer to other integers.

Look at problem 1. Using the integers, +1, –6, +13, and –9, place them on the number line in their correct position.

Allow students time to work. Have a student share his/her number line. As it is shared, other students should check their placement to determine if they agree. Discuss any discrepancies.

Sometimes, we want to order integers in a particular way. If we ordered these integers in the order from least to greatest, what would the order be? *(–9, –6, +1, +13)* **Why are –9 and –6 less than +1?** *(–9 and –6 are negative numbers and +1 is a positive number, –9 falls farther to left than the other numbers and 13 is farther to the right)*

How does the number line help you order the integers? *(answers may vary; for example, a number line shows the order; on* *a horizontal number line, negative numbers are to the left of 0, and positive numbers are to the right of 0.)*

Look at problem 2. Using the integers, 0, –14, 9, and |–4|, place them on the number line in their correct position.

Allow students time to work. Have a student share his/her number line. As it is shared, other students should check their placement to determine if they agree. Discuss any discrepancies.

If we ordered these integers in the order from least to greatest, what would the order be? *(–14, 0, |–4|, -- the absolute value of negative 4, 9)*

Ask students why |–4| comes after 0 in the least to greatest order.

2. Students place the absolute value of integers on the number line.

Give each student a red marker or pencil and a yellow marker or pencil. Write "|–4|" and "|4|" on the whiteboard. Write the absolute value of each number below as the lesson progresses.

Point to the absolute value symbol.

This symbol indicates absolute value. Absolute value of a number is its distance from 0.

On the number line for problem 3, label 0, –4 and 4. With your red and yellow pencils, draw a red line from 0 to –4 and a yellow line from 0 to +4. What do you notice about the lengths of the two line segments? *(same)*

Absolute value is the distance from 0. We can say that –4 is 4 units away from 0 and that +4 is 4 units from 0. The absolute value of 4 and the absolute value of –4 are the same. *(write 4 below "|–4|" and "|4|" written previously on teacher's whiteboard)*

What is the absolute value of –8? *(8)* **+8?** *(8)*

The absolute value of a number is always non-negative. Why would we say non-negative instead of positive? *(answers may vary but students should recognize that the absolute value of 0 is 0 and 0 is neither positive or negative; non-negative is the same as* *"greater than or equal to zero" ≥0; positive is the same as "strictly greater than zero" >0)*

Now look at number 4. Label each number on the number line; then use the number line to place the integers in their order. Remember to do the absolute value first, before you put it on the number line.

Allow students time to work. Have a student share his/her number line. As it is shared, other students should check their placement to determine if they agree. Discuss any discrepancies.

Using your number line as a guide, order the integers from greatest to least. Write the order under the number line. What would the order be? *(5, |–4|, |–3|, 2, –1, –3)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, in small groups, or with the entire class, depending on student needs.
2. Ask each pair of students to provide the answer to one of the problems. Have each pair explain how they figured out the answers.
3. Provide answers for problems that require correction.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students answer the following question.

Write 2 integers that have an absolute value of 4. *(4 and –4)*

Have students share their work and check for understanding.

Integers
Lesson 9A

Lesson 9A: Multiplication of Integers

Lesson Objectives	Students identify patterns in multiplication problems and use the commutative property to solve. Students state and write generalizations related to multiplying integers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Factors, product, generalizations, commutative property, expression	
Misconception(s)	Students may think of multiplication as repeated addition. This does not work with multiplying, for example, –4 x 3.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • Red colored pencil for each student

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Sophia was on a game show that took away points for every wrong answer. She had a score of 800 but then answered 7 questions incorrectly. Each question was worth 300 points. What is her current score?

Allow students to work independently then ask the following questions.

How many points did she lose in all? *(2,100)* **How did you solve?** *(multiply 7 by 300)* **What did you do next?** *(subtracted 2,100 from 800)* **What was Sophia's score after she answered the 7 questions incorrectly?** *(–1,300)*

Is this reasonable? Why is her score negative? *(she lost more points than she had initially)*

Learning to Solve

1. Students identify generalizations when multiplying integers.

Give each student a whiteboard. Turn to the Generalization sheet in the Teacher Masters. Have students turn to the Generalization sheet in the Notes section of their Student Booklets. Write the generalizations on the whiteboard or in the Teacher Masters Generalization sheet for students to copy.

Let's look for patterns. We will write multiplication facts with one factor of 3. Write and simplify the following expressions: 3×4, 3×3, 3×2, 3×1, 3×0.

What is the pattern of the second factor in the expressions? *(decreases by 1)* **Look at the products. What is the pattern?** *(decreases by 3)*

We need to continue the pattern. Looking at the second factor, what is 1 less than 0? *(–1)* **How can we find the product of**

3×-1? *(subtract 0 – 3 or count back 3 from the previous product of 0)* **What is 3 less than 0?** *(–3)*

We can describe the pattern like this: As the second factor decreases by 1, the product decreases by 3.

Continue the pattern by writing the next 3 equations.

Have students write the next 3 equations: $3 \times -2 = -6$, $3 \times -3 = -9$, and $3 \times -4 = -12$.

Think about our pattern. When we decrease the second factor by 1, the product decreases by 3. Is this still true? *(yes)*

We wrote generalizations when we multiplied integers in our previous lessons. What is the sign of both of the factors and the product? *(positive)* **Turn to the Generalization page in the Notes section of your Student Booklets about multiplying integers. Looking at the generalizations you wrote, what generalization can we make when we multiply a positive integer times a positive integer?** *(product will be positive)* **In the third column, write 2 additional examples of a positive integer times a positive integer equals a positive integer as a product next to this generalization.**

Have students share some of the examples they wrote.

Look at the equations when we continued the pattern. What is the sign of the first factor? *(positive)* **What is the sign of the second factor?** *(negative)* **What is the sign of the product?** *(negative)* **What generalization did we write that would explain this pattern?** *(a positive integer times a negative integer equals a negative integer)*

Write 2 additional examples in the third column of a positive integer times a negative integer equals a negative integer as a product next to this generalization.

Have students share some of the examples they wrote.

Does the order of the factors change the product? *(no)* **What property states that we can change the order of the factors without changing the product?** *(the commutative property)*

This time write on your whiteboard 3 as the second factor. Write and solve 4×3, 3×3, 2×3, 1×3, and 0×3.

Have students complete the expressions.

What is the pattern of the factor and the product? *(as one factor decreases by 1, the product decreases by 3)*

What pattern do you notice about the signs? *(positive integer times a positive integer equals positive integer)*

Does changing the order of the factors change the sign of the product? *(no)*

Now, continue the pattern, keeping 3 as the second factor. Write the next 2 equations.

What is the sign of the first factor? *(negative)* **What is the sign of the second factor?** *(positive)* **What is the sign of the product?** *(negative)* **What generalization did we write that would explain this pattern?** *(a negative integer times a positive integer equals a negative integer)*

This pattern makes sense because it is using the commutative property. On the Generalization sheet, in the first column, write "The commutative property reverses the signs of the factors." In the next column, draw a picture if possible to represent the generalization. In the third column, write 2 examples of a negative integer times a positive integer equals a negative integer next to this generalization.

Have students share some of the examples they wrote.

Let's look at 1 more pattern. This time, write the expressions -3×2, -3×1, and -3×0 and give the products.

What pattern can we use to describe the factors and the product? *(as one factor decreases by 1, the product increases by 3)*

Continue the pattern, writing 3 more equations.

Have students write $-3 \times -1 = 3$, $-3 \times -2 = 6$, and $-3 \times -3 = 9$.

What is the sign of the first factor? *(negative)* **What is the sign of second factor?** *(negative)* **What is the sign of the product?** *(positive)*

What generalization about multiplying with integers did we write that would explain this pattern? *(a negative integer times a negative integer equals a positive integer)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete the Practicing Together sheet. Review the sheet as a class when complete. Call on different groups to explain how they found the solutions.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students share their ideas.

> **Write 2 important take-away messages about multiplying integers that you would share with a friend.**

Have students share their work and check for understanding.

Integers
Lesson 10A

Lesson 10A: Division of Integers

Lesson Objectives	Students identify patterns of division with integers. Students state generalizations of dividing with integers. Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Expression, equation, dividend, divisor, quotient, generalization	
Misconception(s)	Students do not see division as a related operation to multiplication and thus do not apply the generalizations from multiplication to division.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Whiteboard • Dry-erase markers • Red colored pencil for each student

Warming Up

Present a word problem. Have students discuss different ways to solve.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Read the problem to the students.

> **McKenzie had 704 tickets. If she shares them with some friends. Each friend got 32 tickets. How many friends did she share her tickets with?**

Have students solve the problem and then ask the following questions:

> **How many friends got tickets?** *(22)* **How did you solve?** ***(division)*** **Is there another way to solve?** *(answers may vary, such as repeated subtraction)*

> **Let's change the word problem slightly. McKenzie has 704 tickets. If she shares them with 22 friends, how many tickets does each friend get?**

> **How could we solve?** *(answers may vary, such as some may notice that there is a relationship to the previous problem and thus can answer without computing; others may divide 704 ÷ 22)* **How do fact families help us solve?** *(there is a relationship between the quantities given in the problems)*

Learning to Solve

TEACHER NOTES

Similar to addition and subtraction, the goal is not for students to merely recite the rules for division of integers, but rather to identify the patterns and form generalizations from the examples. They will use generalizations from the multiplication lesson to form similar generalizations for division of integers.

Write generalizations in words only, not in symbols and words.

1. Students identify generalizations when dividing integers.

Give each student a whiteboard. Have students turn to the Generalization page in the Notes section of their Student Booklets.

There are patterns that can be found in division. Write the following expressions on your whiteboard: 9 ÷ 3, 6 ÷ 3, 3 ÷ 3, 0 ÷ 3. Then solve and simplify.

What is the pattern? *(the dividend decreases by 3 and the quotient decreases by 1)* **What is the sign of all of the numbers, the dividend, divisor, and quotient?** *(positive)* **Turn to the Generalizations page in the Notes section of your Student Booklet. In the previous lesson, what generalization did we write about dividing positive integers by positive integers?** *(a positive integer divided by a positive integer equals a positive integer)* **Write 2 additional examples in the third column of the positive integer divided by a positive integer equals a positive integer.**

Look at the first equation, 9 ÷ 3 = 3. What multiplication equation can be written to create a fact family? *(3 × 3 = 9)* **What multiplication generalization can be stated for this equation?** *(a positive integer times a positive integer equals a positive integer as the product)*

Erase the equations on your whiteboard. Now continue the pattern, decreasing the dividend by 3 and decreasing the quotient by 1. Write 3 more equations: –3 ÷ 3 = –1, –6 ÷ 3 = –2, –9 ÷ 3 = –3.

Allow time for students to write the equations.

What is the sign of the dividend in these equations? *(negative)* **What is the sign of the divisor?** *(positive)* **What is the sign of the quotient?** *(negative)*

What generalization did we write about dividing a negative integer by a positive integer? *(a negative integer divided by a positive integer equals a negative integer)*

Can you apply the commutative property to the dividend and divisor like you can in multiplication? That is, can you reverse the order of the dividend and divisor like you can with the factors when you multiply? *(no)* **Why?** *(accept reasonable answers such as need to start with the whole, not the amount partitioned)*

If we write "9 ÷ –3 = –3", what generalization did we write to explain this equation? *(a positive integer divided by a negative integer equals a negative integer)* **Write 2 additional examples in the third column of the positive integer divided by a negative integer equals a negative integer as the quotient.**

Erase your whiteboard. Now write these 3 expressions on your whiteboard and find the quotients: 6 ÷ –3, 3 ÷ –3, and 0 ÷ –3.

What is the sign of the dividend? *(positive)* **What is the sign of the divisor?** *(negative)* **What is the sign of the quotient?** *(negative, except for 0 ÷ –3. The quotient (0) is neither positive or negative.)* **What generalization did we write about dividing a positive integer by a negative integer?** *(a negative integer divided by a positive integer equals a negative integer)* **Write 2 additional examples in the third column of the negative integer divided by a positive integer equals a negative integer as the quotient.**

Erase your whiteboards. Now continue the pattern, writing the next 3 equations: –3 ÷ –3 = 1, –6 ÷ –3 = 2, –9 ÷ –3 = 3.

What is the sign of the dividend and the divisor? *(negative)* **What is the sign of the quotient?** *(positive)*

What generalization did we write to explain this equation? *(a negative integer divided by a negative integer equals a positive integer)* **Write 2 additional examples in the third column of the negative integer divided by a negative integer equals a positive integer as the quotient.**

Look at the last equation you wrote, –9 ÷ –3 = 3. Write one multiplication equation for this fact family. What equation did you write? *(–3 × 3 = –9 or 3 × –3 = –9)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in pairs, in small groups, or with the entire class, depending on student needs.
2. If done in groups, review the answers, asking students to model how the problems were solved.

Trying It on Your Own

Display the Trying It on Your Own sheet in the Teacher Masters. Have students turn to the Trying It on Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect using a red (or other color) colored pencil.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers. Students mark the total number correct at the top of their page.

Wrapping It Up

Have students turn to the Notes section of their Student Booklets. Have students write 3 to 4 sentences.

Write 3 to 4 sentences explaining how fact families can be used to check your quotients when you divide integers.

Have students share their work and check for understanding.