Progression of Mathematics Skills Toward Algebra Readiness (PMSTAR), From Grades 5–8

Purpose of PMSTAR

In some sense, as middle school teachers, you are all in the business of preparing your students to take and pass a rigorous algebra course and, eventually, a critical examination on the same material.

Where do you, as a teacher, and what you are teaching fit into the framework?

Key Definitions

Estimation is the ability to calculate an approximation of a mathematical result. It might include finding upper or lower bounds of a quantity that cannot readily be computed precisely.

Generalization is the ability to move from a specific or particular case down to a more general case. An example is $2 + 3 = 3 + 2$ to $x + y = y + x$ where x and y are integers.

An **algorithm** is an effective method for solving a problem expressed as a finite sequence of steps.

How to Use the Framework

This framework is designed to show exactly where you fit into the algebra readiness process. The grade levels are less important than the content. First, find the topics that you are teaching this year. (Because this framework is for algebra readiness, some of your topics may not be shown; however, this does not mean that these topics are not critical for student development, just that they are not along the algebra path.)

For each subject area, you will see arrows in and arrows out; and there may be different types of arrows. A simple arrow from one topic to another shows a connection. A circle placed on an arrow is a key input or output that is critical for the next step and should be emphasized. Larger arrows show critical prerequisites.

For example, a student cannot add multiple-digit integers without knowledge of addition facts for single integers. It is also interesting to trace the arrows backward and forward to see where topics lead with algebra readiness as the goal.

Mathematics builds one skill on top of another until there is sufficient knowledge to both solve problems and move to the next level. In the 21st century, algebra will be the gatekeeper for students' mathematics success.

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KEY

- Outcome (hexagon)
- Key input/output (oval)
- Critical prerequisites (red arrow)
- Connection (blue arrow)

ALGEBRA READINESS

Mathematical Procedures and Skills — Real numbers, fractions, decimals, use of algorithms

Problem solving

Generalized Thinking — Multiplicative/Proportional thinking; Generalized notation; Generalization of algorithms

Coordinate plane

EIGHTH GRADE

- Representing, applying, and analyzing proportionality
- Using proportionality and linear equations to solve problems
- Problem solving including similarity, congruency, Pythagorean Theorem, and complex volumes
- Using expressions, equations, functions, and the real number system to represent and solve problems in a variety of contexts

Multiplicative thinking

SEVENTH GRADE

- Representing and applying proportionality
- Using rational numbers and operations in a variety of contexts to solve problems
- Solving problems including determining angles, areas, perimeters, and volumes
- Using expressions and equations in a variety of contexts, including measurement, conversions, probability, and data analysis

Fluency in examining rational numbers

Generalizations

SIXTH GRADE

- Connecting ratios, rates, and proportions to multiplication and division and using these concepts and operations to solve problems involving proportional relationships
- Developing an understanding of and fluency with addition, subtraction, multiplication, and division of fractions and decimals
- Using estimation to examine solutions of arithmetic problems
- Using expressions and equations to represent situations involving rational numbers

Fraction equivalence

Decimal & fraction equivalence

Generalizations in algorithms

Specific & generalized algorithms, place value

Number line

Line graphs

FIFTH GRADE

- Using fractions and decimals to represent, compare and order quantities, and solve problems
- Developing proficient use of whole number division standard algorithms to solve problems
- Using formulas to find area, volume, perimeter
- Understanding number lines, line graphs, and number plots

- Proficiency with addition, subtraction, multiplication, and division of whole number algorithms
- Represent and compare whole numbers on the number line