Ratios and Proportions

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Teacher Lessons

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Ratio and Proportions 1
Lesson 1

Lesson 1: Identifying Relationships

Lesson Objectives	Students identify and write ratios in multiple contexts. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Ratio**: a comparison of two quantities	
Requisite Vocabulary	**None**	
Misconceptions	Students may confuse part-part comparisons (ratios) with part-whole comparisons (fractions). They also may not realize that the order of the ratio makes a difference. For example, they may think that 4:1 is the same as 1:4.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) Projector (or equivalent)	• Student Booklet • Red Colored Pencil

Progress Monitoring Schedule

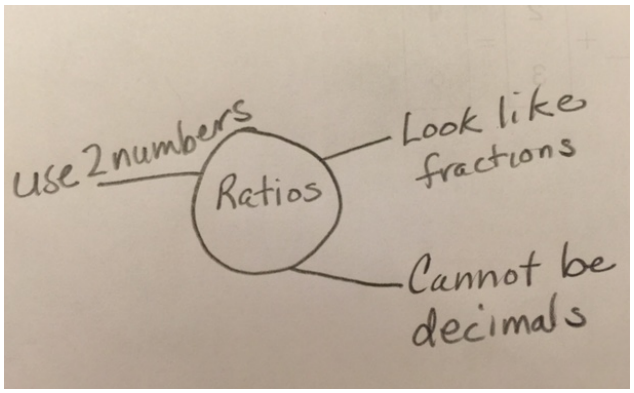
BEFORE	**AFTER**	**AFTER**
Lesson 1: Pre-assessment **Module Check Form A** ✓	Lesson 7: Mid-assessment **Module Check Form B**	Lesson 15: Post-assessment **Module Check Form C**

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. Students will first write their own ideas on this page and then add peer responses. Draw a circle on the chart paper or poster board with "Ratios" in the middle (similar to the one on the student booklet). Display it for students to see.

Today, we will start by thinking about what you know about ratios. Write everything you know about ratios in your Student Booklet.

Ask for student responses. As appropriate, ask for students to clarify or provide examples to aid in the brainstorming of ideas. Add ideas to the concept map.



As you share your ideas, I will record them on our concept map. I may ask you to clarify your responses or provide an example.

Learning to Solve

Teacher Notes

When a ratio is written in fraction format, students need to be aware that the fraction may not indicate the number of equal-sized parts in the numerator and the whole in the denominator. In a ratio, it may represent a part-part comparison. That is why a ratio in fraction form is read as ". . . to . . ." For example, $\frac{4}{5}$ might be read as "4 bananas to 5 oranges."

It is preferable to use the colon format for writing ratios so that students do not confuse fractions and ratios.

1. Students review addition and subtraction of fractions with like and unlike denominators. If you feel students do not need the review, you can skip this section and move to 2.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Turn to the Learning to Solve sheet in your Student Booklet and complete the problems on your own.**

This is a review problem. Students may share both computational (skill) and conceptual ways of solving the problems. As students discuss the problems, consider the following questions:

> **For which problems did you use similar solution methods?**
>
> **How are the solution methods for problems number 2 and 5 alike? How are they different?**
>
> **How are the solution methods for problems 3 and 4 alike? How are they different?**

2. Students recognize relationships that can be described as a ratio.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet located in their Student Booklets.

Have students work with a partner to complete Kara's Problem on the Learning to Solve Sheet.

> **Look at Kara's problem. Kara said, "I'm going to make lemonade. For every 3 cups of water, I need 1 cup of lemon juice. But I'm using 9 cups of water. How many cups of lemon juice will I need?"**
>
> **Work with a partner to solve Kara's problem. Think about what picture or diagram you could use to support your answer. What would you tell Kara?**

Allow students time to decide what they would tell Kara. Ask some pairs to share their answer.

As you describe how you thought about the problem, share any diagram or picture that you used to support your answer.

3. Students are introduced to the term ratio.

There is a special name for the relationship between the water and the lemon juice. We call it a ratio. A ratio is a comparison of 2 quantities. In Kara's problem, what 2 quantities are we comparing? *(the amount of lemon juice to the amount of water)*

Before showing students different ways of writing a ratio, ask if anyone knows a way to write one. If so, use their way of writing. If not, use the script below.

There are different ways that we can write a ratio. In your student booklet, under the Kara Problem, let us write these ways together.

In words, we would say, 3 cups of water to 1 cup of lemon juice. Write this in your booklet.

In ratio format, we would write 3:1. We read this as 3 to 1. In this situation, it means 3 cups of water to 1 cup of lemon juice. Or, for every 3 cups of water, there is 1 cup of lemon juice. Write 3:1 in your booklet.

In fraction format, we would write $\frac{3}{1}$. How would you read this? *(3 cups of water to 1 cup of lemon juice)* **Write $\frac{3}{1}$ in your booklet.**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers to Questions 1 and 2. Note that students may write 6:8 for Question 2 to represent the pattern as it is drawn.

For Question 3, discuss how the same ratio representation (in colon format) can represent different comparisons. Notice that there are multiple answers for that question such as 2:1, 4:2, and so on.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of their Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Describe a situation that can be represented by the ratio 1:4.**

Have students share as time allows.

Ratio and Proportions 1
Lesson 2

Lesson 2: Identifying Relationships

Lesson Objectives	Students identify and write ratios in multiple contexts. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	None	
Requisite Vocabulary	Ratio	
Misconception	Students may believe that ratios are additive relationships because they think of multiplication as repeated addition. They may also think that the order of the numbers in a ratio does not matter.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • 3-in-a-Row Game Cards (1 set per pair, see page 132 in Student Booklet) • 3-in-a-Row Game Sheet (1 per student, see page 131 in Student Booklet) • 2-color counters or equivalent • Red Colored Pencil

Warming Up

Review addition and subtraction of fractions with like and unlike denominators.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Turn to the Warming Up sheet in your Student Booklet and complete the problems on your own.**

This is a review problem. Students may share both computational (skill) and conceptual ways of solving the problems. As students complete the tasks, ask the following questions to encourage discussion of the problems.

> **For which problems did you use similar solution methods?**
>
> **How are problems 2 and 4 alike? How are they different?**
>
> **How are the solution methods for problems 3 and 4 alike? How are they different?**

Learning to Solve

TEACHER NOTES

Thinking multiplicatively is fundamental to proportional reasoning. The focus for middle grades students is to understand that the relationship expressed in a ratio can be expressed as one quantity so many times larger than the other. Encourage students to express relationships in this multiplicative way.

Stress the ratio language of "For every . . . , there are . . ." so that students see relationships between quantities.

1. Students recognize relationships that can be described as a ratio.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Have students work with a partner to solve the Shape Problem on the Learning to Solve Sheet.

> **Look at the shape problem in number 1. Discuss with your partner how you would solve this problem. As you talk, decide if there is a diagram or picture you could use to support your answer.**

Allow students time to determine the ratios in this problem. Select pairs to share their answers.

2. Students are introduced to multiplicative relationships described by ratios.

> **In the first lesson, you solved Kara's problem about lemonade. The ratio of water to lemon juice was 3:1. What does the ratio tell us?** *(For every cup of lemon juice, there are 3 cups of water.)*
>
> **The ratio 3:1 also tells us that there is three times as much water as there is lemon juice. What if the ratio of lemon juice to water is 1:4? How could you describe it using that language?** *(There is 4 times as much water as there is lemon juice. Students could also say there is one-fourth as much lemon juice as there is water.)*
>
> **For problem, 2, look at the ratios of chocolate milk to white milk. How could you describe the ratio of chocolate syrup to white milk if the ratio is 1:5? Write your words to explain this ratio in your Student Booklet**. *(There is 5 times as much white milk as there is chocolate milk. Or, there is one-fifth as much chocolate milk as there is white milk.)*

Give students time to write. Ask them to share. If no one suggests one-fifth as much chocolate milk as there is white milk:

> **I think that if the ratio of chocolate milk to white milk is 1:5, I could say that there is one-fifth as much chocolate milk as there is white milk. Would you agree or disagree? Why?**

Allow students time to talk with a partner. Have students share their thinking. They should agree because one-fifth of 5 is 1 which describes the same relationship as 5 times 1 is 5.

> **We can describe a ratio in 2 ways by thinking about how many times greater one quantity is than the other one. If you have difficulty thinking of both ways, try writing the ratio as a fraction to help you.**
>
> *(1:5 or $\frac{1}{5}$; there is five times as much white milk as there is chocolate milk, and there is $\frac{1}{5}$ the amount of chocolate milk compared to white milk.*

Practicing Together

Play 3-in-a-Row. Distribute a 3-in-a-Row Game Sheet (see page 131 of Teacher Masters) and a set of 3-in-a-Row Game Cards (see page 132 of Teacher Masters) to each student. (Students may play as a pair against a pair, if more appropriate.) Students draw a card and find a picture on the game board that represents the ratio. The first one to get 3 in a row (up, down, or diagonal; 4 in a row takes too much time) wins.

> **We are going to play 3-in-a-Row. You will play in pairs. The object of the game is to be the first one to get 3 counters in a row, up, down, or diagonal. The first person will take a card. Look at the ratio of shaded to unshaded parts. Find a picture on the game sheet that matches the ratio. Place your color counter on the picture. The second person will draw a card and place the counter on your picture. Be careful—some ratios may have more than 1 picture that matches so make your choice carefully.**

Monitor students as they play. Watch for errors in their matches. After they have played 1 (or more games if time allows), discuss their strategies for determining where to place their counters.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Have students work with their partner to complete this activity.

> **With your partner, answer the questions on the Wrapping It Up sheet located in the Student Booklet.**

Once students have completed the sheet, review the answers and extend the discussion with the following questions as time permits.

Question 1, ask students to describe how they would explain the ratios written. For example, for the ratio 1:4, students should indicate that it is for every unit of chocolate chips, there are 4 units of peanuts. They can substitute any measure for unit, such as cup.

For Question 2, discuss the relationship that this ratio represents. Students may have difficulty interpreting the fractional relationship.

Ratio and Proportions 1
Lesson 3

Lesson 3: Creating Equivalent Relationships

Lesson Objectives	Students create equivalent ratios. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Equivalent Ratios**: ratios that can be shown to have the same unit ratio	
Requisite Vocabulary	Ratio	
Misconception	Students may think that ratios are additive rather than multiplicative.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • 3-in-a-Row Game Cards (1 set per pair, see page 132 in Student Booklet) • 3-in-a-Row Game Sheet (1 per student, see page 131 in Student booklet) • 2-color counters or equivalent • Red Colored Pencil

Warming Up

Students write ratios.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet and complete the problems on your own.

Allow students time to complete the problems. Have students share their responses. For example, if the ratio is number of squares to number of sides, as in problem 1, the first number in the ratio must represent the number of squares and the second number must represent the number of sides. Some students, for problem 3, may give equivalent ratios such as 500 pennies: 5 dollars. Discuss the equivalencies if they are shared. It is important that students realize that the order of the quantities in a ratio matters.

Learning to Solve

Teacher Notes

Equivalent ratios will have a multiplicative relationship. As students discuss, emphasize the repetition of the ratio that shows 1 to another quantity, indicating the unit. This is similar to equivalent fractions, when there is a common factor.

Students recognize different ratios can describe the same relationship.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Call on a student to read the fruit salad problem and then have students work with a partner to solve the problem.

Do you agree with Ben? Draw a picture or a diagram to support your answer.

Allow students time to determine the relationship in this problem. Select pairs to share their answers, displaying any diagrams or pictures drawn.

If students draw Derek's salad as 2 apples and 4 oranges without separating it into 2 sets of 1 apple and 2 oranges, ask the next question.

How could you rearrange the drawings for Derek's salad to show another relationship? Is there another way to describe the relationship? *(Both Courtney and Derek's salads have a 1:2 relationship)*

Stress that when ratios are equivalent, you can create groups of the quantities being compared (in this case, apples and oranges) representing the unit rates (1 to x or x to 1). They may also notice that an equivalent ratio will be a multiple of the other ratio.

Courtney's salad has a ratio of 1 apple to every 2 oranges. The ratio of apples to oranges is 1:2. In Derek's salad, you can rearrange the fruit to show a ratio of 1 apple to every 2 oranges. There are two groups of these. Even though Derek's salad uses more apples and oranges, the ratio of apples to oranges is the same.

You can also think of Derek's salad as using <u>twice</u> as many of each fruit. If Derek had used 3 apples and 6 oranges, would it be in the same ratio? *(Yes, because his salad is 3 times as many apples and 3 times as many oranges. They are in the same ratio.)* **When the ratios are the same, we say these are equivalent ratios.**

Write the description of using the comparison of 1 to a quantity on the white board. Have students copy in the Notes section of their Student Booklets.

The simplest way to describe ratios might be to think about the comparison of 1 to a quantity.

1:2, 1:3, 1:4 and so on are ratios like that. This is a drawing that shows a ratio of 1:2.

The ratio 2:4 shows the same relationship. Draw a picture in the Notes section of your Student Booklet that shows why it has the same relationship.

Students should show that 2:4 can be shown to have 2 groups of 1 apple and 2 oranges. The unit of 1:2 can be replicated to create the ratio of 2:4.

Write another ratio that has the same relationship as 1:2 and 2:4 in the Notes section of your Student Booklet. Draw a picture that supports your answer.

Monitor students as they draw, correcting their drawings as needed. Watch for equivalent ratios that use larger numbers such as 50:100.

You may want students to share the ratios they wrote. Record them as they share so that there is a list of them. Ask them to note any patterns.

What do you notice about the ratios you created?
(Answers will vary. For example, students may notice that the second number is twice the first, or the first number is half of the second number. Others may note that there are an infinite number of answers or that some of the equivalent ratios are multiples of others.)

Let us use the ratio 1:3. In the Notes section of your Student Booklet, make a drawing that shows the ratio of 1:3. What is another ratio that has this relationship? Write it and draw a picture to support your answer. *(Accept any ratios that are equivalent to 1:3)*

Monitor students as they draw, correcting their drawings as needed. Watch for equivalent ratios that use larger numbers such as 50:150.

You may want students to share the ratios they wrote. Record them as they share so that there is a list of them. Ask them to note any patterns.

What do you notice about the ratios you created?
(Answers will vary. For example, students may notice that the second number is three times the first, or the first number is a third of the second number. Others may note that there are an infinite number of answers or that some of the equivalent ratios are multiples of others.)

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Once students have completed the sheet, have pairs of students share their answers.

Questions 1 and 2 show the relationship of 1 to a quantity. Students' diagrams or drawing should show that it is possible to see the same ratio in both examples.

Question 3 illustrates an additive relationship, not a multiplicative one. It is not possible to show the ratio or 1 to a quantity in the second example.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Play 3-in-a-Row. Distribute a 3-in-a-Row Game Sheet (see page 131 of Teacher Masters) and a set of 3-in-a-Row Game Cards (see page 132 of Teacher Masters) to each student. (Students may play as a pair against a pair, if more appropriate.) Students draw a card and find a picture on the game board that represents the ratio. The first one to get 3 in a row (up, down, or diagonal; 4 in a row takes too much time) wins.

> **We are going to play 3-in-a-Row. You will play in pairs. The object of the game is to be the first one to get 3 counters in a row, up, down, or diagonal. The first person will take a card. Look at the ratio of shaded to unshaded parts. Find a picture on the game sheet that matches the ratio. Place your color counter on the picture. The second person will draw a card and place the counter on your picture. Be careful—some ratios may have more than 1 picture that matches so make your choice carefully.**

Ratio and Proportions 1
Lesson 4

Lesson 4: Creating Equivalent Ratios

Lesson Objectives	Students create equivalent ratios. Students find missing values in proportions. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Proportions**: two ratios that are equivalent	
Requisite Vocabulary	Equivalent ratios, unit rate	
Misconception	Students may think that equivalent ratios have an additive relationship rather than a multiplicative one.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Deck of cards with face cards removed (1 deck per pair) • Red colored pencil

Warming Up

Students solve problems by writing equivalent ratios.

Play Equivalent Ratios. Give each pair a deck of cards with the face cards removed. Aces in the deck will represent 1, the other cards' values are the number on the card (10 is a 10 for this game). A student will act as dealer and deal 8 cards to each in the pair. The remaining cards are placed in the center of the table, and the top card is revealed.

Students will look at their cards and find 4 cards that can be made into equivalent ratios. For example, if they have the cards, 2, 5, 10, and 1, they can make the ratios 1:2 and 5:10. If they cannot make 2 equivalent ratios, they make take the card in the middle of the table and discard one of their cards. The first one to make all of their cards into ratios is the winner.

> **We are going to play Equivalent Ratios. Each pair will use a deck of cards with 4 of each card; there are no face cards. The aces in the deck represent 1. You have 4 1s, 4 2s, and so on, all the way up to 10. One person in your pair will shuffle the cards and deal 8 cards to each of you. The remaining cards will go in the middle of the table, with the top card turned over.**
>
> **You will look at your cards and try to find 4 cards that will make 2 ratios that are equivalent. For example, if I had the cards 2, 5, 10, and 1, I could make the ratios 1:2 and 5:10. They are equivalent. If you cannot make equivalent ratios, you may take the card that is showing in the middle of the table, and discard one of your cards. You keep playing until one of you makes all of your cards into ratios. If the game ends before no player has no cards left, the player who has made the most matching ratios wins.**

Students can play as time allows.

Learning to Solve

Teacher Notes

Some students may use additive thinking. That is, they may think that 1:2 and 3:4 are equivalent because 1 + 1 = 2 and 3 + 1 = 4. You can add the same amount to both to get the second number. And, 1 + 2 = 3 and 2 + 2 = 4. You can add the same amount to the first ratio to get the second ratio. If you notice students doing this, model with chips or blocks to show that the two ratios do not have the same 1 to a quantity ratio.

Students may think that any two ratios can be equivalent. However, for ratios to be in a proportion they must be equivalent AND they must be comparing the same types of quantities. For example, a ratio of 1:2 comparing the number of cups of sugar to the number of cups of flour is not equivalent to the ratio 2:4 comparing the number of birds to the number of feet.

1. Students create equivalent ratios.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Have students work with a partner to solve the new fruit salad problem on the Learning to Solve Sheet.

   > **Courtney and Derek are making new salads. Discuss with your partner how you would solve the fruit salad in problem 1. As you talk, decide on a diagram or picture you could use to support your answer. Draw your picture and write at least 3 sentences to explain your thinking.**

   Allow students time to determine the relationship in this problem. Select pairs to share their answers. Watch for students who try to use additive thinking to solve the problem. After 1 pair has

explained the problem with a reasonable response, ask if any pair solved it a different way. If so, ask them to explain. Notice that you are asking pairs to share a different way to solve it, even if they got the same answer.

Have students turn to the Notes section of their Student Booklets. Write the equivalent ratios on the board as the lesson progresses.

> **The 2 ratios 1:3 and 4:12 are equivalent ratios. Why do you think they are equivalent?** *(Both ratios represent the same ratio of 1:3.)* **Watch as I write on the white board. We can write the relationship between the two ratios like this 1:3::4:12 or like this,** $\frac{1}{3} = \frac{4}{12}$**.**
>
> **When the relationship is written like 1:3::4:12, we read it as the ratio of 1 to 3 is equivalent to the ratio of 4 to 12. When two ratios are equivalent, we say they form a proportion. A proportion shows two ratios that are equal or equivalent.**
>
> **In the Notes section of your Student Booklet, write 1:3::4:12 and** $\frac{1}{3} = \frac{4}{12}$**. Underneath that, write, "These ratios are in a proportion." That means they are equivalent or equal.**

2. Students will find missing values in proportions.

> **If you know that 2 ratios are equivalent, you can use that idea to find missing parts of a ratio. For example, look at the ratio** $\frac{1}{3} = \frac{4}{12}$**. Is it possible to multiply the numerator and the denominator in the ratio** $\frac{1}{3}$ **by the same number to get the ratio** $\frac{4}{12}$**?** *(yes)* **What number would you multiply the numerator and denominator by?** *(4)*

Watch for students who think that the relationship is $\frac{1}{3} \times 4$ rather than $\frac{1}{3} \times \frac{4}{4}$.

Even though the ratio $\frac{4}{12}$ represents 4 times the amount of pears and apples than the ratio $\frac{1}{3}$, they are still equivalent because the ratio of comparing 1 to a quantity is the same in both ratios.

Let's use that idea to solve problem 2 in your Student Booklet in the Learning to Solve section.

Give students time to work in their pairs to solve problem 2 on the Learning to Solve sheet. Have pairs share their answers to each part of problem 2. Be sure that pairs write the proportions using the colon format and the fraction format. Emphasize the multiplicative relationship (part A represents 3 times the unit amount and part C represents 5 times the unit amount.)

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers. As they share their answers, have them explain how they found it.

Pay close attention to problem 5. While the numerical ratio appears to be equivalent, the contexts of the ratios are not the same so the ratios cannot be equivalent. Stress to students that the context of the ratio does make a difference.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Decide whether this proportion is a true statement: 1:4::4:12

Allow time for students to respond. If time allows, have the students share their responses.

Ratio and Proportions 1
Lesson 5

Lesson 5: Using Multiplicative Relationships to Find Equivalent Ratios

Lesson Objectives	Students find equivalent ratios in a proportion. Students find missing values in proportions. Students use multiplicative relationships to determine equivalence. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	**Proportions, unit rate, ratio**	
Misconception	**Students misapply the multiplicative relationship we often call "cross multiplying."**	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Deck of cards with face cards removed (1 per pair) • Red colored pencil

Warming Up

Students determine if two ratios are equivalent.

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet and complete the problems on your own.

Ask students to share their answers; have them write the proportion for the first one in their Student Booklets. The second proportion is not true but students can write a statement to show that: $\frac{1}{8} \neq \frac{2}{9}$.

Learning to Solve

TEACHER NOTES

Some students may use additive thinking; that is, they may think that 1:2 and 3:4 are equivalent because 1 + 1 = 2 and 3 + 1 = 4. You can add the same amount to both of the numbers in each ratio to get the second number. And, 1 + 2 = 3 and 2 + 2 = 4. You can add the same amount to the first ratio to get the second ratio. If you notice students doing this, model with chips or blocks to show that the two ratios do not have the same 1 to a quantity ratio.

The Means Extremes Product Property of Proportionality is often called cross-multiplying. Students often over generalize the use of the cross multiplication and misapply any time they see 2 fractions in problem, such as $\frac{1}{2} + \frac{3}{4}$ where they might multiply 1 x 4 and 2 x 3 to get an answer of $\frac{4}{6}$

1. Students find equivalent ratios that do not have a unit ratio.

In the last lesson, we used the unit rate or ratio as a way of finding equivalent ratios in a proportion and of finding missing values. A unit rate or ratio is a comparison of 1 quantity to another, like 1:2 or 1:4.

What is another example of a unit ratio? *(a unit ratio compares 1 to another quantity. It will always have a 1 as one of the numbers.)*

When we have a unit rate, we can multiply each quantity by the same amount to find an equivalent ratio. What if we started with a ratio like 2:3? How could we find an equivalent ratio? Let's explore that in the next problem.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Turn to the Learning to Solve sheet in your Student Booklet.

Pat made a fruit punch. For every 3 cups of pineapple juice, he used 2 cups of orange juice.

Write the ratio of the number of cups of orange juice to the number of cups of pineapple juice.

Give students a few seconds to write the ratio. Ask a student to share how he or she wrote it. If students do not write both 2:3 or $\frac{2}{3}$, have them do so in their Student Booklets.

Pat wants to make a larger quantity of fruit punch. Work with a partner to solve problems 2, 3, 4, and 5 in the Learning to Solve section. Be sure you can explain your answer.

Allow students 2–3 minutes to determine the relationship for each problem. When students are finished, select pairs to share their answers for one problem. Watch for students who try to use additive thinking to solve the problem (see teacher note). After 1 pair has explained the problem with a reasonable response, ask for another way to solve the problem. It is important to ask pairs to share a different way, even if they got the same answer because the thinking used in the solutions may be different.

Ratios can be equivalent even if we are not given a unit rate. Notice that the equivalent ratios have a relationship that shows both quantities have been multiplied by the same factor or number. When Pat

used 9 cups of pineapple juice for the fruit punch, that was 3 times more than the initial amount. This means that he had to use 3 times more orange juice as well. To keep the ratios equivalent, you have to multiply both quantities by the same amount.

2. Students find missing values in proportions.

Look at problem 6 on the next sheet. Cora made fruit punch using 3 cups of cranberry juice for every 4 cups of orange juice. Write the ratio in your Student Booklet that represents this relationship.

What ratio did you write? *(3:4 or $\frac{3}{4}$)*

For problem 7, she used 16 cups of orange juice to make a large batch of the punch. How many cups of cranberry juice does she need? What proportion would you write to represent the problem? Write it in problem 7 in your Student Booklet. *(3:4::x [or box or other symbol]:16 OR $\frac{3}{4} = \frac{x}{16}$ Students may use other symbols rather than a variable.)*

Have students share the proportion they wrote. Be sure that they write a proportion and not a single ratio.

For number 8, find the number of cups of cranberry juice that she needs. Decide how many cups you think she will need, then talk with your partner about how you found it. Record your work and write your answer in your Student Booklet.

Give students time to solve and talk with their partners. Ask pairs to share their solutions. Most students will indicate that the proportion is based on a multiplicative relationship of 4 times the quantity. As you debrief on the problem, refer to the answers on the Teacher Masters.

As you were sharing your solution to the problem, you may have noticed that the number of cups of orange juice used was 4 times the original ratio. You could use that relationship to find the number of cups of cranberry juice.

There is another way that you could solve the problem. It is a special property of proportions that some of you may have called cross-multiplying.

Work though the example for number 8 in the Teacher Masters. Discuss as needed with students.

Let's try that property with another proportion. If the ratio of pounds of peanuts to pounds of almonds is 4:5 and we use 25 pounds of almonds, how many pounds of peanuts will we use to keep it in the same ratio? Use the cross-multiplication strategy.

Be sure to circulate and check students' work for accuracy in using the property. You may want a student to demonstrate how to use the property to find the missing amount.

Students may ask why this property works. Unfortunately, while this property is often used in middle grades, it is difficult to justify because of the algebraic thinking required.

(Note: This information is for teacher use -- not to be done with students.) Here is the algebraic "proof" of why this property (Means Extremes Product Property) allows us to cross multiply. (Note: This is here for teachers' benefit only – do not use as an activity with students.)

$\frac{a}{b} = \frac{c}{d}$ Start with any proportion.

$\frac{a}{b} \cdot b = \frac{c}{d} \cdot b$ Multiply each side by the same amount.

$a = \frac{cb}{d}$ Simplify.

$a \cdot d = \frac{cb}{d} \cdot d$ Multiply each side by the same amount.

$ad = bc$ Simplify.

This is an example with numbers that illustrates why this works.

$$\frac{3}{4} = \frac{6}{8}$$

$\frac{3}{4} \cdot 4 = \frac{6}{8} \cdot 4$ Multiply each side by the same amount.

$3 = \frac{24}{8}$ Simplify.

$3 \cdot 8 = \frac{24}{8} \cdot 8$ Multiply each side by the same amount.

$3 \cdot 8 = 24$

$24 = 24$ Simplify.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

> **Work with a partner to solve each problem. Use multiplication or unit rates to decide whether the following ratios are equivalent. Show your work.**

Have pairs of students share their answers. As they share their answers, have them explain how they found it. Focus on the use of cross multiplication.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Play Equivalent Ratios. Give each pair a deck of cards with the face cards removed. Aces in the deck will represent 1, the other cards' values are the number on the card. A student will act as dealer and deal 8 cards to each in the pair. The remaining cards are placed in the center of the table, and the top card is revealed.

Students will look at their cards and find 4 cards that can be made into equivalent ratios. For example, if they have the cards, 2, 4, 8, and 1, they can make the ratios 1:2 and 4:8. If they cannot make 2 equivalent ratios, they make take the card in the middle of the table and discard one of their cards. The first one to make all of their cards into ratios is the winner.

> **We are going to play Equivalent Ratios. Each pair will use a deck of cards with 4 of each card; there are no face cards. The aces in the deck represent 1. You have 4 1s, 4 2s, and so on, all the way up to 10. One person in your pair will shuffle the cards and deal 8 cards to each of you. The remaining cards will go in the middle of the table, with the top card turned over.**
>
> **You will look at your cards and try to find 4 cards that will make 2 ratios that are equivalent. For example, if I had the cards 2, 4, 8, and 1, I could make the ratios 1:2 and 4:8. They are equivalent. If you cannot make equivalent ratios, you may take the card that is showing in the middle of the table, and discard one of your cards. You keep playing until one of you makes all of your cards into ratios. If the game ends before no player has no cards left, the player who has made the most matching ratios wins.**

Students can play as time allows.

Ratio and Proportions 1
Lesson 6

Lesson 6: Applying Multiplicative Relationships

Lesson Objectives	Students find missing values in proportions. Students use multiplicative relationships to determine equivalence. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, proportions	
Misconception	Students misapply the multiplicative relationship we often call "cross multiplying."	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Prop 4 Game Cards (1 set per student, see pages 133- 136 in Student Booklet) • Prop 4 Game Board (1 per student, see page 137 in Student Booklet)

		• 20 2-color counters (or appropriate counters) per pair • Red colored pencil

Warming Up

Display the Warming Up sheet located in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet and complete the problems on your own.

Allow students time to work. When they are finished, ask the following questions for each problem:

Are the ratios equivalent? How do you know? What method did you use to decide if they are equivalent?

(For question 1, students could use cross multiplication as one method. Some students may notice that 9:16 does not have the same unit ratio or cannot be simplified as 6:8 can. For question 2, students might think about unit ratios or they may notice that 7 is 1 x 7 and 42 is 6 x 7. The same constant was used to multiply both quantities. Others may use cross multiplication.)

Learning to Solve

TEACHER NOTES

Students often over generalize the use of the cross multiplication. The lessons involving cross multiplication should focus on appropriate use of that algorithm. Other methods are possible to determine equivalency or to find a missing value in a proportion, such as multiplicative relationships. Alternative methods to cross multiplication may be preferable.

Students find missing values in proportions.

Since we started this module, we have found 3 important ways to think about finding equivalent ratios: 1) unit ratios, 2) multiplicative relationships, and 3) cross multiplication. Today, we're going to use the cross multiplication method to find missing values in proportions.

Display the Learning to Solve sheet located in the Teacher Masters. Have students turn to the Learning to Solve sheet located in their Student Booklet.

> **Turn to Learning to Solve in your Student Booklet. How would you describe a proportion?** *(A proportion tells us that two ratios are equivalent or equal.)*
>
> **If you use cross multiplication, how will you know the ratios are equivalent?** *(The products will be equal.)* **In your Student Booklet, use cross multiplication to show that 2 ratios are equivalent. You can choose the ratios you use.**

Give students time to give an example. Have students share their ideas. Discuss as needed. Students' examples should indicate the same multiplicative relationships if the ratios are equivalent. If students have difficulty, or they create examples that are not equivalent, give the ratios 2:3 and 6:9. Have students show that the products of 2 x 9 and 3 x 6 are equal.

> **If the two ratios are written as a proportion, then we know that they are equivalent. Let us use cross multiplication to find missing values in proportions.**
>
> **Look at problem 1 in your Student Booklet in the Learning to Solve section. Write the steps to find the missing value, *x*, as we do them together.**

Go through the steps shown in the Student Booklet and your Teacher Masters. Have students do the step and then you share a correct or accurate step. Discuss each step as needed. When solving the equation $84 = 3x$, students may divide both sides of the equation by 3 or consider what times 3 equals 84, depending on their familiarity with equations. Either approach would be acceptable.

> **Look at problem 2 in your Student Booklet in the Learning to Solve Section. Write the steps to find the missing value, *t*, as we do them together.**

Go through the steps shown in the Student Booklet and your Teacher Masters. Have students do the step; then you share the correct step. Discuss each step as needed. When solving the equation

$3t = 48$, students may divide both sides of the equation by 3 or consider what times 3 equals 48. Either approach would be acceptable.

Using cross multiplication can help you find the missing value in a proportion.

Practicing Together

Play Prop 4. Distribute a set of Prop 4 Game Cards (see pages 133- 136 of Teacher Masters), a Prop 4 Game Board (see page 137 of Teacher Masters), and counters to each student. The first student will take a card, match it to an equivalent ratio on the board, and place a counter on the board space. Students continue matching until someone has 4 counters in a row (up, down, or diagonal).

We are going to play Prop 4. You will shuffle your cards. The first player will take a card and match it to an equivalent ratio on your board by covering it with your color counter. Then the next player takes a turn and does the same. Continue playing until someone has 4 counters in a row, up, down or diagonally. Some ratios may have more than one match and there may be some that have no match.

Allow students to play as long as time allows. Monitor their play. Watch for errors in matching. After they have time to play at least one game, discuss their strategies for determining equivalent ratio matches.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 6A to provide extended practice before proceeding to Lesson 7.

Wrapping It Up

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

> **With your partner, answer the questions on the Wrapping It Up sheet located in the Student Booklet.**

Have pairs of students share their answers. As they share their answers, if they used cross multiplication, have them show the equation they used and how they solved the equation. Other solution methods may be possible, such as determining the multiplicative relationship between corresponding numerators or denominators. If this method is not shared, discuss it as a possible strategy. For example, in problem 1, 9 x 4 = 36. Therefore, the numerator must be multiplied by 4 to maintain the relationship.

Ratio and Proportions 1
Lesson 7

Lesson 7: Determining Unit Rate

Lesson Objectives	Students determine unit rates. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**Unit rate:** a ratio that tells how many units of the first quantity corresponds to 1 unit of the second quantity	
Requisite Vocabulary	Equivalent ratios, proportions, denominator	
Misconception	Students often think that a unit rate of *x*:1 is the same as a unit rate of 1:*x*.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Red colored pencil

Warming Up

Display the Warming Up sheet located in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet. Solve the problems and show your thinking.

As students share their answers, watch for multiple ways of solving the problems. Some students may notice the multiplicative relationship (4 times in both) and use that to find the missing value. Others may use cross multiplication.

Discuss the methods as they share out.

What method did you use to find each missing value?
(answers will vary, for example some students may use the cross-multiplication method while others may use a multiplicative approach)

Learning to Solve

TEACHER NOTES

Unit rates are useful in being able to compare ratios, especially those types that are relative to finding the best deal, the best rate, and so on.

1. Students find the unit rate.

Ratios can help us in our everyday life. Can you think of any ratios that you have used or seen others use?

Have students think independently for a short time, then ask them to share their idea(s) with a partner. Have the pairs report out as time permits? Record their ideas on the whiteboard or by projecting their work as they share. Discuss as appropriate.

Shopping is one way to use a ratio. For example, if you are at the Buy-It-Here store, and you see candy bars selling for 3 for $1.50, that is a ratio. How could we write this ratio? *(We could write it as $1.50:3; for every $1.50, you can buy 3 candy bars.)*

But what if I wanted to only buy 1 candy bar? How much would 1 candy bar cost?

Give students time to decide. Ask them to share their answers.

Every candy bar would cost 50¢. How did you decide that? What method did you use?

2. Turn to the Learning to Solve sheet located in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Some students will use division while others may use a multiplicative approach. Discuss both methods if students use them. As they move through problems, they will find that division will be more generalizable than the multiplicative methods, but both methods are appropriate.

   When we find out how much 1 of a quantity is, we can write that as a unit ratio, or a unit rate. We could write it as $.50:1 or, as a fraction, $.50 over 1. The unit, or 1, is the second number in the ratio, or, as a fraction, the denominator. In your Student Booklet, write the unit ratio for problem 1.

   Have students record the unit ratio for the first problem.

   What if candy bars were selling 4 for $1.00 as in problem 2? What is the unit ratio?

   Have students solve. Ask them to share the answer and the method they used. Be sure that students are recording it as $.25:1, the unit rate, or, if written as a fraction, $.25 over 1.

   Would your method for finding the unit ratio work on problem 3? Try it and record the unit ratio.

Have students solve. Ask them to share the answer and the method they used. Be sure that students are recording it as $4:1, the unit ratio.

Show the meaning of unit ratio in your Teacher Masters and have students copy it into their Student Booklet.

> **When you see a unit ratio or rate, it tells us the quantity that compares to 1 of the other quantities. Write the meaning of unit ratio, the quantity that compares to 1 of the other quantities, in your Student Booklet.**

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets. Have students work with their partner to complete this activity.

> **With a partner, solve the problems on the Practicing Together sheet located in the Student Booklet. Show your work so that you can share your method with the class.**

Have pairs of students share their answers. As they share their answers, make sure that they record the unit rate as a quantity compared to 1. In problems 3 and 4, students may not be comfortable putting $7.50 as the numerator in the ratio written as a fraction. If students do not show this as one way to write the unit rate, point out that this is another way to show the unit rate, $7.50 over 1.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

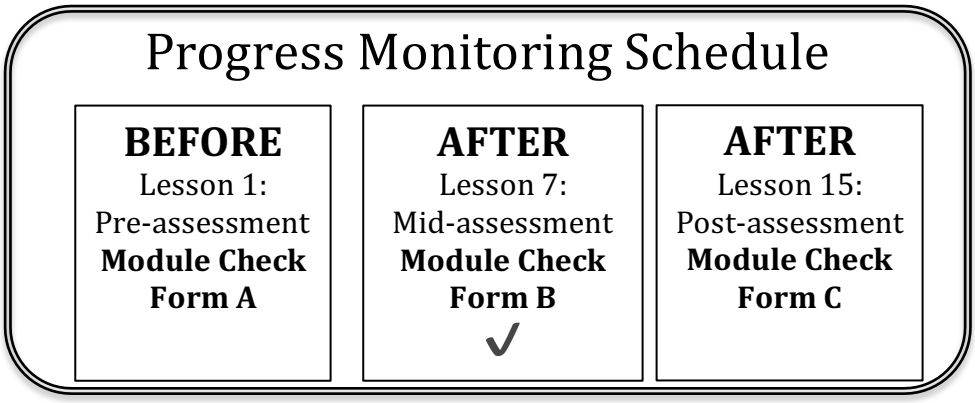
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet located in their Student Booklet.

Which is a better buy: 3 apples for $0.99 or 4 apples for $1.20? Why?

Allow time for students to respond. Point out that unit rates can help us find the best deal. If time allows, have the students share their responses.



Ratio and Proportions 1
Lesson 8

Lesson 8: Determining Unit Rate

Lesson Objectives	Students determine unit rates. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Unit rate, equivalent ratios	
Misconception	Students often think that a unit rate of *x*:1 is the same as a unit rate of 1:*x*.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Create the Rate Game Sheet (1 per student, see pages 138-141 of Teacher Masters) • Regular decks of cards (minus face cards, 1 deck per pair) • Calculator • Red colored pencil

Warming Up

Display the Warming Up sheet located in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet and complete the problems on your own.

Have the students share their answers; watch for multiple ways of solving the problems. Some students may notice the multiplicative relationship such as 30 is 6 times larger than 5 so 72 must be 6 times larger than the unknown value and use that to find the missing value. Others may use cross multiplication.

Discuss the methods as they share out. Note that unit rates are written as *x*:1.

What method did you use to find the missing value?

Learning to Solve

Teacher Notes

Unit rates are useful in being able to compare ratios, especially those types that are relative to finding the best deal, the best rate, and so on.

Students find the unit rate.

In the last lesson, we started thinking about unit rates. I would like for you think quietly about how you would describe a unit rate. Then, I am going to ask you to share with a partner.

Have students think independently, then ask them to share their idea(s) with a partner. Have the pairs report out as time permits. Record their ideas on the whiteboard or by projecting their work as they share. Discuss as appropriate.

We are going to continue thinking about unit rates today by creating the largest unit rate possible using a deck of cards.

Turn to the Learning to Solve sheet located in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Play Create the Rate. Distribute the appropriate version of the Create the Rate Game Sheet (see pages 138-141 of Teacher Masters) to each student.

As students play the game, monitor their method for finding the unit rate. In some cases, they will have a decimal compared to 1, depending on the cards that they draw.

Give each student a calculator if enough are available or provide a calculator for each pair of students.

Give each pair of students a deck of cards. Tell them to remove the Jack, Queen, and King cards.

Read the Version 1 of Create the Rate directions to the students. Note: If time permits, pass out Version 2 of the sheets and have them play again, this time drawing four cards.

Now you are going to play, Create the Rate. First, decide who will be Player A and who will be Player B. We will use a regular deck of cards, but you need to remove all of the face cards. The deck contains the 2 through 10 cards. The 10 card represents 0 and the ace represents 1. There are four of each number in the deck. When you are finished removing the face cards, shuffle the deck.

Give students time to remove the face cards and shuffle.

Look at the Version 1 directions and find your Create the Rate Game Sheet, Version 1 in your Student Booklet; follow along as I read the directions to you. As I read each step of the game, follow the directions and write your answers on Round 1 of your Player A or Player B game sheet. When we're done, you will do the remaining rounds on your own and sum and

record your Total Scores to determine who has more points.

1. **Shuffle the cards and spread them out face down on the desk.**
2. **Each player draws 3 cards.**
3. **Each player arranges the cards in an order to create the largest unit ratio.**
4. **Each player writes their card arrangement on the Create the Rate Game Sheet and then uses the calculator to determine the unit rate (if necessary).**
5. **The player with the largest unit ratio receives 1 point. He or she circles "1" for the score for that round.**
6. **Play the next four rounds using 3 cards.**
7. **Calculate the Total Score. The player with the most points wins.**
8. **If time permits, play another 5 rounds, this time with 4 cards and using the Version 2, Create the Rate Sheets I will pass out.**
9. **Calculate the Total Score. The player with the most points wins.**

Follow along as students play the game and check for understanding. If students struggle with finding unit rates, provide error correction as needed. Some students may not have time to play using Version 2.

Practicing Together

There is no Practicing Together for this lesson.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Have students turn to the Notes page located in their Student Booklets.

> **Turn to the Notes page in your Student Booklet. Write 1 new idea you have learned about ratios in this module.**

Have students share their ideas as time allows.

Ratio and Proportions 1
Lesson 9

Lesson 9: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, unit rate	
Misconception	When students use a table, they sometimes will notice a pattern that is focused on noting the changes in either a row or a column, but it does not generalize for the entire table.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Set of You Say, I Say Game Sheets (see page 142 of Teacher Masters)	• Student Booklet • Red colored pencil

Warming Up

Play You Say, I Say to support students' development of identifying rules represented in tables. Turn to the Warming Up sheet located in the Teacher Masters. Project the You Say, I Say Game Sheets (see page 142 of Teacher Masters). Select a rule from below or make up your own rule. Using the first game sheet, call on students to give you a number. Apply your rule to the number and place the result in the I Say column on the whiteboard. When students think they know the rule, have them raise their hand. As numbers continue to be called, ask those that think they know the rule to supply the result in the I Say column. You may want to play a practice round. Play as time allows, using up to four game sheets. Throughout the game and when done, check for understanding.

We are going to play a game called You Say, I Say. I will call on some of you to give me a number. I will put it in the You Say column of the table. Then, I have a rule in my mind that I am going to use on your number. I will record the result in the I Say column. When you think you know the rule, raise your hand but don't say the rule out loud. As we keep going, when you are sure you know the rule, write the rule in the I Say column.

Rules that you can use (You Say number is x, the result is the I Say number):

1. $2x + 1$
2. $3x - 2$
3. $4x$
4. $2x + 5$
5. $3x + 2$
6. $(x + 1) \div 2$
7. $x + 6$
8. $3x + 1$
9. $3x$
10. $2x + 10$

Learning to Solve

TEACHER NOTES

When students are asked to find a pattern in a table, they often will only look at the difference from row to row (recursive thinking) rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model the covariational aspects so that students move to thinking about the relationships of the quantities.

Students make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Up to this point, we have been identifying ratios, creating equivalent ratios, finding missing values in ratios, and determining unit rates. These have involved using different computations like multiplication or division. Today, we are going to show ratios using a table.**
>
> **Turn to the Learning to Solve section in your Student Booklet. Mona started a table in problem 1 that shows the number of triangles and the number of angles. Decide what should go into the blank cells of the table.**

Give students time to find the missing values. Have students share the values they found. Record them in the table as they share.

> **How did you determine the missing values? Did anyone use a different method?**
>
> **Let us look at problem 2. This problem asks you to describe the relationship(s) that you see in the table. Write what you think best describes the relationship between the number of triangles and the number of angles.**

What do you think is the relationship between the number of triangles and the number of angles? *(Accept reasonable answers such as the number of triangles increases by 1, the number of angles increases by 3. There are multiple statements or descriptions that are equivalent.)*

Ask pairs to share the relationships they wrote. As they share, record their responses on the whiteboard or by projecting their work.

It would be possible to write ratios to show the relationship(s) that you found. In problem 3, write at least 1 ratio that would show the relationship(s) between the 2 quantities.

Have students share the ratios they created. As they share, ask them to say the ratio in the form of: There are _____ for every _____. For example, they would say there are 3 angles for every triangle as the unit rate or ratio is 3:1. Watch for students writing the ratio in the same order as they say the relationship.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers in the table. Check for accuracy and discuss any discrepancies. You may find computational errors when students are working with the fractional amounts. For questions 2 and 3, focus on the covariational thinking aspects of identifying the relationships.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Students find unit rates.

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet in the Student Booklets.

Turn to the Wrapping It Up sheet in your Student Booklet and find the unit rates on your own.

As students share their answers, watch for multiple ways of solving the problems. You may notice that some methods are used more often than others.

Discuss the methods as they share out as time permits.

What methods did you use to find the unit rates?

Did you notice that the ratio was to be written to show cost per pound? That means that we write the cost first so that it fits the order of the ratio. As you continue to work with unit rates and ratios, you will

want to pay close attention to the order of the comparison.

Ratio and Proportions 1
Lesson 10

Lesson 10: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, unit rate, ratio	
Misconception	When students identify patterns in a table, they may think that you only need to look at one pair of entries or focus on the change in the output column.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Ratio Round-Up Mat (1 per pair, see page 143 of Teacher Masters) • Set of Ratio Round-Up Cards (1 per pair, see page 144 of Teacher Masters) • Red colored pencil

Warming Up

Divide students in pairs to play Ratio Round-Up. Distribute a Ratio Round Up Mat (see page 143 of Teacher Masters) and a set of Ratio Round-Up Cards (see page 144 of Teacher Masters) to each pair. Tell the pairs they are to sort the cards under each of the unit rates at the top of each column.

> **We are going to play Ratio Round-Up. In your pair, you will sort the cards under the mat heading that is the correct unit rate for your ratios.**

Allow students time to work. Debrief as time allows by asking students to describe the strategy they used for sorting the cards.

Learning to Solve

Teacher Notes

When students are asked to find a pattern in a table, they often will only look at the differences from row to row (recursive thinking) rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model the covariational aspects so that students move to thinking about the relationships of the quantities. This can be modeled by saying, for example, as the number of dogs increases by 1, the number of legs increases by 4.

Students make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **In the last lesson, you completed tables and found relationships. Today, we are reversing it so that you will be given a relationship and you make the table.**

Turn to the Learning to Solve section in your Student Booklet. With your partner, decide on the entries that you will make in your table. Make sure that your entries show the relationship that is given.

Give students time to make their table. Have students share the values they put in the table. Record them in the table as they share. Ask them to explain how they found their answers. If an entry is incorrect, do not correct at this time. After the pairs have shared all their entries in the table, discuss.

This table shows all the entries you made in your tables. We need to check to see if all the entries represent equivalent ratios. Let us start by first finding the unit rate. Determine or calculate the unit rate and write it in your booklet.

Give students time to determine the unit rate.

What is the unit rate? $(\frac{1}{2} : 1)$ **In your pairs, check the entries that I recorded in the table to make sure that they are equivalent to the unit rate.**

The unit rate represents the relationship that says for every cup of granola, there is a half cup of banana chips.

If there are a large number of entries in the table, you may want to divide the entries among the pairs to save time. Allow pairs time to work. When you notice that they have completed their checks, ask them to report out.

Before we look at any entries with which you do not agree, I would like you to share your method for deciding if the entry was correct or not.

Select a pair to demonstrate how they used the unit rate (or another method) to determine if an entry was correct. After they share, ask if any pair used a different method. If so, ask them to share. Continue in this way until all methods have been shared. Discuss as needed.

Were there any entries that you do not agree with? If so, be prepared to defend your answer.

Have pairs share any with which they do not agree. Have pairs support their answer by indicating what method they used to determine that they were not equivalent. Discuss as needed.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

> **With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.**

Have pairs of students share their answers in the table. Discuss as before in Learning to Solve. Check for accuracy and discuss any discrepancies. You may find computational errors when students are working with the fractional amount given in problem 3. The relationship is that she uses 3 times as many tablespoons as she uses cups of milk.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

> If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It On Your Own, branch to Lesson 10A to provide extended practice before proceeding to Lesson 11.

Wrapping It Up

Students find the missing value in proportions and ratios.

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Turn to the Wrapping It Up sheet in your Student Booklet and find the missing values on your own.

As students share their answers, watch for multiple ways of solving the problems.

Discuss the methods as they share out.

How did you find the missing values? What method did you use?

Ratio and Proportions 1
Lesson 11

Lesson 11: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, unit rate, ratio	
Misconception	When students compare tables, they often will look only for common entries. They neglect focusing on relationships such as a consistent unit rate.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Match the Unit Rate Cards (1 set for the class, see page 145 of Teacher Masters) • Red colored pencil

Warming Up

Play Match the Unit Rate. Distribute the Match the Unit Rate Cards (see page 145 of Teacher Masters), 1 per student. If you have less than 30 students, you may want to give some students more than 1 card. Plan ahead to have your unit rates ready. Call out the unit rates. If students have a ratio with that unit rate, they stand up and share their ratio. Below, the X stands for the unit rate you say, and the *x:x* stands for the ratio the student should say, based on your unit rate.

> **We are going to play Match the Unit Rate. There are cards with ratios on them. Teacher will call out a unit rate; if you have a card with a ratio that matches the unit rate, stand up and share the ratio. For example, if the unit rate is 9 and you have a ratio 9 to 1 or 18 to 2, stand up, show the card, and state the ratio.**
>
> **The unit rate is X** *(ratio = x:x)*

Learning to Solve

TEACHER NOTES

When students are asked to find a pattern in a table, they often will only look at the different from row to row (recursive thinking) rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model the covariational aspects so that students move to thinking about the relationships of the quantities. This can be modeled by saying, for example, as the number of dogs increases by 1, the number of legs increases by 4.

Students make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

In the last 2 lessons, we have extended tables and created tables, given a ratio. In this lesson, we are going to continue to work with tables and practice what you have learned.

You will work in pairs to solve problems. The problems will require you to think about all the things you know about ratios and tables. Be prepared to share your solution methods and defend your answers.

Give students time to complete the problems. Ask pairs to share their solutions and solution methods. After a pair presents their solution and its method, ask if anyone solved it a different way OR got a different answer. Discuss the methods and solutions. You may find that there are computational errors that will need some review as students share their solutions.

The tables do not have the same entries. How did you decide if the tables represented equivalent ratios?

Practicing Together

There is no Practicing Together as this is merged with Learning to Solve for this lesson.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Students find the missing value in proportions and ratios.

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Turn to the Wrapping It Up sheet in your Student Booklet and find the missing values on your own.

As students share their answers, watch for multiple ways of solving the problems. Ask them to share the method they used to determine the missing values.

What method did you use to find the missing value?

Discuss their strategies as time allows.

Ratio and Proportions 1
Lesson 12

Lesson 12: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Dependent variable:** a quantity whose value depends on another quantity **Independent variable:** a quantity that affects another variable (the dependent variable)	
Requisite Vocabulary	Equivalent ratios, unit rate	
Misconception	Students often confuse the two variables and cannot see the dependency relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Colored pencils or crayons (orange, blue, red, green, yellow, 1 of each per group of 4)

		• Reba's Rectangle Game Cards (1 set per group of 4, see page 146 of Teacher Masters) • 1 Reba's Rectangle Grid Sheet per group of 4 (see page 147 of Teacher Masters)

Warming Up

Divide students into groups of 4. Distribute Reba's Rectangle Game Cards (page 146 of Teacher Masters), one to each team member. Provide each group with one Reba's Rectangle Grid Sheet (see page 147 of Teacher Masters) and a set of colored pencils or crayons. Each student in the group should have 1 card, and each group will have a grid sheet. Allow time for them to work. Share out the rectangles as time allows, there should be multiple answers.

> **We are going to play Reba's Rectangle. You will solve Reba's Rectangle as a 4-person team. Each of you should have 1 card. You will take turns reading your card to your group. Your group must decide how to color in the blocks on your grid sheet so that your final product matches all the clues. Be sure that you read all the clues first before you start coloring your grid. Once you find a solution, decide if there are other solutions.**

There are multiple answers. For example, one rectangle could have 6 orange, 2 blue, 3 red, 6 yellow, and 3 green blocks. Other rectangles are possible such as 12 orange, 4 blue, 6 red, 12 yellow, and 6 green blocks.

Learning to Solve

Teacher Notes

When students are describing the dependent and independent variables, they should note that it is the <u>number of</u>, rather than the name of the object. This is in preparation for describing the meaning of the variable in some of the next lessons and module. It is important that students realize that the variable is generalizing about quantities, and it is not used as an object.

Students identify dependent and independent variables.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **What is a ratio?** *(A ratio is a comparison of two quantities; it tells us about a relationship or comparison between two quantities or amounts.)* **If we look closely at the relationship, we can see that one of the quantities affects the other quantity.**
>
> **Turn to the Learning to Solve sheet in your Student Booklet. Let us look at some examples together. In the first example, we are comparing the number of dogs to the number of legs. What is the ratio of the number of dogs to the number of legs?** *(1:4 or any equivalent ratio)*
>
> **What can you say about the relationship between the number dogs and the number of legs? As we increase the number of dogs, what can we say about the number of legs?** *(the number of legs increases by 4)* **We say that the number of legs <u>depends on</u> the number of dogs. Write that in your Student Booklet.**
>
> **Because the number of legs depends on the number of dogs, we say that the number of legs is the <u>dependent</u> variable. It depends on something else.**
>
> **The number of dogs is the <u>independent</u> variable because it affects or controls the number of legs, the dependent variable. Write that relationship in your Student Booklet.**
>
> **Look at example 2. Think about the relationship and fill in the blanks. Make sure that you include "number of" when you describe the dependent and independent variables.**

Give students time to complete the questions. Discuss their responses as needed to make any corrections. Make sure that as they write the independent and dependent variables, the students include the NUMBER OF. They should not write just triangles and angles, for example.

If you know the dependent and independent variables, you can describe the relationship using words. Let us look at example 3 in your Student Booklet. How could we say the relationship in words if we know the dependent and independent variables? Decide and write your idea in your Student Booklet.

Give students time to complete the questions. Discuss their responses as needed to make any corrections.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers. Check for accuracy and discuss any discrepancies. Students may try to use the order of the numbers in the statements to determine which one is the dependent or independent variable. Focus on establishing the meaning of the relationship rather than order in the statements.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Students find unit rate.

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet in the Student Booklet.

> **Turn to the Wrapping It Up sheet in your Student Booklet and find the unit rate for the 3 problems on your own.**

As students share their answers, discuss the meaning of unit rate.

Ratio and Proportions 1
Lesson 13

Lesson 13: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Dependent variable, independent variable	
Misconception	Students confuse the two variables and cannot see the dependency relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent • Set of You Say, I Say Game Sheets (see page 142 of Teacher Masters)	• Student Booklet • Red colored pencil

Warming Up

Play You Say, I Say to support students' development of finding rules in tables. Turn to the Wrapping It Up sheet located in the Teacher Masters. Project the You Say, I Say Game Sheets (see page 142 of Teacher Masters). Select a rule from below or make up your own rule. Using the first game sheet, call on students to give you a number. Apply your rule to the number and place the result in the I Say column on the whiteboard. When students think they know the rule, have them raise their hand. As numbers continue to be called, ask those that think they know the rule to supply the result in the I Say column. You may want to play a practice round. Play as time allows, using up to four game sheets. Throughout the game and when done, check for understanding.

We are going to play a game called You Say, I Say. I will call on some of you to give me a number. I will put it in the You Say column of the table. Then, I have a rule in my mind that I am going to use on your number. I will record the result in the I Say column. When you think you know the rule, raise your hand but don't say the rule out loud. As we keep going, when you are sure you know the rule, write the rule in the I Say column.

Rules that you can use (You Say number is x, the result is the I Say number):

1. $2x + 1$
2. $3x - 2$
3. $4x$
4. $2x + 5$
5. $3x + 2$
6. $(x + 1) \div 2$
7. $x + 6$
8. $3x + 1$
9. $3x$
10. $2x + 10$

Learning to Solve

TEACHER NOTES

When students are describing the dependent and independent variables, they should note that it is the number or amount of, rather than the name of the object. This is in preparation for describing the meaning of the variable in some of the next lessons. It is important that students realize that the variable is generalizing about quantities, and it is not used as an object.

Students identify dependent and independent variables.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **In your Student Booklet, turn to the Learning to Solve sheet. Look at problems 1, 2, and 3. Write your descriptions and give an example of dependent and independent variables.**

Give students time to complete the questions. Ask some students to share their responses. Discuss their responses as appropriate. If any students give an example of dependent and independent variables that is not accurate, ask them to help find a way to revise the example so that it is accurate.

> **Sometimes relationships can be found in descriptions of situations. Follow along with example 1 in your Student Booklet.**

Read the situation in the Teacher Masters, or have a student read it. then give students time to think about a relationship they might find in the problem.

The sixth-grade class sponsored a movie to make money for a class trip. They charged $3.25 per person to watch the movie.

Think about the situation. What relationships can you find? Write your idea in your Student Booklet. Then, share your idea with the person sitting next to you. Compare what the two of you wrote. Did you find similar relationships?

Monitor students' work. When you see students finishing, pair them up with a student next to them. Have them share their ideas and compare and contrast them. Then, ask some pairs to share with the class. Watch for relationships that focus on the fact that the total amount earned from the movie is dependent upon the number of people who attend. This relationship requires students to make inferences about the situation. For some, this may be a new skill. Be explicit in the discussion that follows.

You may have noticed that you have to think about the situation in order to find the relationships. One relationship that you can find is that for every person attending the movie, the 6th grade class makes $3.25. What is the independent variable? *(The number of people attending the movie)* **What is the dependent variable?** *(The total amount of money made by the class)*

Why is the amount of money the 6th grade class makes the dependent variable? *(Because it depends on the number of people attending the movie)*

To find the dependent variables in a situation, you have to think about what quantity depends on the other one. The quantity that it depends on will be the independent variable.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers. As students share, record their descriptions of the relationships they find. Ask if any other pairs had another way of saying the relationship. Check for accuracy and discuss any discrepancies. Students may try to use the order of the numbers in the statements to determine which one is the dependent or independent variable. Focus on establishing the meaning of the relationship rather than order in the statements.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

As students share their answers, you may want to note their methods. Some review of dividing decimals or fractions may be needed.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It On Your Own, branch to Lesson 13A to provide extended practice before proceeding to Lesson 14.

Wrapping It Up

Students find the unit rate.

Display the Wrapping It Up sheet located in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Turn to the Wrapping It Up sheet in your Student Booklet and find the unit rates on your own.**

Ask students to share their answers.

> **What method did you use to find the unit rate? How would you read the unit rate?**

Ratio and Proportions 1
Lesson 14

Lesson 14: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways, including writing an expression or equation. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Dependent variable, independent variable	
Misconception	Students confuse the independent and dependent variables.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Set of You Say, I Say Game Sheets (see page 142 of Teacher	• Student Booklet • Red colored pencil

	Masters)	

Warming Up

Students identify patterns in a table.

Play You Say, I Say to support students' development of finding rules in tables. Turn to the Wrapping It Up sheet located in the Teacher Masters. Project the You Say, I Say Game Sheets (see page 142 of Teacher Masters). Select a rule from below or make up your own rule. Using the first game sheet, call on students to give you a number. Apply your rule to the number and place the result in the I Say column on the whiteboard. When students think they know the rule, have them raise their hand. As numbers continue to be called, ask those that think they know the rule to supply the result in the I Say column. You may want to play a practice round. Play as time allows, using up to four game sheets. Throughout the game and when done, check for understanding.

We are going to play a game called You Say, I Say. I will call on some of you to give me a number. I will put it in the You Say column of the table. Then, I have a rule in my mind that I am going to use on your number. I will record the result in the I Say column. When you think you know the rule, raise your hand but don't say the rule out loud. As we keep going, when you are sure you know the rule, write the rule in the I Say column.

Rules that you can use (You Say number is x, the result is the I Say number):

1. $2x + 1$
2. $3x - 2$
3. $4x$
4. $2x + 5$
5. $3x + 2$
6. $(x + 1) \div 2$
7. $x + 6$
8. $3x + 1$
9. $3x$
10. $2x + 10$

Learning to Solve

TEACHER NOTES

Students may not remember some of the algebraic symbolism for showing multiplication. You may have to review, depending on the needs of your students. These symbols would include ×, ·, $(x)(y)$, and so on.

The relationships shown in this module will focus on direct proportions, which are written in the format of $y = mx$.

Students identify dependent and independent variables, and write an expression or equation to describe the relationship.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **In your Student Booklet, turn to the Learning to Solve section. As we learned in previous lessons, tables can be used to show equivalent ratios. Look at Henna's table. With a partner, complete the table but leave the bottom row of the table blank. Then identify the independent and dependent variables.**

Give students time to complete the example. Ask some students to share their responses. Discuss their responses as appropriate.

> **How would you describe the relationship in the table? Write your description in your Student Booklet.** *(Answers will vary, but focus on students saying that the total amount of money earned is 5 times the number of hours worked.)*

As students describe the relationships, focus on those that indicate the multiplicative nature.

Let us revisit your table and find another way to describe the relationship. Look at the bottom row of Henna's table in your Student Booklet. We can write the relationship in the table by using a variable so that it gives us the rule for any number we might use. This is called a generalization. Turn to the Generalization section in your Student Booklet. A generalization is a statement that describes patterns and relationships. Write this generalization on the Generalizations page: For a term, $5x$ for example (5 is the coefficient, x is the variable), you can change the value of x, but the amount will always be a factor of 5, or whatever coefficient is with the variable. In the next column, draw a picture if possible to represent the generalization. In the last column, write an equation for the generalization.

Now look at the bottom of the first column. We are going to use a variable, in this case x, to help us write the generalization. The variable x represents the number of hours worked.

We want to write an expression that describes the relationship or the computation that is used on x to give the amount in the second column. You told me that the amount in the second column is 5 times the amount in the first column. We will write 5 times x in the second column.

If students are unclear or do not remember how to show 5 times a variable, you may want to review the symbols for showing multiplication with a variable. There are multiple ways to write the multiplication. These include $5x$, $5 \cdot x$, or $(5)(x)$. It is not advisable to use the symbol × to represent times because students can confuse the multiplication symbol and the variable.

What, if for every hour worked, the pay was $6.00? How would that change the generalization $5x$ that we wrote? What would be the generalization now? *(6x)*

You can use the generalization to find the amount of pay for any hours worked. If the generalization is $6x$, what is the pay for working 12 hours? *($72)*

Continue with similar examples if needed.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

> **With your partner, complete the tables in the Student Booklet.**

Have pairs of students share their answers. As students share their answers, you may want to note how they identify generalizations with a variable. As appropriate, you may want to have them look for equivalent ways. For example, there are multiple ways of writing $2x$ such as $2(x)$ or $2 \cdot x$.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of their Student Booklets and graph their number of correct answers.

Wrapping It Up

Ask students to turn to the Wrapping It Up in their Student Booklet while you display the prompt from the Teacher Masters.

Write the generalization, using a variable, that shows the following:

For every cup of rice, it takes 3 cups of water to cook it.

Discuss student responses as time allows.

Ratio and Proportions 1
Lesson 15

Lesson 15: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways, including writing an expression or equation. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Dependent variable, independent variable, unit rate	
Misconception	Students may interpret multiplicative relationships in tables as additive because they are using recursive thinking.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator • Red pencils

Warming Up

Students determine unit rates.

Display the Warming Up sheet located in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Turn to the Warming Up sheet in your Student Booklet and find the unit rates on your own.

As students share their answers, discuss the meaning of unit rate as needed. Ask students to describe the process they used to find the unit rate. Students may need some help in working with rational numbers in the problem. If you notice that the numbers selected in the problems are too difficult for students to compute, change the numbers so they are whole number value and will lessen the complexity of the problem.

How did you find the unit rates?

Learning to Solve

TEACHER NOTES

Students may not remember some of the algebraic symbolism for showing multiplication. You may have to review, depending on the needs of your students.

The relationships shown in this module will focus on direct proportions, which are written in the format of $y = mx$.

Students identify dependent and independent variables, and write an expression or equation to describe it.

In the last lesson, we began to find ways of writing an algebraic expression to generalize a relationship we found in a table of equivalent ratios.

Display the first table in the Teacher Masters.

> **Remember this problem from Trying It On Your Own in the last lesson? What are the generalizations?** *(the number of tablespoons of chocolate syrup is 3 times the number of cups of milk)*
>
> **What is the dependent variable?** *(number of tablespoons of chocolate syrup)* **What is the independent variable?** *(number of cups of milk)* **How do you know which is the independent and dependent variable?** *(accept reasonable answers that describe the relationship and how the number of tablespoons of chocolate syrup is dependent on the number of cups of milk)*
>
> **How can we write the relationship between the independent and dependent variables?** *(3x)* **In this expression, the *x* represents the number of cups of milk. The expression 3*x* represents the number of tablespoons of chocolate syrup.**

Display the second table in the Teacher Masters.

> **Look at the second table in your Student Booklet in Learning to Solve. Draw lines to match the items in the left column to independent or dependent variable in the right column. How do you know which variable is independent or dependent?** *(accept reasonable answers)* **What is the relationship or the generalization of number of math problems to the number of minutes to complete?** *(2x)*

Give students time to solve. Ask them to share with a partner. You can have pairs share their answers or show the matching and have them compare.

Practicing Together

Display the Practicing Together sheet located in the Teacher Masters. Have students turn to the Practicing Together sheet located in their Student Booklets.

Have students work with their partner to complete this activity.

With your partner, answer the questions on the Practicing Together sheet located in the Student Booklet.

Have pairs of students share their answers. Watch for equivalent ways of writing the expression using variables.

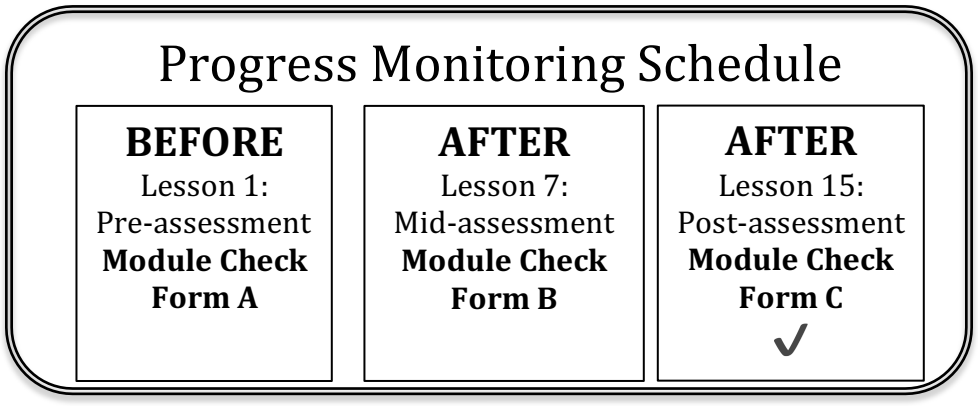
Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets. You may want students to have access to a calculator as they work.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of their Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Concept Map created in Lesson 1. Have students review the map and add to, delete, or revise the map.



Appendices

Ratios and Proportions

1

Ratios and Proportions 1
Lesson 6A

Lesson 6A: Applying Multiplicative Relationships

Lesson Objectives	Students find missing values in proportions. Students use the means-extremes product property of proportionality to determine equivalence. Students reason abstractly and quantitatively. (SMP 2)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, proportions	
Misconception	Some students misapply the multiplicative relationship we often call "cross-multiplying."	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	Student Booklet • Prop 4 Game Cards (1 set per student, see pages 133- 136 in Teacher Masters) • Prop 4 Game Board (1 per student, see page 137 in Teacher Masters) • 20 2-color counters (or appropriate counters) per pair • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Complete the 4 problems on your own.

Allow students time to work. Discuss each problem using the following questions.

How did you find the values? What method did you use? Did anyone solve it a different way?

Learning to Solve

Teacher Notes

The means-extremes product property of proportionality is often called cross-multiplying. Students often overgeneralize the use of cross-multiplication. In this module, we use the more formal name as an effort to maintain the integrity of the property.

Students will find missing values in proportions.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Look at problems 1 through 4. Write the steps to find the missing value, x, as we complete the steps together.

Have students do each step first and then correct or discuss as needed. When solving the equation $2x = 140$, students may divide both sides of the equation by 2 or consider what times 2 equals 140. Either approach is acceptable.

Look at the next problem, the football field.

Select a student to read the problem.

Follow along as [student] reads the problem.

Pause for the student to read.

Do you think the dimensions in the model are the same ratio as the dimensions of a regulation size football field? Think about it to yourself first.

Pause for students to think.

Turn to your partner and share your idea.

After students have shared with each other, ask them to share their answer with the class.

Let's determine whether the dimensions are the same. What is the ratio of width to length of the actual football field? *($\frac{160}{360}$)* **What is the ratio of Ron's model?** *($\frac{2}{5}$)* **This problem is different because we do not have to find a missing value. Rather, we need to determine whether the dimensions are in the same ratio. There are a few ways to solve.**

One way to simplify the first ratio, $\frac{160}{360}$. What is $\frac{160}{360}$ simplified? *($\frac{4}{9}$)* **Because it did not simplify to $\frac{2}{5}$, it is not the same ratio. What is another way to solve?** *(accept reasonable answers, such as those in the Teacher Masters)*

We can also use the steps above to check whether it is the same ratio. What product do we find first? *(160 × 5)* **What is 160 × 5?** *(800)* **What is the next product to solve?** *(360 × 2)* **What is 360 × 2?** *(720)* **Is 800 equal to 720?** *(no)* **Because the products are not equal, the ratio is not the same.**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity.

Have pairs of students share their answers. If they used cross-multiplication, have them show the equation they used and how they solved the equation. Other solution methods may be possible.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of their Student Booklets and graph their number of correct answers.

Wrapping It Up

Play Prop 4. Distribute a set of Prop 4 Game Cards (see pages 133- 136 of Teacher Masters), a Prop 4 Game Board (see page 137 of Teacher Masters), and counters to each student. The first student will take a card, match it to an equivalent ratio on the board, and place a counter on the board space. Students continue matching until someone has 4 counters in a row (up, down, or diagonal).

> **We are going to play Prop 4. You will shuffle your cards. The first player will take a card and match it to an equivalent ratio on your board by covering it with**

your color counter. Then the next player takes a turn and does the same. Continue playing until someone has 4 counters in a row, up, down or diagonally. Some ratios may have more than one match and there may be some that have no match.

Allow students to play as long as time allows.

Ratios and Proportions 1
Lesson 10A

Lesson 10A: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Equivalent ratios, unit ratios and rates	
Misconception	When students identify patterns in a table, they may think that you only need to look at one pair of entries or focus on the change in the output column.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Set of You Say, I Say Game Sheets (see page 142 of Teacher Masters)	• Student Booklet • Calculator • Red colored pencil

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets. If you feel the computations are too difficult, allow students to use calculators.

Turn to the Warming Up sheet in your Student Booklet. Find the missing values on your own.

Have students share their answers. Watch for multiple ways of solving the problems, such as finding equivalent fractions or using cross-multiplication. Discuss the methods as students share.

How did you find the missing values? What methods did you use? Did anyone use a different method?

Learning to Solve

Teacher Notes

When students are asked to find a pattern in a table, they often look at only the differences from row to row (recursive thinking), rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model the covariational aspects, so that students move to thinking about the relationships of the quantities. This can be modeled by saying, for example, as the number of dogs increases by 1, the number of legs increases by 4.

Students will make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the table as the lesson progresses.

We need to create a table to show that for every 6 pounds a person weighs on Earth, they weigh 1 pound on the moon.

What is the unit rate or unit ratio? *(6:1)* **Write this ratio on the line for number 2.**

What value should we write in the first row of the weight on Earth column? *(6)* **When you weigh 6 pounds on earth, how much do you weigh on the moon?** *(1 pound)*

What if you weighed 24 pounds on Earth, how much would you weigh on the moon? *(4 pounds)* **How did you determine your weight on the moon?** *(accept reasonable answers, such as divide by 6, thinking that 6 times 4 is 24)*

With your partner, decide on the entries that you will make in your table. Make sure that your entries are equivalent to the unit rate of 6:1.

Give students time to make their table.

Have students share their values. Record them in the table as students share. Ask them to explain how they found their answers. If an entry is incorrect, do not correct at this time. After the pairs have shared their entries in the table, discuss.

We need to check whether the entries are equivalent. Let's start by finding the unit rate of each entry.

Give students about a minute to determine the unit rate.

What is the unit rate? *(6:1)* **In your pairs, check whether the entries I recorded in the table are equivalent to the unit rate.**

Allow pairs time to work.

Do you not agree with any entries? Why? How did you determine the values? Did anyone solve it differently?

Have pairs share any values with which they do not agree.

Look at problem 3. Using our unit rate of 6:1, if you weigh 138 pounds on Earth, how much will you weigh on the moon? *(23 pounds)* **How did you solve?** *(accept reasonable answers, such as divide by 6)*

Look at problem 4. How could I write this as a proportion? What value are we trying to find? *(weight on Earth)* **Under the problem, write the proportion $\frac{6}{1} = \frac{x}{2.5}$. If you weigh 2.5 pounds on the moon, how much do you weigh on Earth?** *(15 pounds)* **How did you solve?** *(accept reasonable answers, such as multiply by 6)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete the activity sheet.

Have pairs of students share their answers in the table. Discuss as before in Learning to Solve. Check for accuracy and discuss any discrepancies. Problem 3 requires students to multiply 12 by 2 to get 24 ounces so that they can determine the ratio.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Play You Say, I Say to support students' development of finding rules in tables. Turn to the Wrapping It Up sheet located in the Teacher Masters. Project the You Say, I Say Game Sheets (see page 142 of Teacher Masters). Select a rule from below or make up your own rule. Using the first game sheet, call on students to give you a number. Apply your rule to the number and place the result in the I Say column on the whiteboard. When students think they know the rule, have them raise their hand. As numbers continue to be called, ask those that think they know the rule to supply the result in the I Say column. You may want to play a practice round. Play as time allows, using up to four game sheets. Throughout the game and when done, check for understanding.

We are going to play a game called You Say, I Say. I will call on some of you to give me a number. I will put it in the You Say column of the table. Then, I have a rule in my mind that I am going to use on your number. I will record the result in the I Say column. When you think you know the rule, raise your hand but don't say the rule out loud. As we keep going, when you are sure you know the rule, write the rule in the I Say column.

Rules that you can use (You Say number is x, the result is the I Say number):

1. $2x + 1$
2. $3x - 2$
3. $4x$
4. $2x + 5$
5. $3x + 2$
6. $(x + 1) \div 2$
7. $x + 6$
8. $3x + 1$
9. $3x$
10. $2x + 10$

Ratios and Proportions 1
Lesson 13A

Lesson 13A: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Dependent variable, independent variable	
Misconception	Students confuse the two variables and cannot see the dependency relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent) • Set of You Say, I Say Game Sheets (see page 142 of Teacher Masters)	• Student Booklet • Red colored pencil

Warming Up

Play You Say, I Say to support students' development of finding rules in tables. Turn to the Wrapping It Up sheet located in the Teacher Masters. Project the You Say, I Say Game Sheets (see page 142 of Teacher Masters). Select a rule from below or make up your own rule. Using the first game sheet, call on students to give you a number. Apply your rule to the number and place the result in the I Say column on the whiteboard. When students think they know the rule, have them raise their hand. As numbers continue to be called, ask those that think they know the rule to supply the result in the I Say column. You may want to play a practice round. Play as time allows, using up to four game sheets. Throughout the game and when done, check for understanding.

We are going to play a game called You Say, I Say. I will call on some of you to give me a number. I will put it in the You Say column of the table. Then, I have a rule in my mind that I am going to use on your number. I will record the result in the I Say column. When you think you know the rule, raise your hand but don't say the rule out loud. As we keep going, when you are sure you know the rule, write the rule in the I Say column.

Rules that you can use (You Say number is x, the result is the I Say number):

1. $2x + 1$
2. $3x - 2$
3. $4x$
4. $2x + 5$
5. $3x + 2$
6. $(x + 1) \div 2$
7. $x + 6$
8. $3x + 1$
9. $3x$
10. $2x + 10$

Learning to Solve

TEACHER NOTES

When students describing dependent and independent variables, they should note that the variables represent the number or amount of, rather than the name of, the object. This understanding prepares students for describing the meaning of variables in the next lessons. It is important that students realize that the variables generalize about quantities and are not used as an object.

Students will identify dependent and independent variables.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Sometimes, relationships can be found in descriptions of situations. Look at the relationship on the Learning to Solve sheet.**

Read the situation in the Teacher Masters or have a student read it. Complete the questions as the lesson progresses.

> **The sixth-grade class is selling hot dogs at the softball game to raise money for their field trip. For every 10 hot dogs they sell, they earn $25.**
>
> **Think about the situation. What relationships can you find?** *(accept reasonable answers, such as the number of hot dogs to the amount of money students earn)* **What is the ratio?** *(10:$25)*
>
> **If the ratio is 10 hot dogs sold earns $25, what is the unit rate?** *(1:$2.50)* **How did you determine the unit rate?** *(divide 25 by 10)*

Think about hot dogs and amount of money earned. What is the independent variable? *(number of hot dogs sold)*

What is dependent on the number of hot dogs sold? *(the amount of money earned)*

Look at the table. What should be written on the blank in the column title "Number of hot dogs sold?" *(Independent)* **What can we label amount of money earned?** *(Dependent)*

Write in the first values, 10 and $25. *(pause)* **If they sold 15 hot dogs, how much money would the sixth-grade students earn?** *($37.50)* **How did you solve?** *(multiply 15 by 2.5)*

If they earned $50, how many hot dogs were sold? *(20)* **How did you solve?** *(divide by 2.5)*

Have students complete the last row on their own. Have students share the values.

How many hot dogs were sold? How much money was earned? What method did you use? How do you know that it is equivalent to the ratio 10:$25? *(accept answers that are equivalent to the ratio 10:$25, such as 25 and $62.50)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

Have students work with their partner to complete this activity.

Have pairs of students share their answers. Check for accuracy and discuss any discrepancies. Students may try to use the order of the numbers in the statements to identify the dependent or independent variable. Focus on establishing the meaning of the relationship, rather than order in the statements.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of their Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **For each statement, write "IV," which stands for "independent variable," above what you think is the independent variable. Write "DV," which stands for "dependent variable," above what you think is the dependent variable.**

Discuss student responses as time allows.