Ratios and Proportions

2

Teacher Lessons

The Meadows Center for Preventing Educational Risk—Mathematics Institute
The University of Texas at Austin

Ratios and Proportions 2
Lesson 1

Lesson 1: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Ratio**: A relationship between 2 quantities or numbers	
Requisite Vocabulary	None	
Misconception(s)	As students interpret relationships in tables, they may use the relationships only in rows without attending to the changes in the columns. This is called recursive thinking.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Progress Monitoring Schedule

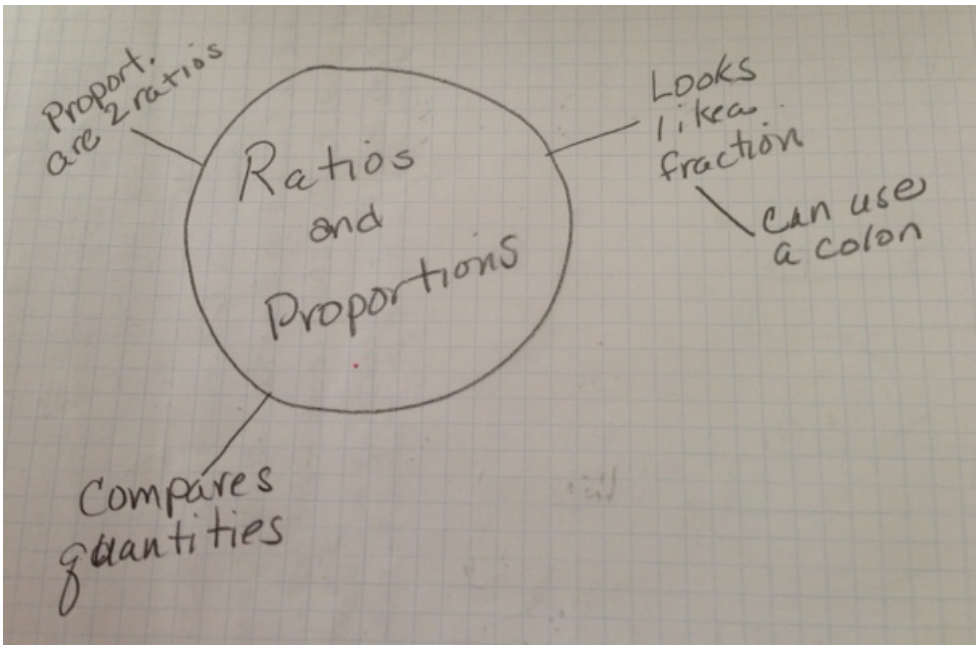
BEFORE	AFTER	AFTER
Lesson 1: Preassessment **Form A** ✔	Lesson 7: Midassessment **Form B**	Lesson 15: Postassessment **Form C**

Warming Up

Create a concept map about ratios and proportions on a class poster or chart paper and display it for the duration of the module. Have students turn to the Warming Up sheet in their Student Booklets. Students will first write their own ideas on this page and then add peer responses. Draw a circle on the board with "Ratios and Proportions" in the middle.

> **Today, we will start by thinking about what you know about ratios and proportions. Write everything you know about ratios and proportions in your Student Booklet.**

Ask for student responses. Make a concept map similar to the photo below as students share ideas. Ask for students to clarify or provide examples to aid in the brainstorming of ideas.



> **What are some important ideas? What is a ratio? What is a proportion? Can you add an example to any of your words?**

When students finish sharing their ideas, display the map for the duration of this module. The concept map will be used again at the end of this module. Displaying the concept map during the module allows both teachers and students to add ideas, change ideas, or make connections across lessons.

Learning to Solve

Teacher Notes

Using tables to determine relationships such as unit rates provides a foundation for more complex work with ratios and proportions. These tables should not be called "function tables" so that students do not develop misconceptions regarding functions. When students are asked to find a pattern in a table, they often look at only the changes from row to row (recursive thinking) rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model the covariational aspects, so that students move to thinking about the relationships of the quantities.

1. Students will make and extend tables of ratios.

   Have students turn to the Notes section of their Student Booklets.

   > **In the warming up, you shared things that you know about ratios and proportions. Let's focus first on ratios. If someone asked you what a ratio is, how would you describe or define it?**
   > *(students may give characteristics of a ratio, such as it is multiplicative in nature, it uses a colon or can be written in fraction form, and so on)*

   Write the characteristics or definition on your whiteboard as students share. As lesson progresses, model what students should write.

   > **A ratio is relationship between 2 quantities or numbers. Write this definition in your Notes section.**
   >
   > **One way to show relationships is by using a table.**

   Display the Learning to Solve sheet from the Teacher Masters. Have students turn to the Learning to Solve sheet in the Student Booklet.

Mona started a table in problem 1 that shows the number of triangles and the number of angles. Fill in the missing values.

Give students time to find the missing values.

What are the missing values? *(9, 12, 21, 10)* **How did you find these values? Did anyone solve differently? Why did you choose that way to find the values?**

Look at problem 2. What is the relationship between the number of triangles and the number of angles? Write the relationship between the number of triangles and the number of angles.

Write student responses on the board.

What relationships did you write? *(answers will vary)* **How are your relationships similar? Different?**

Words are one way that we can represent the relationship. Another way to represent the relationship is to show the ratio with numbers, like 1:3. For example, there is 1 triangle for every 3 angles.

When we represent the relationship or the ratio, we need to make sure that we match the numbers or words to the relationship being described. In our table, triangles are in the first column and angles are in the second column, so we need to write the relationship between triangles and angles. To match this statement, we would need to write 1 triangle for every 3 angles. It would be incorrect to write 3 to 1. Why? *(answers will vary. For example, students may say that 3 to 1 could incorrectly show 3 triangles for every angle.)*

In problem 3, write at least 1 ratio that would show the relationship between the 2 quantities. Remember that there are different ways to write a ratio.

Have students share the ratios they created.

As we share the ratios that you wrote, we need to read them in a similar way. When you share your ratio, you will read it like this: There is (blank) for every (blank). As you read them, I will record them. Remember that order is important when representing ratios.

As students share, record them on the whiteboard. Have students compare and contrast the ratios. Make sure that the ratios are represented in the "colon" format, such as 1:3 or 3:1, and that the order is correct.

What do you notice about the ratios? *(students should notice that the ratios all represent the same relationship)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students complete the Practicing Together sheet in small groups or as an entire class if appropriate.

2. Review the sheet as a class when complete, asking students to describe or explain how they used the table to find the missing values.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 15A to provide extended practice.

Wrapping It Up

Have students respond in their Student Booklets in the Notes section.

In your Notes section, describe a time that you used a ratio or describe how a ratio could be used to show a relationship.

Ratios and Proportions 2
Lesson 2

Lesson 2: Making Tables to Show Ratios With Unit Rates

Lesson Objectives	Students make and extend tables to show ratios. Students find the unit rate. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Unit rate**: a ratio expressed as *x*:1; the relationship is shown such that the second quantity or number is 1	
Requisite Vocabulary	Ratio; equivalent	
Misconception(s)	Students may think that a unit rate can be written as 1:*x* rather than *x*:1.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **You are asked to find the cost of 1 pound of 2 different foods. Solve each problem, and be sure you can explain the process you used during problem solving.**

Allow students to complete the sheet.

> **How did you solve the problems? What method did you use? Did anyone solve it differently?**

Learning to Solve

Teacher Notes

Creating a table, given a relationship, gives students an opportunity to find equivalent ratios. The task then leads into finding the unit rate.

1. Students will make and extend tables of ratios.

> **A way to show relationships is to use a table. In Lesson 1, you were given a table and you completed or extended it, using the relationship given in the table. Today, we will reverse that idea. You will make a table from a given relationship.**

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **What does the relationship tell you?** *(3 cups of granola for every 2 cups of banana chips)* **How many cups of granola are given in the relationship?** *(3 cups)* **Record that in the first blank row and column in your table.**

How many cups of banana chips would be used if we used 3 cups of granola? *(2 cups)* **Record that in the second row and column in your table.**

What if you had 6 cups of granola? How many cups of banana chips would you use? *(4 cups)* **How do you know?** ***(twice as many as before)***

Complete the table, using the relationships of 3 cups of granola for every 2 cups of banana chips. Select the numbers to complete the table.

Have students complete the table.

Have students share the values they put in the table. Record them in your displayed table. Ask students to explain how they found their answers. Do not correct incorrect answers at this time.

What values did you use to complete the table? How did you determine how many banana chips were needed? Did anyone solve differently?

This table shows all of the entries you made in your tables. We need to check whether all of the entries are equivalent to the relationship of 3 cups of granola to 2 cups of banana chips. Let's start by first finding how many cups of granola we would use if we had 1 cup of banana chips.

Give students time to determine the quantity.

How many cups of granola for 1 cup of banana chips? *(1.5)* **How did you solve?**

We could write this ratio as 1.5:1, which says that for every cup of banana chips, we use 1.5 cups of granola. In this ratio, the second number or quantity is 1. This ratio has a special name. It is called a unit rate. All unit rates have 1 as the second quantity or number.

You can use the unit rate to determine equivalent ratios. What does "equivalent" mean? *(the same as or equal)* **Let's check the**

entries recorded in the table to make sure that they are equivalent to the unit rate.

Let's find how we can use the unit rate to check our work. We will do 1 together. On my table, I wrote 4 cups of granola and 20 cups of banana chips. I wanted to find how many cups of banana chips I would need if I had 4 cups of granola. I think I need 20 cups of banana chips. Do you agree with me? Why or why not? *(Disagree because this is in a different ratio.)*

Thinking about the unit rate, what should it be? *(1.5:1 or equivalent)*

How many cups of granola to 20 cups of banana chips? *(30 cups)* **How did you use the unit rate to solve? Did anyone solve it differently?**

Continue asking for different ways of using the unit rate until all methods have been shared. Discuss as needed.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner, in small groups, or as a class if applicable to complete the sheet.

2. Have students share their answers and their reasoning for how they completed the table. Discuss as before in Learning to Solve. Check for accuracy and discuss any discrepancies. You may find computational errors when students work with the fractional amount given in problem 3. The relationship is that she uses 3 times as many tablespoons of chocolate syrup as she uses cups of milk. Remind students about the unit rate as needed.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and have them mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

In the Notes section of your Student Booklet, think about which method you prefer: creating your own table with a given relationship like we did today or completing a table like we did in Lesson 1? Why?

Discuss student responses as time allows.

Lesson 3: Using Ratio Tables

Lesson Objectives	Students make, extend, and use tables to show ratios and determine equivalency. Students find unit rates and other relationships from tables. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**None**	
Requisite Vocabulary	Unit rate	
Misconception(s)	When students are asked to find unit rate, a pattern, or relationship in a table, they often will look at only the changes from row to row (recursive thinking), rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). It is important to continue to model covariational aspects, so that students move to thinking about the relationships of the quantities.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

If students in your class have difficulty working with fractions, do only problems 1 and 2. These problems use a multiple of a quantity and result in whole numbers. Problem 3 results in a fraction.

> **We need to find the missing values. Follow along as I read number 1. For every 8 tablespoons of drink mix, 14 cups of water are needed. How many cups of water are needed for 24 tablespoons of drink mix?**
>
> **So, you need 8 tablespoons of drink mix for 14 cups of water. How many cups of water are needed for 24 tablespoons of drink mix?** *(42 cups)*
>
> **How did you solve? What method did you use?** *(accept reasonable answers, such as 8 times 3 is 24, so 14 times 3 is 42)* **Did anyone solve it differently?**
>
> **Read and solve numbers 2 and 3 on your own.**

Give students time to work, then call upon students to share their answers.

> **How did you solve? What method did you use?** *(accept reasonable answers)* **Did anyone solve it differently?**

Learning to Solve

1. Students will make, extend, and use tables to show ratios and determine equivalency.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **In the last lesson, we extended and created tables when given a ratio. In this lesson, we will continue to work with tables and practice what you have learned.**

Select a student to read the first problem.

> **Look at the first problem and table. Follow along as [student] reads.**

Pause for the student to read.

> **What is the relationship that Brent and Tim created a table for?** *(cups of carob chips to cups of banana chips)*
>
> **Both boys started their tables with the same values: 5 cups of carob chips are needed for 2 cups of banana chips. How could we write this ratio?** *(5 to 2, 5:2)*
>
> **After this initial relationship, the values in the tables are different. We have to determine whether the tables represent the same ratio. Turn Problem 1.**
>
> **First, what is the unit rate of the cups of carob chips for every cup of banana chips in Brent's table. If we had 1 cup of banana chips, how many cups of carob chips do we need?** *(2.5)* **How did you determine the amount of carob chips needed?** *(accept reasonable answers)*
>
> **Write the unit rate, 2.5:1, for the cups carob chips to the cups of banana chips.**
>
> **Using the unit rate ratio, we can now determine whether Brent's and Tim's tables represent the same ratio. How can we do that?** *(accept reasonable answers, such as multiplying the cups of banana chips by 2.5 or dividing the cups of carob chips by 2.5)*
>
> **Brent and Tim chose different values to represent the cups of carob chips; do Brent's table and Tim's table represent the same ratio?** *(yes)* **Mark your booklet.**
>
> **Tim is going to extend his table. If Tim used 30 cups of banana chips, how many cups of carob chips will he need? Write your answer.** *(75 cups of carob chips)* **How did you solve?** *(multiply 30 by 2.5)* **Did anyone solve it differently?**

Brent is also extending his table. If Brent used 30 cups of carob chips, how many cups of banana chips would he need? Write your answer. *(12 cups of banana chips)* **How did you solve?** *(divide by 2.5)* **Did anyone solve it differently?**

Practicing Together

This lesson has no Practicing Together sheet.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

What ideas about fractions have you used in this module on ratios and proportions?

Discuss student responses as time allows.

Lesson 4: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Dependent variable:** a quantity whose value depends on another quantity (the independent variable) **Independent variable:** a quantity that affects another variable (the dependent variable)	
Requisite Vocabulary	Unit rate	
Misconception(s)	Students sometimes use the order of the ratio to determine which is the dependent variable and which is the independent variable.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **These problems ask you to find the unit rate. How do we write a unit rate as a ratio?** *(x:1)* **How do we find unit rates?** *(accept reasonable answers, such as answers that involve multiplication or division)* **How would you describe a unit rate?** *(accept reasonable answers, such as that it is the ratio of something to 1 or that it represents the relationship of the first quantity or number to the 1 of the second quantity or number)*

Allow students time to complete the problems.

> **What is the unit rate for each problem? How did you find the unit rates? Did anyone solve differently?**

Learning to Solve

Teacher Notes

When students describe a dependent or independent variable, they should note that it is the number of, rather than the name of, the object. This understanding prepares students for describing the meaning of the variable in the next lessons. It is important that students realize that variables generalize about quantities and are not used as objects.

1. Students will identify dependent and independent variables.

> **Ratios tell us about relationships between 2 quantities or amounts. If we look closely at the relationship in a ratio, we can see that 1 of the quantities affects the other quantity.**

Display the Learning to Solve sheet from the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets. Complete the sheet as the lesson progresses.

In the first problem, we are comparing the number of dogs to the number of legs. What is the ratio of the number of dogs to the number of legs? *(1:4 or any equivalent ratio)* **Write your ratio in the blank. Today we will better understand the relationship and label each variable in the ratio as independent or dependent.**

As we increase the number of dogs, the number of legs increases. So the number of legs depends on the number of dogs. Fill in those blanks in your Student Booklet.

Because the number of legs depends on the number of dogs, we say that the number of legs is the dependent variable. The dependent variable depends on another quantity, the independent variable. Write "number of legs" on the dependent variable line.

The number of dogs is the independent variable because it affects or controls the number of legs, the dependent variable. Write "number of dogs" on the independent variable line.

Look at the first sentence. This sentence can help us understand and identify each variable as independent or dependent.

Could I say that the number of legs causes a change or controls the number of dogs? *(no)* **Why?** *(doesn't make sense)*

Look at problem 2. Think about the relationship, use the sentence to help you identify the independent and dependent variables, and fill in the blanks.

Give students time to work, checking that they write "number of triangles" and "number of angles," not just "triangles" and "angles."

What is the independent variable? *(number of triangles)* **What is the dependent variable?** *(number of angles)*

So the number of angles depends on what? *(number of triangles)*

If you know the dependent and independent variables, you can describe the relationship by using words.

Let's look at problem 3. How could we say the relationship in words if we know the dependent and independent variables? Decide and write your idea in your Student Booklet.

Give students time to work. Discuss their responses and make any corrections.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets

1. Have students complete the sheet with a partner or as a whole class if more appropriate.

Read each ratio problem and identify the independent and the dependent variables.

2. Have students share their answers. Check for accuracy and discuss any discrepancies. Students may try to use the order of the numbers in the statements to determine the dependent and independent variables. Focus on establishing the meaning of the relationship, rather than the order.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping it Up sheet in the Teacher Masters. Have students turn to the Wrapping it Up sheet in their Student Booklets.

> **Describe a situation that shows a relationship between dependent and independent variables.**

Discuss student responses as time allows.

Ratios and Proportions 2
Lesson 5

Lesson 5: Determining Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Dependent variable, independent variable, unit rate	
Misconception(s)	Students often interpret the dependent and independent variables as objects, rather than quantities.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Resource Booklet • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Some students may have difficulty with computation of decimals. You may need to help students follow the steps for computation or allow them to use a calculator.

> **Each of these problems asks you to find the unit rate. Write each of the answers as a ratio. What is important to remember when you write a unit rate as a ratio?** *(the second number must be 1)*

Give students time to complete the problems.

> **What is the unit rate for each ratio? How did you solve? Did anyone solve differently?**

Learning to Solve

Teacher Notes

When students describe a dependent or independent variable, they should note that it is the number of, rather than the name of, the object. This understanding prepares students for describing the meaning of the variable in the next lessons. It is important that students realize that variables generalize about quantities and are not used as objects.

1. Students will identify dependent and independent variables.

   Display the Learning to Solve sheet from the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   > **Write your answers to problems 1, 2, and 3 in your own words. For 1 and 2, write a description of a dependent and**

independent variable. For number 3, give a specific example that shows a dependent and independent variable. Identify each variable.

Give students time to work. You may complete as a whole class if students are struggling to write and identify dependent and independent variables.

Ask some students to share their responses and write their responses on the board. If a student gives an example of a dependent or an independent variable that is not accurate, ask a different student to help revise the example so that it is accurate.

How did you describe dependent and independent variables? What example did you write? Let's check our examples. Does your dependent variable depend on the independent variable?

Select a student to read problem 4.

Sometimes, relationships can be found in descriptions of situations. Follow along as [student] reads the situation in problem 4.

Pause while the student reads.

Think about the situation. What relationship can you find?

Describe the relationship and identify the independent and dependent variables in your Student Booklet. Then, share your responses with the person sitting next to you. Compare what you wrote. Did you find similar relationships?

Try applying this sentence to the relationship, "As the number of _____, the number of _____."

Monitor students' work. Ask some pairs to share with the class. Watch for relationships that focus on the total amount earned from the movie is dependent upon the number of people who attend. This relationship requires students to make inferences about the situation. For some, this may be a new skill.

How did you describe the relationship from the example? *(answers will vary; write what students share)*

You may have noticed that you have to think about the situation to find the relationships. One relationship that you can find is that for every person attending the movie, the sixth-grade class makes $3.25. What is the independent variable? *(the number of people attending the movie)* **What is the dependent variable?** *(the amount of money the sixth-grade class makes)*

Why is the amount of money the sixth-grade class makes the dependent variable? *(because it depends on the number of people who attend the movie)*

To find the dependent variables in a situation, you have to think about what quantity depends on the other. The quantity that the dependent variable depends on is the independent variable.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner or as a class if more appropriate to complete the sheet.

2. Have pairs share their answers. As students share, record their descriptions of the relationships. Ask whether any other pairs had another way of saying the relationship. Check for accuracy and discuss any discrepancies. Students may try to use the order of the numbers in the statements to identify the dependent and independent variables. Focus on establishing the meaning of the relationship rather than the order in the statements.

Trying It On Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Circle how you feel about the statement, "I feel comfortable identifying independent and dependent variables."**

Discuss students' answers as time allows.

Lesson 6: Determining and Describing Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways, including writing an expression or equation. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**Variable:** a letter or symbol that represents an unknown quantity	
Requisite Vocabulary	Dependent variable, independent variable	
Misconception(s)	Students often think a variable represents an object, rather than a quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Identify the independent and dependent variable in each problem.

Allow students time to work.

How did you decide which was the independent or dependent variable? How would you describe the relationships?

Learning to Solve

Teacher Notes

Students may not remember some of the algebraic symbolism for showing multiplication. You may have to review these symbols, depending on the needs of your students. Some students may think that $5x$, for example, represents a 2-digit number, rather than multiplication. The symbols are $5 \cdot x$, $(5)(x)$, or $5x$. Note that when using a variable, using the multiplication symbol (×) is not recommended because it can be confused with the variable.

The relationships shown in this module focus on direct proportions, which are written in the format of $y = mx$.

1. Students will identify dependent and independent variables and write an expression or equation to describe the variables.

   Display the Learning to Solve sheet from the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   As we learned in previous lessons, tables can be used to show equivalent ratios. Look at the example.

How would you describe the relationship? *(Sam makes $5 an hour for helping his dad)* **What is the independent variable?** *(number of hours worked)* **What is the dependent variable?** *(amount of money made)* **Write these at the bottom of the page.**

Look at the table. We need to find the unit rate. If Sam works 1 hour for his dad, how much money does he make? *($5)* **What is the unit rate?** *(5:1)*

If he works for 2 hours, how much money has he made? *($10)* **What about when he works for 2.5, 3, and 6 hours?** *($12.50, $15, $30)* **How did you solve?** *(multiply the hours by 5)*

Look at the next value Sam made: $22.50. How many hours did he work? *(4.5)* **How did you solve?** *(divide 22.50 by 5)*

Work with your partner to complete the next 2 missing values, and at the bottom of the page, note the relationship of the number of hours worked and the amount of money earned.

Watch for computational errors with decimals in determining the missing values. You may want to review some decimal computations or allow students to use a calculator.

Let's find another way to describe the relationship. We can represent the relationship by using a variable. A variable is a letter or a symbol that represents an unknown value. What is a variable? *(a letter or symbol that represents an unknown quantity)*

We will use a variable—in this case, *x*—to write a rule or generalization that describes the relationship in the table. We will write this rule or generalization in the last row.

The variable, *x*, will represent the independent variable, the number of hours Sam worked. Record *x* in the last row of the number of hours worked column.

We want to write an expression that describes the relationship or the computation that is used on *x* to give the amount in the second column, or the amount of money earned. You told me that the amount in the second column is 5 times the amount in

the first column. We will write "5*x*," which means 5 times *x*, in the second column.

What if for every hour worked, the pay was $6? How would that change the generalization, 5*x*, that we wrote? What would be the generalization now? *(6x)*

Continue with similar examples if needed.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the sheet.
2. Have pairs of students share their answers. Watch for equivalent ways of writing the expression, using variables. For example, some students may write "2*x*," and others may write "2(*x*)" or "2 × *x*."

If you notice that students are writing inappropriate expressions, such as 2 *x x*, correct it.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Write a generalization, using a variable, that shows the following: For every cup of rice, it takes 3 cups of water to cook it.**

Discuss student responses as time allows. Answers will vary but they should be in the form of $3x$, where x is the number of cups of rice. Make sure the variable is identified as the number of cups of rice. Some students may mistakenly say it is the number of cups of water. $3x$ represents the number of cups of water.

Ratios and Proportions 2
Lesson 7

Lesson 7: Determining and Describing Dependent and Independent Variables

Lesson Objectives	Students determine the dependent and independent variables in a situation. Students describe relationships between the dependent and independent variables in multiple ways, including writing an expression or equation. Students reason abstractly and quantitatively. (SMP 2) Students look for and express regularity in repeated reasoning. (SMP 8)	
Vocabulary	**None**	
Requisite Vocabulary	Variable, dependent variable, independent variable, generalization, algebraic expression	
Misconception(s)	Students often think a variable represents an object, rather than a quantity.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet • Calculator (optional)

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **On your own, find the unit rates. How do we write a unit rate as a ratio?** *(the second number is 1, x:1)*

Allow students time to complete. Allow students who struggle with decimal computations to use calculators.

> **How did you determine the unit rate? How would you describe the unit rate of problem 1 in words?** *(steak is $7.49 per pound)*
>
> **How would you describe the unit rate of problem 2 in words?** *(eggs are $2.50 per dozen)*

Learning to Solve

Teacher Notes

Students may not remember some of the algebraic symbolism for showing multiplication. You may have to review these symbols, depending on the needs of your students. Some students may think that 5*x*, for example, represents a 2-digit number, rather than multiplication. The symbols are $5 \cdot x$, $(5)(x)$, or $5x$. Note that when using a variable, using the multiplication symbol (x) is not recommended because it can be confused with the variable.

The relationships shown in this module focus on direct proportions, which are written in the format of $y = mx$.

1. Students will identify dependent and independent variables and write an expression or equation to describe the variables.

In the last lesson, we began to find ways to write an algebraic expression to represent a relationship or generalization we found in a table of equivalent ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

This problem is from Trying It on Your Own in the last lesson. We can look at the quantities and label them as dependent and independent variables. What is the dependent variable? *(number of tablespoons of chocolate syrup)* **What is the independent variable?** *(number of cups of milk)* **Label the variables in each column at the bottom of the page.**

What is the generalization, or the algebraic expression written to describe the relationship? *(3x)* **In this expression, the *x* represents the number of cups of milk. The expression 3*x* represents the number of tablespoons of chocolate syrup.**

Look at problem 2. At the bottom of the page, draw a line from the items in the left column to "independent variable" or "dependent variable" in the right column.

Give students time to solve. Have students share with a partner.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the sheet.

2. Have pairs of students share their answers. Watch for equivalent ways of writing the expression, using variables.

Trying It on Your Own

Display the Trying It On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Read the statement, "I can use a variable to write a generalization about a table," and circle how you feel.

AFTER this lesson: Administer Progress Monitoring
FORM B

Lesson 8: Graphing Relationships From a Table

Lesson Objectives	Students graph ordered pairs from tables of equivalent ratios. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP 4)	
Vocabulary	**Line graph:** a graph that can be used to show how something changes over time **Scatter plot**: a graph in which plotted points show a relationship but the points are not connected	
Requisite Vocabulary	Expression, ratio, relationship	
Misconception(s)	Students often do not understand the differences between graphs or understand when to use particular graphs.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

This is a line graph that shows the height of a river in Delaware on certain days. A line graph can be used to show how something changes over time. There is a gauge in the river that looks like a ruler. 0 would represent the floor of the river. The height of the river can be measured, so that flooding can be detected.

Answer the questions. What is the height of the river on November 24? *(2.80 feet or slightly less)* **How did you read the scale?**

On what date was the river's height about 3.10 feet? *(November 27 or 30)* **How did you read the scale?**

What could cause the spike in the graph between November 26 and November 28? *(heavy rainfall)*

Learning to Solve

Teacher Notes

Some graphs should be represented only as a scatter plot. These discrete graphs indicate that the values between each point are not appropriate for the context of the problem. Line graphs indicate that any value on the line can represent an appropriate value for the context of the problem.

1. Students will graph ordered pairs from tables of equivalent ratios.

In earlier lessons, we used tables to write expressions that show the relationships between the independent and dependent variables. Another way to show the relationship is to graph the information in a table.

Display the Learning to Solve sheet from the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

> **Look at problem 1. This table is similar to tables that we have completed before, except it has a third row. We will complete the third row later. First, read the problem and write in the missing values for cups of orange sherbet.**

Ask students to share the completed row. At this point, students should be able to complete the row with little discussion.

> **Now look at the third row at the bottom of the table. Complete this row by writing a fraction where cups of ginger ale is the numerator and cups of orange sherbet is the denominator.**
>
> **In this row, you will write the ratio of each of the different measures. We will write the ratio in a fraction form. Start in the column with 6 cups of ginger ale. When we started the table, these were the only values given.**
>
> **What is the ratio or the relationship of ginger ale to orange sherbet?** *(6 cups of ginger ale to 3 cups of orange sherbet)* **We can write the fraction $\frac{6}{3}$ to show that relationship. What is $\frac{6}{3}$ in simplified form?** *($\frac{1}{2}$)* **Write "$\frac{6}{3} = \frac{2}{1}$" in the last row below the 3. What is another way to write $\frac{2}{1}$?** *(2)*
>
> **Complete the table, showing the relationship as a fraction for each cup of ginger ale to cups of orange sherbet.**

Give students time to complete the ratios. As you monitor their work, check for students who are having difficulty.

> **What do you notice about all of the ratios?** *(they all represent the unit rate and all of the ratios are equivalent to 2)*

Use o for the number of cups of orange sherbet and g for the number of cups of ginger ale. What equation or rule could you write to show how to find the number of cups of orange sherbet for each cup of ginger ale? *(answers will vary—examples: $o = \frac{1}{2}g$ where g = the number of cups of ginger ale and o = the number of cups of orange sherbet; or students may write words, such as "the number of cups of orange sherbet is equal to half the number of cups of ginger ale")*

Ensure that all students have the correct values in the table before proceeding to problem 2.

Corey decided to graph the relationship between the number of cups of the ginger ale and the orange sherbet. You can see her graph in problem 2. Her graph has a special name. It is called a scatter plot. A scatter plot is a graph in which plotted points show a relationship but the points are not connected.

Look at the points on her graph. Talk with your partner. What do the points represent? What would the point (0, 0) mean as compared to $(1, \frac{1}{2})$? *(the point (0, 0) indicates that you do not use any of the ingredients, but the other points show $\frac{1}{2}$ cups of orange sherbet to 1 cup of ginger ale)*

Using the table we just completed, graph the remaining points on the scatter plot.

Have a student share the graph. Ask the other students to compare their graphs to the one shared. Correct any points that are incorrect. At this point, if students are unsure how to graph a point, you may want to discuss the process before proceeding.

How do the points on the scatter plot relate to the ratio of the number of cups of ginger ale to the number of cups of orange sherbet? *(the ratios are all the same at each point because they are equivalent)*

It would be possible for us to connect all of the points to make a line graph. If you connected the points, would they be on the same line? *(yes)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with their partner to complete the sheet.
2. Have pairs of students share their answers.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Turn to the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Circle your answer to the following statement: I know how to graph an ordered pair.

Lesson 9: Graphing Relationships From a Table

Lesson Objectives	Students graph ordered pairs from tables of equivalent ratios. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Scatter plot	
Misconception(s)	Because the points students graph are linear, students often believe that every scatter plot can be represented by a continuous line.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Look at the graph. Where is the *x*-axis?** *(across the bottom)* **Where is the *y*-axis?** *(to the left, vertical)* **How do you graph points on the graph that are shown in the table?** *(accept reasonable explanations)* **Graph the points given in the table.**

Allow time for students to work. As you monitor, if you notice a student having difficulty graphing, help the student graph the first point and then gradually fade your assistance for the rest of the points.

When students are finished, ask them to share their graphs and show on the graph how they found each point.

> **What do you notice about the points?** *(they form a line)*

Learning to Solve

Teacher Notes

Some graphs should be only represented as a scatter plot. These discrete graphs indicate that the values between each point are not appropriate for the context of the problem. Line graphs indicate that any value on the line can represent an appropriate value for the context of the problem.

1. Students graph ordered pairs from tables of equivalent ratios.

> **In the last lesson, we graphed ratios after completing or showing the relationship in a table.**

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

This is the table that you graphed during the Practicing Together section of the last lesson.

What do you notice about the ratios that you wrote? *(the ratios are the same or equivalent)* **Think about how these ratios look on a graph. If a table shows equivalent ratios, will the points make a line? Write your thoughts on the Learning to Solve sheet.**

Give students time to discuss with a partner and then ask students to share their ideas. Equivalent ratios will always form a line. However, students may have different ideas at this stage. Allow students to share while you write their ideas on the board. At this point, do not tell students that the points will form a line.

As we go through the lesson, think about whether you think these points form a line. We will refer back to this idea to see whether your ideas fit with the tables and graphs that we create in this lesson.

Turn to the next page of Learning to Solve. In the last lesson, Corey wanted to make her famous punch for the school carnival. She found that the orange sherbet she needed for her recipe was on sale at another store in town. What is the ratio? What is the price of orange sherbet to half-gallons of orange sherbet? *($\frac{3}{1}$ or \$3 to 1 half − gallon)*

How much would 2 gallons of orange sherbet cost? *(\$6)* **Write \$6 in its space in the second row. How did you solve? What is the ratio?** *($\frac{6}{2}$)* **Can this fraction be simplified?** *(yes, $\frac{3}{1}$)* **Write "$\frac{6}{2} = \frac{3}{1}$" in the last row under \$6. Complete the table.**

Allow time for students to complete the table. Check the table when completed.

If you graphed the data in your table, would the points form a line? *(yes)* **Write your answer in your Student Booklet. Why?** *(the ratios are equivalent)*

Graph your data to check your prediction. The number of half-gallons of orange sherbet is represented by the *x*-axis. The cost is represented by the *y*-axis.

As you monitor their work, watch for students who are having difficulty graphing. Allow time for students to graph the ordered pairs. Ask students to share their graphs.

What do you notice about all of the points on the graph? *(they form a line)*

There are many types of graphs, including scatter plots and line graphs. Line graphs indicate that any value on the line can represent an appropriate value for the context of the problem.

Think about our problem. The sherbet is sold in half-gallon containers. You cannot split a container; you must buy the full container. So, for example, is it possible to buy 1.5 half-gallons of sherbet? *(no)* **Because that value is not possible, this graph is a scatter plot.**

If students have difficulty with this idea, connect the points on the graph in the Teacher Masters and work through another value on the line.

Look at the points on your graph. What do the points represent? What does the point (0, 0) represent? *(no orange sherbet was bought)* **What about the point (1, 3)?** *(1 half-gallon of orange sherbet was bought and $3 was spent)* **What about (2, 6)?** *(2 half-gallons were bought and $6 was spent)*

What equation or rule could you write to show how to find the cost of the half-gallons of orange sherbet? If writing an equation, use *p* for the price of half gallons of orange sherbet and use *o* for the number of half gallons of orange sherbet. *(accept reasonable answers—examples: p = 3o where p = price of half-gallons of orange sherbet and o = the number of half-gallons of orange sherbet or students may use words such as "the cost of the half-gallons of orange sherbet is equal to 3 times the number of half-gallons of orange sherbet")*

Have students share the equations or rules, writing on the board. Compare the rules students wrote. If there are different rules, have students check to whether the different rules are equivalent.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with their partner to complete the sheet.
2. Have pairs of students share their answers. Focus on question 5. (2, 110) is an example of a nonequivalent ratio. All the other points represent equivalent ratios of 50:1.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Complete this sentence:

The points on a graph will be on the same line if ...

Discuss student responses as time allows. Revisit their initial predictions or conjectures from the first activity in Learning to Solve.

Ratios and Proportions 2
Lesson 10

Lesson 10: Creating a Table From a Graph

Lesson Objectives	Students create a table from a graph. Students determine a proportional relationship. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**Proportional relationship**: a relationship in which 1 quantity is equal to a constant multiplied by another quantity ($y = mx$)	
Requisite Vocabulary	None	
Misconception(s)	Students often think that when ordered pairs are graphed, they always form a proportional relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Graph the points in the table on the graph. Label each point.

As you monitor students working, watch for students who continue to have difficulty graphing points.

When students are finished, ask them share their answers and describe how they found the points.

When we graph points and do not connect them, what do we call the graph? *(a scatter plot)* **What do you notice about the points in our graph?** *(they form a line)*

Learning to Solve

Teacher Notes

Proportional relationships share some common characteristics on a graph. They always go through the origin and all of the points representing the relationship have the same ratio.

Rules or equations can be written so that they express the relationship in multiple ways. However, the standard form is $y = mx$.

1. Students will create a table of equivalent ratios from a graph.

In the last 2 lessons, you have graphed points from a table. Today, we will reverse this process by using a graph to create a table.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

This is a graph that shows the number of pepper plants and the number of tomato plants that Bob planted in his garden. The *x*-axis represents the number of pepper plants. Write "number of pepper plants" in the blank.

The *y*-axis represents the number of tomato plants. Write "number of tomato plants" in that blank. What do you notice about the points? *(the points are on a line)*

What do you notice about the ratio of the number of pepper plants to the number of tomato plants for each point? *(all are equivalent or the same)*

What kind of graph is this? *(scatter plot)* **Could we connect these lines and make it a line graph?** *(no)* **Why?** *(the points cannot be connected because it would not be possible, for example, to have a half of a plant)*

There are 6 points on the graph. Look at the first point. What is the *x*-axis, or number of pepper plants? *(1)* **What is the *y*-axis, or number of tomato plants?** *(3)* **Write these points in the table on the next page. What is the ratio?** *($\frac{3}{1}$)* **Write it.**

With your partner, determine the coordinates of the remaining 5 points and complete the table.

Give students time to discuss and then ask students to share their table.

Write an equation or a rule that shows the relationship found in the graph -- between the number of pepper plants and the number of tomato plants. If you write an equation, use *h* for the number of tomato plants and use *m* for the number of pepper plants. *(accept reasonable answers—examples: the number of tomato plants is 3 times the number of pepper plants, or there are one-third as many pepper plants as there are tomato plants, or h = 3m when h represents the number of tomato plants and m represents the number of pepper plants)*

Record the equations or rules on the board.

What do you notice about all of the equations or rules we found? *(each rule shows that the number of tomato plants is 3 times as many as the number of pepper plants)*

When you can multiply a number by another quantity and get an equivalent ratio, it is called a proportional relationship. In this problem, you can multiply the number of pepper plants by 3 to find the number of tomato plants. Or you can multiply the number of tomato plants by one-third to find the number of pepper plants. Write, "This is a proportional relationship" under your rule.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner or as an entire class if appropriate to complete the sheet.

2. Have students share their answers. If students have difficulty finding the coordinate of the points, review.

3. Have students share multiple rules. Students should notice that there are different ways of describing the rule for the table and the graph. Watch for students to notice that the relationship in Karen's point appears at first glance to be the same that Sarah used. However, students should notice that the relationship is different from Sarah's and does not give the same ratio.

Trying It on Your Own

This lesson has no Trying It on Your Own section.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Jacob, a fifth-grader, was not sure how to graph the point (3, 5) on the coordinate grid. Write a description of how you could help him.

Discuss student responses as time allows.

Ratios and Proportions 2
Lesson 11

Lesson 11: Creating Tables From Graphs

Lesson Objectives	Students create a table from a graph. Students determine a proportional relationship. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Proportional relationship	
Misconception(s)	Students often think that proportional relationships can be additive.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **How would you describe a proportional relationship? Write your description of a proportional relationship.**

Have students share their descriptions of a proportional relationship. Look for ideas about a relationship that can be described as a number or constant multiplied by a quantity.

> **A proportional relationship between 2 quantities can be represented as a number or constant multiplied by a quantity. Think about the relationship of number of dogs to number of legs. If you had 1 dog, how many legs?** *(4)* **If h represents the number of dogs and p represents the number of legs, what would be the relationship? p = 4 times what?** *(h)* **So we could write the relationship as $p = 4h$.**

Write "$p = 4h$" on the board. Provide similar examples as needed.

> **Use your description of a proportional relationship to decide whether each table represents a proportional relationship.**

Monitor students as they work.

When they finish, ask students to share their answers.

> **How did you decide whether the table represented a proportional relationship?**
>
> **Now, write a rule or equation to show the relationship in each table, if there is one.**

Monitor students as they work. When they finish, discuss and write each rule. Be sure to identify what each variable represents.

> **For tables 1 and 2, if you graphed that data, what would the graphs look like?** *(the points would form a line)*

Learning to Solve

TEACHER NOTES

Proportional relationships are only multiplicative. Any relationship that is additive is not be proportional. A graph that represents a proportional relationship should pass through the point (0, 0).

1. Students will create a table of equivalent ratios from a graph.

   In the last lesson, you found the coordinates of points on a graph and put them into a table.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   You worked on this graph in the last lesson. By looking at the graph, how can you tell that this is a proportional relationship?

   Give students time to discuss and then ask a student to share an idea.

   Think about how we could describe the values. When the value of the *x*-coordinate increases by 1, what happens to the value of the *y*-coordinate? *(the value of the y-coordinate increases by 0.5)* **Does the value of the *y*-coordinate increase or decrease?** *(increase)* **Do the points form a line?** *(yes)*

   Think about this question: If a graph forms a line, is it always a proportional relationship? We will not discuss your answer to that question yet, but write your prediction in your Student Booklet. We will return to it later.

   Not all graphs that form lines are proportional relationships. To be a proportional relationship, (0, 0) must be a point on the graph.

Look at problem 2 in your Student Booklet. Use this graph to complete the table.

Allow time for students to complete the table. Ask a student to share the completed table with the class.

What is the ratio of *y* to *x* for these points?

Allow students time to work with a partner to determine the ratio, $\frac{3}{4}$. Ask students to stand when they think they know the ratio. When all students are standing, ask a student to share the ratio and explain how he or she found the answer.

What do you notice about all of the points on the graph? *(they form a line)* **If you extended the line, would it go through (0, 0)?** *(yes)*

We just saw that the ratio for each point is the same. Is this a proportional relationship? *(yes)* **How do you know?** *(accept reasonable answers, such as if you multiply the value of x by $\frac{3}{4}$, you can find the y value)*

What is an equation or rule you could write that would show this relationship? *(accept reasonable answers, such an equation that is equivalent to $y = \frac{3}{4}x$)*

Encourage students to write the rule as an equation rather than in words.

Let's recap what we found. The ratio for each of the points is the same. The points form a line on the graph.

Keep those ideas in mind as we look at another example. Look at problem 3. Use this graph to complete the table.

Allow time for students to complete the table. Ask a student to share the completed table with the class.

What do you notice about all of the points on the graph? *(they form a line)* **What is the first data point?** *(1, 4)*

What is an equation or rule you could write that would show this relationship? *(accept any equation equivalent to y = x + 3)*

What is the ratio of *y* to *x* for these points?

Allow students time to work with a partner to determine the ratio. This is an additive relationship, and thus, there is no consistent ratio. Students may be puzzled about this fact.

Were you able to determine a consistent ratio? *(no)* **For this graph and table, we could not find a consistent, multiplicative ratio. If we do not have a consistent ratio, we cannot have a proportional relationship. Another way to check a proportional relationship is to see whether the data point (0, 0) is included. Is it?** *(no)*

Is this a proportional relationship? *(no)* **How do you know?** *(there is no consistent ratio and the graph did not include the point (0, 0) nor would it be if the line were extended)*

Practicing Together

This lesson has no Practicing Together section.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Turn to the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **Earlier in the lesson, you wrote a prediction on the Learning to Solve sheet about whether all lines represent a proportional relationship. Read your prediction. Is your prediction accurate? Why or why not?**

Discuss student responses as time allows.

Ratios and Proportions 2
Lesson 12

Lesson 12: Representing Relationships in Multiple Ways

Lesson Objectives	Students create a table from a graph. Students graph relationships from a table or equation. Students write an equation from a graph or table. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Proportional relationship	
Misconception(s)	Students are sometimes confused by a multiplicative relationship and think it is additive. They may also think that any line represents a proportional relationship.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

What determines a proportional relationship? *(a consistent, multiplicative relationship that includes the point (0, 0))*

For problems 1, 2, and 3, decide whether each relationship is proportional. For each problem, also answer the question, "How do you know?"

Provide time for students to work.

Ask students to share their answers.

How did you decide whether the representation was proportional?

What is the rule for proportional relationships? What equation can we write to show the rule?

If you graphed the equation in problem 1, what would the graph look like? *(it would be a line)*

Learning to Solve

Teacher Notes

Be sure to use the vocabulary of "substitute," rather than "plug in to." Some students may need to be reminded about the symbolism. $1.5x$ means to multiply the value of x by 1.5.

1. Students will graph a relationship from an equation.

In previous lessons, you found the coordinates of points on a graph, put them into a table, and then found a rule for the

relationship, using the table. Today, we will use those rules to make a graph.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

$y = 1.5x$ is an equation that shows us a relationship. Is this a proportional relationship? *(yes)* **How would you decide?**
(answers will vary. For example, some students may say that one point on the graph of this equation would be (0, 0))

Allow students to share their ideas with a partner.

What operation is used in a proportional relationship? *(multiplication)* **A proportional relationship means that a quantity is multiplied by a constant to find the value of the other quantity. This is considered a multiplicative relationship and thus a proportional relationship.**

If you wanted to graph the equation, you would need to find ordered pairs that make the equation true. How could you do that?

Ask students to share their ideas. The most productive response is to create a table by substituting values into the equation to find the ordered pairs.

One way is to create a table. In your Student Booklet, a blank table has been made for you. Under this table are steps. Step 1 is to create a table. Write, "Create a table" on the line.

Next, we have to select values for x. We can use easier numbers to practice. Write in 1, 2, 3, and 4 in the x row. Step 2 is to substitute values into the equation. Write, "Substitute values into the equation" for Step 2.

Step 3 is to complete the table. Write, "Complete the table" for Step 3.

Complete the table before we discuss Step 4. We have to substitute the values of x—1, 2, 3, and 4—into our equation.

What is 1.5 times 1? *(1.5)* **Write it. What is 1.5 times 2?** *(3)* **Write it and complete the table.**

What are the next 2 values for *y*? *(4.5 and 6)*

Step 4 is to graph the points. Write, "Graph the points" for Step 4.

Now we can use this table to graph the points on the coordinate grid. Graph the points in your Student Booklet.

Ask a student to share his or her graph.

What do you notice about all of the points on the graph? *(they form a line, extend to go through the point (0, 0), and the ratio of each is the same)*

If the equation represents a proportional relationship, we would expect that the graph would form a line and go through the origin (0, 0). That is a way that we can check our work. Is this a proportional relationship? *(yes)*

What is the process or steps to graph an equation? *(create a table, substitute values into the equation, complete the table, graph the points)*

We now have practiced 3 different ways to represent a proportional relationship. We can use a table, a graph, and an equation.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner or as a class if more appropriate to complete the sheet.

2. Have students share their answers, reporting the values selected for *x* and how they used the equation to solve and graph. Watch for students who have difficulty doing the multiplication.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

> **In your Student Booklet, write your response to Hannah's question: "My teacher asked me to graph an equation. What should I do?"**
>
> **What would you tell Hannah?**

Discuss student responses as time allows.

Ratios and Proportions 2
Lesson 13

Lesson 13: Representing Relationships in Multiple Ways

Lesson Objectives	Students create a table from a graph. Students graph relationships from a table or equation. Students write an equation from a graph or table. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Proportional relationship	
Misconception(s)	Students may confuse additive and multiplicative relationships.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

> **Follow the instructions and do problems 1 and 2. To complete the tables, you first need to determine the rule for each table. Make sure that you can explain the rules you used to decide what went into the tables.**

Allow students time to work. As they finish, ask students to share and justify their answers by explaining how they decided what rule to use (such as $y = 4x$ for the first problem).

> **If you graphed the points from the tables, what would the points look like?** *(they would form a line)*
>
> **How do you know they would form a line?** *(accept reasonable answers, such as the ratio for each point is the same)*

Learning to Solve

Teacher Notes

In this lesson, students continue to refine their use of multiple representations. By now, you should notice changes in their proficiency to graph points and determine the relationship.

1. Students will graph a relationship from an equation.

> **In the last lesson, you used an equation to find points to graph. What were the four steps we used to find the points?** *(create a table, substitute values into the equation, complete the table, graph the points)*

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

The equations we graphed in the last lesson all went through the origin, or the point (0, 0). Will the graphs of all equations will go through (0, 0)? Think about that question and then write your prediction in your Student Booklet.

Ask students to share their prediction with a partner. If time permits, you may want them to share some of their ideas as you record them for later discussion.

To help us think about this, let's look at the 2 equations on the Learning to Solve sheet, $y = 0.5x$ and $y = 2x + 1$. How are the equations alike? How are they different? Write a few ideas in your booklet.

Give students time to compare the equations. Ask students to share their ideas, writing them on the displayed Learning to Solve sheet. As students share, check their ideas by comparing the equations. For example, if students say that the *x* is multiplied by different numbers, have students check that the equations have different coefficients. Students should especially notice that in 1 equation, a constant or number is being added to the *x* term.

Before graphing these equations to determine how the differences you found affect the graph, complete the blank tables on the sheet. First, write in values for *x*. What is the next step? *(substitute the x values to determine y)*

Work with a partner to complete each table.

Allow students time to work. Have student pairs share the completed tables.

After the table is completed, what is the next step? *(graph the equations, using the data points from the table)* **Work with your partner to graph the 2 equations in your Student Booklet.**

Allow students time to work. Have some pairs share their graphs.

What do you notice about the graphs? How are the 2 graphs alike? How are they different? *(1 graph goes through the origin and the other graph does not; 1 graph is steeper than the other)*

Do either of the equations represent a proportional relationship? *(yes)* **How does the graph help you decide?** *(if a line goes through the origin, it is a proportional relationship; proportional relationships are only multiplicative in nature)*

Look back at the prediction that you wrote at the beginning of the lesson. Would you revise your prediction? If so, how? *(accept reasonable answers)*

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner or as a whole class if more appropriate to complete problem 2 on the Learning to Solve sheet.
2. Have students share their answers and their reasoning with the group.

Trying It On Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

In your Student Booklet, complete the sentence, "The points on a graph will be on the same line if…".

Discuss student responses as time allows.

Lesson 14: Solving Rate Problems

Lesson Objectives	Students solve rate problems in multiple contexts. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**Proportions:** 2 equivalent ratios	
Requisite Vocabulary	Unit rate	
Misconception(s)	Students sometimes overgeneralize the "cross-multiplying" strategy to find a missing quantity in a proportion, thinking that they can use the strategy anytime 2 ratios appear together. Students may use the strategy when they see fractions in a computational problem, such as in addition.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the problem. Determine the price of each T-shirt.

Allow time for students to solve the problem.

Ask students to share their answers. As students share, ask them to show how they found the answer. This problem is a good review of dividing a decimal number by a whole number.

You found the cost of 1 T-shirt in this problem. Does this remind you of anything you have done in this module?
(answers may vary and may include a comparison to unit rate)

Learning to Solve

Teacher Notes

Some students may have learned "cross-multiplying" as a strategy for finding an unknown amount in a proportion. However, using the multiplicative relationship is often as intuitive and allows students to make sense of the ratios in the problem context.

1. Students will solve rate problems.

   Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

   Finding the unit rate is 1 of the important things we have done in this module.

   You have used equations, tables, and graphs to solve problems. In this lesson, we will use the different ways we have represented ratios to solve problems.

Let's think about how we can use these ideas to solve problems, starting with the multiplication property of ratios.

Select a student to read problem 1.

Look at problem 1 and follow along as [student] reads.

Allow the student time to read.

What does the problem ask you to find? *(how many tablespoons of syrup he should add)*

What do we already know from the problem? *(3 tablespoons of chocolate syrup for 1 cup of milk)*

To solve this problem, we need to find an equivalent ratio. A way to think about it is to look at relationships in the given values. Writing a proportion may help you see the relationship. A proportion is 2 equivalent ratios. What is a proportion? *(2 equivalent ratios)*

What proportion could we write for this problem? *(accept any of the following responses: 1:3::3:x. Or 3:1::x:3. Or,* $\frac{1}{3} = \frac{3}{x}$ *or the equivalent)*

What strategy would you use to solve the problem? Decide what you would do to find the missing quantity and write it in your booklet.

Use your strategy to find the tablespoons of chocolate syrup to add to 3 cups of milk. How many tablespoons of syrup are needed? *(9 tablespoons)* **How did you solve?** *(student responses will vary and may include 1 cup was multiplied by 3 to get 3 cups, so the 3 tablespoons must be multiplied by 3; or multiply 1 by* x *and 3 • 3 to find that* x = 9*)*

Discuss each of the solution strategies. Have students record them in their Student Booklet. Ask whether other strategies were used.

Select a student to read problem 2.

Follow along as [student] reads the next problem.

What does the problem ask you to find? *(the number of carrots to feed 12 rabbits)*

How is it similar to problem 1? *(accept reasonable answers, such as it has a similar structure and/or it asks students to find an equivalent ratio)* **What proportion could we write for this problem?** *(5:2::x:12 or 2:5::12:x or,* $\frac{5}{2} = \frac{x}{12}$ *or the equivalent)*

What strategy would you use to solve the problem? Decide what you would do to find missing quantity and write it in your booklet.

Use your strategy to determine the number of carrots needed to feed 12 rabbits. How many carrots are needed for 12 rabbits? *(30 carrots)* **How did you solve?** *(student responses will vary and may include that the 2 rabbits were multiplied by 6 to get 12 rabbits, the 5 carrots must be multiplied by 6; or multiply 2 by x and 5 by 12 to find that 2x = 60 or x = 30)*

Discuss each of the solution strategies. Have students record them in their Student Booklet. Ask whether other strategies were used.

Both of these problems involved equivalent ratios. Finding the number to multiply a quantity in a ratio by to create a quantity in a second ratio is a strategy to solve for an unknown amount. You also identified other strategies.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the sheet.

2. Have pairs of students share their answers. Have students share how they determined the missing values. If they used a proportion, have students write the proportion. Ask students whether anyone used a different method and whether the table was useful.

Trying It on Your Own

This lesson has no Trying It on Your Own section.

Wrapping It Up

Display the Warming It Up sheet in the Teacher Resource Booklet. Have students turn to the Warming It Up sheet in their Student Booklets.

> **In your Student Booklet, respond to the questions, "How are a ratio and a proportion alike? How are they different?".**

Discuss student responses as time allows.

Ratios and Proportions 2
Lesson 15

Lesson 15: Solving Rate Problems

Lesson Objectives	Students solve rate problems in multiple contexts. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Proportions, unit rate	
Misconception(s)	Students sometimes overgeneralize the "cross-multiplying" strategy to find a missing quantity in a proportion, thinking that they can use the strategy anytime 2 ratios appear together. Students may use the strategy when they see fractions in a computational problem, such as in addition.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Determine how much Josie will pay for the apples.

Allow time for students to complete.

How much will Josie pay for the apples? *($1.78)* **How did you solve?** *(accept reasonable answers, such as finding the unit rate by dividing, creating a table, or using a proportion)* **Did anyone solve differently?**

Learning to Solve

1. Students will solve rate problems.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

You have used proportions, unit rates, tables, and graphs to solve problems. How would you describe to someone the process you use to find a unit rate? Write your description in your Student Booklet.

Give students time to write. Ask some students to share their process. Discuss the process as needed to make it explicit and specific. Have students correct their writing as appropriate.

In previous lessons, you developed strategies for finding missing quantities in ratios. Today, you will use the strategies you have been practicing to solve more problems.

We will do the first problem together and then you will complete the remaining problems with a partner.

Select a student to read problem 1.

Follow along as [student] reads.

Allow time for the student to read.

What does the problem ask you to find? *(which is the better buy)*

What is a way to solve? Does anyone have a different strategy?

Discuss student ideas.

One strategy is to find the unit rate, or how much the cereal costs per ounce? How can we figure out the unit rate? *(accept reasonable answers, such as dividing)*

Let's try dividing. What is $4.44 divided by 12? *($0.37)* **What is $7 divided by 20?** *($0.35)*

Which is the better buy? *(the 20-ounce box)* **Write that the 20-ounce box is a better buy because the price per ounce is cheaper. What is the difference in price per ounce?** *(2 cents)* **Add that to your answer, so you can accurately defend your answer.**

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in their Student Booklets.

1. Have students work with a partner to complete the sheet.
2. Have pairs of students share their answers and how they found the answers. Ask students whether anyone used a different method.

Trying It On Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 15A to provide extended practice.

Wrapping It Up

Display the concept map created in the first lesson of this module.

Have students review their ideas. Ask students to think about things they would revise, add, or delete and to write these ideas in their Student Booklet.

Discuss as time permits, making changes as students suggest them.

AFTER this lesson: Administer Progress Monitoring
FORM C

Appendices

Ratios and Proportions

2

Ratios and Proportions 2
Lesson 1A

Lesson 1A: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Ratio**: a relationship between 2 quantities or numbers	
Requisite Vocabulary	None	
Misconception(s)	As students interpret relationships in tables, they may use only the relationships in rows without attending to the changes in the columns. This is called recursive thinking.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the three problems on your own.

Provide time for students to work.

When they are done, have students share their answers. Watch for multiple ways of solving the problems and discuss the various methods.

Learning to Solve

Teacher Notes

Using tables to determine relationships such as unit rates is a foundation for more complex work with ratios and proportions. Do not call these tables function tables, so that students do not develop misconceptions regarding functions. When students are asked to find a pattern in a table, they often will look at only the changes from row to row (recursive thinking), rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model covariational aspects, so that students move to thinking about the relationships of the quantities.

1. Students will make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Tables are helpful in organizing information so that we can see patterns. You can use a pattern to complete a table or make a rule for the table.

For problem 1, Kara made a table to show a relationship between the number of bicycles and the number of wheels, but

she is missing some entries in the table. Decide what should go into the blank cells of the table.

Give students time to find the missing values. Have students share the values. Record them in the table as students share.

How did you determine the missing values? What relationship or pattern do you see between the number of bicycles and the number of wheels?

Students should notice that there are 2 wheels for every bicycle. This is an opportunity to introduce the language of "for every ___, there are ___." Write the pattern on the whiteboard.

To describe the pattern, we can say that for every bicycle, there are 2 wheels. Write the pattern or relationship in your Student Booklet next to the table.

Let's look at problem 2. Beau made a table to show the relationship between the number of hexagons and the number of sides. This table shows a relationship that Beau found. Complete the table. Then, fill in the blanks to complete the relationship that you see in the table.

It may be necessary to note that a hexagon has 6 sides. Ask pairs to share the table entries. As they share, record their responses on the whiteboard or by using the projector. Ask students to describe how they found the missing values.

What method did you use to decide what would go in the blank cells in the table? *(answers will vary—some students may use a numerical approach such as noticing that the entries in the right column are 6 times that of the left column)*

We can also describe the relationship by saying that there are 6 sides for every hexagon. It describes the same relationship.

We can write this relationship as "1:6." This means that for every 1 hexagon, there are 6 sides.

Have students write the ratio in their Student Booklet.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in the Student Booklet.

1. Have students work in pairs to complete the sheet.
2. Have student pairs show on the board how they solved the problems. Ask students to describe or explain how they used the table to find the missing values.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.
2. Give the answers to the students and have them mark their answers as correct or incorrect.
3. Have the students sum their correct answers and mark the total number correct at the top of their page.
4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

If the majority (51% or greater) of your class answers fewer than 3 questions correctly on Trying It on Your Own, branch to Lesson 1B to provide extended practice before proceeding to Lesson 2.

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets.

Share your answer to this problem with a partner: For every 3 apples, I pay $1.83. How much do I pay for 1 apple?

Have students share their answers as time allows.

Ratios and Proportions 2
Lesson 1B

Lesson 1B: Making Tables to Show Ratios

Lesson Objectives	Students make and extend tables to show ratios. Students make sense of problems and persevere in solving them. (SMP 1) Students reason abstractly and quantitatively. (SMP 2) Students attend to precision. (SMP 6)	
Vocabulary	**Ratio**: a relationship between 2 quantities or numbers	
Requisite Vocabulary	None	
Misconception(s)	As students interpret relationships in tables, they may use only the relationships in rows without attending to the changes in the columns. This is called recursive thinking.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in their Student Booklets.

Solve the three problems on your own.

Provide time for students to work.

When they are done, have students share their answers. Watch for multiple ways of solving the problems and discuss the various methods.

Learning to Solve

Teacher Notes

Using tables to determine relationships such as unit rates is a foundation for more complex work with ratios and proportions. Do not call these tables function tables, so that students do not develop misconceptions regarding functions. When students are asked to find a pattern in a table, they often will look at only the changes from row to row (recursive thinking), rather than thinking in more general terms to show how the 2 quantities vary together (covariational thinking). Continue to model covariational aspects, so that students move to thinking about the relationships of the quantities.

1. Students will make and extend tables of ratios.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in their Student Booklets.

Tables have been used to organize information and help us find patterns.

Dylan made a table in problem 1 to show the relationship between the number of boxes of books and the number of books. He is missing some entries in the table. Decide what should go into the blank cells of the table and write your answers. Then, describe the relationship for problem 2.

Give students time to find the missing values and describe the relationship. Have students share the values. Record them in the table as students share.

How did you determine the missing values? What relationship or pattern do you see between the number of boxes and the number of books?

Students should notice that there are 8 books for every box of books. Model the language of "for every ___, there are ___" if students do not use it. Write the pattern on the whiteboard.

What ratio could you write to describe the relationship? *(1:8)*

Look at problems 3 and 4. Complete Jess' table and write a ratio.

Give students time to work. Ask pairs to share the table entries. As they share, record their responses on the whiteboard or by using the document camera. Ask students to describe how they decided what the entries would be. Notice that the first row does not give the scale for the rest of the table. Students will have to use entries further down the table to discern the pattern.

What method did you use to decide what would go in the blank cells in the table? *(answers will vary—some students may use a numerical approach such as noticing that the entries in the right column are 4 times that of the left column)*

We can also describe the relationship by saying that there are 4 legs for every chair. It describes the same relationship.

We can write this relationship as "1:4." This means that for every 1 chair, there are 4 legs.

Be sure that students write the ratio in their Student Booklet.

Practicing Together

Display the Practicing Together sheet in the Teacher Masters. Have students turn to the Practicing Together sheet in the Student Booklet.

1. Have students work in pairs to complete the sheet.

2. Have student pairs show on the board how they solved the problems. Ask students to describe or explain how they used the table to find the missing values.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

3. Have the students sum their correct answers and mark the total number correct at the top of their page.

4. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers..

Wrapping It Up

Display the Wrapping It Up sheet in the Teacher Masters. Have students turn to the Wrapping It Up sheet in their Student Booklets

> **Write the ratio that shows this relationship: For every spider, there are 8 legs.**

Have students share their answers as time allows.

Ratios and Proportions 2
Lesson 15A

Lesson 15A: Solving Rate Problems

Lesson Objectives	Students solve rate problems in multiple contexts. Students reason abstractly and quantitatively. (SMP 2) Students model with mathematics. (SMP4)	
Vocabulary	**None**	
Requisite Vocabulary	Proportions, unit rate	
Misconception(s)	Students sometimes overgeneralize the "cross-multiplying" strategy to find a missing quantity in a proportion, thinking that they can use the strategy anytime 2 ratios appear together. Students may use the strategy when they see fractions in a computational problem, such as in addition.	
Instructional Materials	**Teacher**	**Student**
	• Teacher Masters • Whiteboard (or equivalent) • Projector (or equivalent)	• Student Booklet

Warming Up

Display the Warming Up sheet in the Teacher Masters. Have students turn to the Warming Up sheet in the Student Booklet.

Determine how much Marc will pay for the notebooks.

Allow students time to complete.

How much will Marc pay for the notebooks? *($4.20)* **How did you solve?** *(accept reasonable answers, such as finding the unit rate by dividing, creating a table, or using a proportion)* **Did anyone solve it differently?**

Learning to Solve

Teacher Notes

As students solve the problems, watch for different strategies for finding unit rates. Typically, students use division to find unit rates.

1. Students will solve rate problems.

Display the Learning to Solve sheet in the Teacher Masters. Have students turn to the Learning to Solve sheet in the Student Booklet.

You have developed strategies for finding missing quantities in ratios or for finding the best buy. There are different ways of finding the best buy and for determining rates or missing quantities in ratios.

We will do the first problem together and then you will complete the remaining problems with a partner.

Select a student to read problem 1.

Follow along as [student] reads.

Allow time for the student to read.

What is the problem asking you to find? *(which is the better buy)*

One strategy is to find the unit rate. How much is each shirt? How can we figure out the unit rate? *(accept reasonable answers, such as dividing)*

Let's try dividing. Divide $21 by 4 to find how much 1 T-shirt costs at that price. What is $21 divided by 4? *($5.25)* **Now, what is $24 divided by 5?** *($4.80)*

Which is the better buy? *(the 5 T-shirts for $24)* **Write that 5 t-shirts for $24 is the better deal because each T-shirt costs less than the 4 for $21 deal. What is the difference in price per T-shirt?** *($.45)* **Include that information in your answer so you can accurately defend our answer.**

Practicing Together

This lesson has no Practicing Together sheet.

Trying It on Your Own

Display the Trying it On Your Own sheet in the Teacher Masters. Have students turn to the Trying It On Your Own sheet in their Student Booklets.

1. Have students work on their own to complete the problems on the sheet.

2. Give the answers to the students and have them mark their answers as correct or incorrect.

2. Have the students sum their correct answers and mark the total number correct at the top of their page.

3. Have the students turn to the Graphing Your Progress section of the Student Booklets and graph their number of correct answers.

Wrapping It Up

Display the concept map created in the first lesson of this module.

Have students review their ideas. Ask students to think about things they would revise, add, or delete and to write these ideas in their Student Booklet.

Discuss as time permits, making changes as students suggest them.

AFTER this lesson: Administer Progress Monitoring
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