Lesson 6: 4s, 8s, and 10s

Lesson Objective

- Students will apply the distributive property of multiplication over addition to visually represent and solve multiplication problems.

Instructional Materials

Material	Quantity	Description
Timer	1	
How Am I Doing? graph	1 per student	
Fact Practice graph	1 per student	
Colored Pencils	1 per student	
Display Masters	1 each	• Preview: Key Idea: 4s, 8s, and 10s • Demonstrate: Partially Completed Multiplication Table • Demonstrate: 10 x 6 Array A • Demonstrate: 10 x 6 Array B • Demonstrate: 10 x 6 Array C • Demonstrate: 10 x 6 Array D • Demonstrate: 10 x 6 Array E • Demonstrate: 4s Facts A • Demonstrate: 4s Facts B • Demonstrate: The 2s Doubled Are the 4s A • Demonstrate: The 2s Doubled Are the 4s B • Demonstrate: 8s Facts A • Demonstrate: 8s Facts B

Instructional Materials (cont.)

Material	Quantity	Description
		• Demonstrate: The 4s Doubled Are the 8s A • Demonstrate: The 4s Doubled Are the 8s B • Demonstrate: The Doubling Strategy
Handouts	1 per student	• Timed Fact Practice 6 • Cumulative Review • Multiplication Table (optional) • 4s Facts • The 2s Doubled Are the 4s • 8s Facts • The 4s Doubled Are the 8s • Practice 1 • Practice 2 • Independent Practice
Answer Keys	1 each	• Timed Fact Practice 6 • Cumulative Review • Practice 1 • Practice 2 • Independent Practice

Timed Fact Practice

Distribute the Timed Fact Practice 6 handout of the chosen set of facts; multiplication, division, or mixed. Remember to use the same set of facts throughout the module.

Say: *When I say, "begin," you will have one minute to complete the 20 multiplication/division/mixed facts. Start with the first one, going across the rows. If you make a mistake, cross out the wrong answer and write the correct answer next to it. When I say, "stop" or the timer goes off, put your pencil down.*

Say: *Ready? Begin.*

After the timer goes off, display the Timed Fact Practice 6 Answer Key and have the students use a colored pencil or marker to check their work and write the number correct on the score line on the Facts Practice Graph.

Then have students graph the number correct. As the lessons proceed, connect the new point with the previous lesson's point.

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of the 2s, 3s, 5s, 7s, 10s, and 11s facts. Students will apply their knowledge of these known facts to determine unknown facts (4s, 8s) by doubling. Students will use the knowledge

taught in this lesson to continue building fact fluency and develop alternate strategies for determining unknown multiplication facts.

Display and introduce through a brief explanation of the key idea for this lesson:

- Unknown facts can be derived from known facts using the doubling strategy.

Use the Key Idea: 4s, 8s, and 10s DM display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge related to the content to be taught in this lesson. Ask students questions such as:

- What does it mean to double a number? (multiply a number by 2, or add the number to itself)
- What are all of the combinations of 2 positive numbers that can be added together to equal 2? (1 + 1) 4? (1 + 3, 2 + 2) 8? (1 + 7, 2 + 6, 3 + 5, 4 + 4) 10? (1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5)
- What property is illustrated by the equation 1 x 5 = 5 x 1? (commutative property of multiplication)
- What is the missing factor for the mathematical equations shown below?
- 2 x _?_ = 4 (2) 2 x _?_ = 8 (4) 2 x _?_ = 10? (5)
- What are the factors of 15? (1, 3, 5, 15) 20? (1, 2, 4, 5, 10, 20) 24? (1, 2, 3, 4, 6, 8, 12, 24)

Display the numbers 1–20.

- Which of the numbers are even? (2, 4, 6, 8, 10, 12, 14, 16, 18) Which are odd? (1, 3, 5, 7, 9, 11, 13, 15, 17, 19)

If students cannot correctly answer these questions, stop and explicitly teach the material.

Demonstrate

1. Identify the facts that students have not yet been taught explicitly.

Display the Partially Completed Multiplication Table DM display master with the 1s, 2s, 3s, 5s, 7s, 9s, 10s, and 11s completed.

Say: *We have part of a 12 by 12 multiplication table completed. Which facts are still missing? (4s, 6s, 8s, and 12s) Today, we are going to fill in the 4s and 8s using the doubling strategy.*

2. Visually model the doubling strategy to determine the 10s facts, which are already completed. 1

Say: *Doubling a known fact to find an unknown fact is a strategy we can use to find even facts. I can use doubling for the 4s, 6s, 8s, 10s, and 12s. The doubling strategy is similar to the near facts strategy because we break apart one factor into two factors that we know automatically. The doubling strategy is different from the near facts strategy because the two factors are the same. For example, 10 is taken apart into 5 + 5.* 2

Display a 10 x 6 array. Use the 10 x 6 Array A DM display master as needed.

Say: *What multiplication problem is represented by the array? (10 x 6)*

Say: *I can divide the array into two equal parts. "Equal parts" means that the two factors are the same.*

1 **TEACHER NOTE**
Since the doubling strategy is similar to the strategy for finding the unknown facts for the 3s and 7s, students may benefit from relating the two strategies and identifying similarities and differences.

2 **TEACHER NOTE**
You may want to draw attention to the fact that you know many of the students already know the 10s facts automatically or from previous lessons. However, you will be using the 10s facts as an example to illustrate the doubling strategy.

TEACHER NOTE
Some students may be confused when relating the array model to the partial product expressions. If so, use colored markers to identify the relationship between each divided array section, its partial product, and the resulting product in the addition expression.

Relate partitioning the array to the strategy used for 3s and 7s. Highlight similarities (partitioning an array into two parts) and differences (equal or unequal parts).

Display a 10 x 6 array with a line drawn to divide the array into two 5 x 6 arrays. Use the 10 x 6 Array B DM display master as needed.

Say: *What multiplication problem is represented by each array? (5 x 6)*

Display (5 x 6) next to each array. Use the 10 x 6 Array C DM display master as needed.

Draw attention to which factor changed (10) and which factor did not change (6).

Say: *When you break the 10s apart into 5s, you can think of the 10s facts as double the 5s facts. I can think of 10 x 6 as 5 x 6, doubled. I can write an expression to represent this.*

Display (5 x 6) + (5 x 6). Use the 10 x 6 Array D DM display master as needed.

Say: *5 x 6 = 30 for each part of the expression. 30 doubled, or 30 + 30, equals 60. So, 10 x 6 equals 60. Use the 10 x 6 Array E display master as needed.*

3. Identify the facts for which the doubling strategy can be used.

Draw attention to the top row or left column of the partially completed multiplication table DM display master.

Say: *10s can be found by doubling 5s. What other facts can*

be found by doubling another multiple? (2s, 4s, 6s, 8s, 12s) 4

Say: *4s can be found by doubling the 2s. 6s can be found by doubling the 3s. 8s can be found by doubling the 4s. 12s can be found by doubling the 6s. I can use the doubling strategy to find the 4s, 6s, 8s, 12s. In this lesson we will concentrate on the 4s and 8s. In a following lesson we will learn how to apply the doubling strategy to the 6s and 12s.* 5

4. Visually model the doubling strategy for the 4s.

Repeat step 2 to visually model doubling the 2s facts to find the 4s facts. Emphasize examples that are not easy facts or for which an alternative strategy has not yet been taught. These facts are 4 x 4, 4 x 6, 4 x 8, and 4 x 12.

Distribute a copy of the 4s Facts handout to each student. Use the 4s Facts A DM display master as needed to show the arrays.

As you work through each example, use the language described in step 2. Be sure to emphasize that you are dividing the array into two equal parts. We take apart the factor 4 into 2 parts. Connect this change to the strategy in which the 2s are doubled to determine the 4s.

Use the 4s Facts B DM display master as needed to show the arrays partitioned with the calculations shown.

5. Calculate the 4s facts by doubling the 2s facts.

4 **TEACHER NOTE**
Some students may observe that the facts for which the doubling strategy can be used are even. An even number is evenly divisible by 2. This occurs because doubling can be written as adding the same number two times (n + n) or multiplying the number by 2 (2n).

5 **TEACHER NOTE**
Many students already know the 2s and 10s facts automatically. Some students may choose to find the 2s and 10s by doubling the 1s and 5s respectively. Although this strategy is applicable for the 2s, it is not explicitly taught in this lesson.

TEACHER NOTE
For more advanced students, you may want to use the Multiplication Table handout instead of The 2s Doubled Are the 4s handout, which is an adapted version of the multiplication table with additional scaffolding. Display 2 + 2 above the 4s column and to the left of the 10s row before performing the think aloud. If students use the Multiplication Table handout, they should continue to use it when calculating the 8s.

Distribute The 2s Doubled Are the 4s handout.

Have students complete the 2s column and row and any 4s facts they automatically know.

Display The 2s Doubled Are the 4s A DM display master with the 2s facts completed.

Say: *What 4s facts do you automatically know?*

Display the facts that the majority of students automatically know.

Acknowledge that students automatically know some of the 4s facts and encourage them to use the doubling strategy only to find facts they do not know.

Say: *I want to complete the 4s facts. To find the 4s facts that I do not automatically know, I can double the 2s facts.*

Think aloud as you use the doubles strategy to complete the 4s column for the remaining facts using the language described in step 3. As you complete each step, have students do the same. Use The 2s Doubled Are the 4s B DM display master as needed.

TEACHER NOTE
Students may choose to use the doubles strategy for more facts than are explicitly taught or know more 4s facts automatically and not need to use the strategy. As needed, explain to students that either is acceptable and the goal is for all students to be able to automatically answer a fact.

Remind students that by the commutative property of multiplication, if they know a fact in the 4s column, they also know it in the 4s row. For example, if they know 4 x 3, they also know 3 x 4. The products are equal. 3 x 4 equals 12.

6. Repeat the process in steps 2 and 3 to double the 4s to find the 8s.

Repeat step 2 to visually model doubling the 4s facts to find the 8s facts. Emphasize examples that are not easy facts or for which an alternative strategy has not yet been taught. These facts are 8 x 6, 8

x 8, and 8 x 12.

Distribute the 8s Facts handout. Use the 8s Facts A and B DM display masters as needed to show the arrays.

Repeat step 3 to calculate the 8s facts by applying the doubling strategy.

Distribute The 4s Doubled Are the 8s handout.

Have students complete the 4s column and any 8s facts they automatically know.

Use The 4s Doubled Are the 8s A and B DM display masters as needed.

7. Connect the facts learned in this lesson using the doubling strategy to the multiplication table.

Display a multiplication table. Use the Partially Completed Multiplication Table DM display master as needed.

Draw attention to the facts that were completed in this lesson as the multiplication table filled up from facts that we already know.

Display the doubles above and to the left of each row for which the strategy was used. Use the Doubling Strategy DM display master as needed. Have students do the same on their multiplication table.

TEACHER NOTE
Students may choose to use the doubles strategy to first find the 4s.

Practice

WATCH FOR

Some students believe that performing a strategy, such as doubling, changes the total number in the array. Teach students that the doubling strategy changes how the arrays look, but not the total number. Use visualizations and manipulatives as needed.

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice 1 handout. Select a few students to verbalize their reasoning and each step in the process.

Activity 2: Have students work in pairs or small groups to complete the activity on the Practice 2 handout. Have students verbalize their reasoning and each step in the process to their partners.

Circulate to monitor student progress. Randomly stop, draw attention to a completed cell and ask: How did you get this answer? If you know this fact, what other fact do you know? Why?

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.

2. Go over the answers (students self-check and correct using a colored pencil).

3. Have students record the number correct in the box and complete their How Am I Doing? graph.

4. Collect the papers to review and monitor student progress.

Closure

Review the key idea. Have students provide examples from the lesson.

Have students discuss their answer to the following questions.

- What is a strategy you can use to find the 4s, and 8s facts that you do not automatically know?
- What is the doubling strategy?
- Why does the strategy work?

Clear up any misconceptions. Students who believe performing a strategy changes the total number in the array need additional instruction.