

Lesson 8: 6s and 12s

Lesson Objective

- Students will visually model the doubling strategy and apply it to determine the unknown 6s and 12s facts.

Instructional Materials

Material	Quantity	Description
Timer	1	
How Am I Doing? graph	1 per student	
Fact Practice graph	1 per student	
Colored Pencils	1 per student	
Display Masters	1 each	<ul style="list-style-type: none">• Preview: Key Idea: 6s and 12s• Demonstrate: Partially Completed Multiplication Table• Demonstrate: 6 x 6 Array A• Demonstrate: 6 x 6 Array B• Demonstrate: 6 x 6 Array C• Demonstrate: 6 x 6 Array D• Demonstrate: 6 x 6 Array E• Demonstrate: 6 x 6 Array F• Demonstrate: 6 x 12 Array A• Demonstrate: 6 x 12 Array B• Demonstrate: 6s Multiplication Table• Demonstrate: 12 x 12 Array A• Demonstrate: 12 x 12 Array B• Demonstrate: 12 x 12 Array C

Instructional Materials (cont.)

Material	Quantity	Description
		<ul style="list-style-type: none"> • Demonstrate: 12 x 12 Array D • Demonstrate: 12 x 12 Array E • Demonstrate: 12 x 12 Array F • Demonstrate: 12s Multiplication Table • Demonstrate: Facts Strategies
Handouts	1 per student	<ul style="list-style-type: none"> • Timed Fact Practice 8 • Cumulative Review • 6 x 12 Array • Partially Completed Multiplication Table • The 3s Doubled Are the 6s (optional) • The 6s Doubled Are the 12s (optional) • Practice 1 • Practice 2 • Independent Practice
Answer Keys	1 each	<ul style="list-style-type: none"> • Timed Fact Practice 8 • Cumulative Review • Practice 1 • Practice 2 • Independent Practice

Timed Fact Practice

Distribute the Timed Fact Practice 8 handout of the chosen set of facts; multiplication, division, or mixed. Remember to use the same set of facts throughout the module.

Say: *When I say, “begin,” you will have one minute to complete the 20 multiplication/division/mixed facts. Start with the first one, going across the rows. If you make a mistake, cross out the wrong answer and write the correct answer next to it. When I say, “stop” or the timer goes off, put your pencil down.*

Say: *Ready? Begin.*

After the timer goes off, display the Timed Fact Practice 8 Answer Key and have the students use a colored pencil or marker to check their work and write the number correct on the score line on the Facts Practice Graph.

Then have students graph the number correct. As the lessons proceed, connect the new point with the previous lesson's point.

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of the 1s through 5s and 7s through 11s facts. Students will apply their knowledge of these known facts to determine unknown facts (6s and 12s) by doubling. Students will use the knowledge taught in this lesson to continue building fact fluency and develop alternate strategies for determining unknown multiplication facts.

Display and introduce through a brief explanation of the key idea for this lesson:

- Unknown facts can be derived from known facts using the doubling strategy.

Use the Key Idea: 6s and 12s  display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge related to the content to be taught in this lesson. Ask students questions such as:

- What does it mean to double a number? (multiply a number by 2, or add the

number to itself)


- If you do not know 4×4 , how could you use facts you know to solve the problem?
- If you know the fact 6×12 , what other fact do you know? (12×6)
- What is the missing factor for the mathematical equations shown below?

$$3 \times _? = 12 \text{ (4)} \quad 3 \times _? = 9 \text{ (3)} \quad 3 \times _? = 24 \text{ (8)}$$

If students cannot correctly answer these questions, stop and explicitly teach the material.

Demonstrate

1. Identify the facts that students have not yet been taught explicitly.

Display the Partially Completed Multiplication Table  display master with the 1s, 2s, 3s, 4s, 5s, 7s, 8s, 9s, 10s, and 11s completed.

Say: *We have part of a 12 by 12 multiplication table completed. Which facts are still missing? (6s and 12s) Today, we are going to complete the table by filling in the 6s and 12s using the doubling strategy for the facts we do not already know automatically. This is the same strategy we used to find the 4s and 8s.*




TEACHER NOTE

Since the doubling strategy is similar to the strategy used for the near facts to find the 3s, 7s, 9s, and 11s, students may benefit from relating the two strategies and identifying similarities and differences. Be sure to relate this lesson to the previous lesson on the doubling strategy to find the 4s, 8s, and 10s.

2. Visually model the doubling strategy to determine the 6s facts. 


Say: *In the previous lesson, we doubled a known fact to find an unknown fact. I can use this strategy to find even facts. In this lesson, I am going to apply it to find the 6s and 12s. Doubling is similar to the near facts strategy because I take apart one factor into two factors that I*

know automatically. The doubling strategy is different from the near facts strategy because in the doubling strategy, the two factors are the same. For example, 10 can be taken apart into $5 + 5$. How can 4 be taken apart? ($2 + 2$) How can 8 be taken apart? ($4 + 4$)

Display a 6 x 6 array. Use the 6 x 6 Array A  display master as needed.

Say: *What multiplication problem is represented by the array? (6×6)*

Say: *I can divide the array into two equal parts. The equal parts mean that the two factors are the same.*


Relate partitioning the array to the strategy for 3s and 7s and the previous lesson on the doubling strategy. Highlight similarities (partitioning an array into two parts) and differences (equal or unequal parts). 

Display a 6 x 6 array with a line drawn to divide the array into two 3 x 6 arrays. Use the 6 x 6 Array B  display master as needed.

Say: *What multiplication problem is represented by each array? (3×6)*

Display (3×6) next to each array. Use the 6 x 6 Array C  display master as needed.

Say: *When you take the 6 apart into 3s, you can think of the 6s facts as double the 3s facts. I can think of 6×6 as 3×6 , doubled.*


Display $(3 \times 6) + (3 \times 6)$. Use the 6 x 6 Array D  display master as needed.




TEACHER NOTE

Some students may be confused when relating the array model to the partial product expressions. If so, use colored markers to identify the relationship between each divided array section, its partial product, and the resulting product in the addition expression.


Say: $3 \times 6 = 18$ for each part of the expression. 18 doubled, or $18 + 18$, equals 36. So, 6×6 equals 36.

Display $18 + 18 = 36$ under the previous expression. Use the 6 x 6 Array E  display master as needed.


Say: *I could have divided the array differently to solve the fact.*

Divide the array horizontally into two 6 x 3 arrays. Use the 6 x 6 Array F  display master.

Say: *What multiplication fact is represented by each array? (6×3)*

Say: *If I know 3×6 , I know what 6×3 is because of the commutative property of multiplication. I could have also divided the array this way to find 6×6 is double 6×3 , which is 18, doubled. $6 \times 6 = 36$.* 

Repeat with additional examples of the 6s facts as needed.

Emphasize facts that have not been explicitly taught (6×12) and facts that students find difficult. Use the 6 x 12 Array A and B  display masters and 6 x 12 Array handout as needed.

Say: *What is the doubling strategy? (doubling a known fact to find an unknown fact for even facts)*

3. Calculate the 6s facts by doubling the 3s facts. 

Distribute one copy of Partially Completed Multiplication Table handout to students. 

Have students complete as many of the 6s facts as they know and share their answers. Remind students to complete both the row and

**TEACHER NOTE**

As needed, spend additional time walking through the example in which the commutative property of multiplication is applied to solve the fact 6×6 .


**TEACHER NOTE**

This lesson applies the same strategy—the doubling strategy taught in the previous lesson—to new facts. Since students have had experience with the strategy, the scaffolding provided by the adapted multiplication table is removed. Relate the previous lesson's handouts to the multiplication table as needed.

**TEACHER NOTE**

For students who continue to need additional scaffolding, use **The 3s Doubled Are the 6s** handout and subsequently **The 6s Doubled Are the 12s** handout.

column.

Display the Partially Completed Multiplication Table  display master with the 3s highlighted.

Display $3 + 3$ above the 6s column and to the left of the 6s row.


Say: *I can double the 3s to find the 6s. $3 + 3$ is double 3. When using a multiplication table, writing this strategy helps me remember it.*

Think aloud as you use the doubles strategy to complete the 6s column. Draw attention to where on the multiplication table the fact $3 \times n$ is found.

Say: *6×1 is double 3×1 . 3×1 equals 3. 3 doubled is equal to 6×1 . $3 + 3 = 6$ so $6 \times 1 = 6$. Many of you already know this fact. If you know this fact, you do not need to use the strategy. The strategy is to help you with the facts you do not know automatically.*

Display 6 in the first row for 6s column.

Say: *If I know 6×1 , what other fact do I automatically know? (1×6).*

Display 6 in the cell for 1×6 . Use the 6s Multiplication Table  display master as needed.

Repeat for each fact by selecting students to help complete the 6s column. Have students complete their table as you complete yours. Emphasize the use of this strategy for the facts that have not been explicitly taught (6×6 and 6×12) and those with which students struggle.

Encourage students to use scratch paper to double the 3s as needed.

Say: *Are all of the 6s facts double the 3s facts? (yes) Double means to add the same number twice. Double 3 is equal to 3 plus 3.*

Say: *What is the doubling strategy? (doubling a known fact to find an unknown fact for even facts)*



TEACHER NOTE

Some students may choose to use the distributive property of multiplication over addition, in which 6 is taken apart into $5 + 1$ or $4 + 2$. Some students may choose to triple the 2s, adding the $2 \times n$ fact three times to find the 6s. These strategies are applicable and may help students find the 6s more quickly; however, they are not explicitly taught in this lesson.

4. Visually model the doubling strategy for the 12s.

Repeat step 2 to visually model doubling the 6s facts to find the 12s facts. Emphasize examples that have not yet been explicitly taught, such as 12×12 and facts with which students struggle.

Use the 12 x 12 Array A-F  display masters as needed to show the arrays.

Work through the example using the language described in step 2. Be sure to emphasize that you are dividing the array into two equal parts. We take apart the factor 12 into 2 parts.

Repeat with additional examples of 12s facts that students find difficult.



TEACHER NOTE

For students who continue to need additional scaffolding, use The 6s Doubled Are the 12s handout.

5. Calculate the 12s facts by doubling the 6s facts. 

Distribute one copy of the Partially Completed Multiplication Table handout to students.

Have students complete as many of the 12s facts as they know on their Partially Completed Multiplication Table. Remind students to complete both the row and column.


Display $6 + 6$ above the 12s column and to the left of the 12s row.

Repeat step 3 to calculate the 12s facts by doubling the 6s facts. Use the same language, adapted for the example. Use the 12s Multiplication Table  display master as needed. 

6. Connect the facts learned in this lesson using the doubling strategy to the multiplication table.

Draw attention to the facts that were completed in this lesson.

Say: *I now know a strategy to solve the 3s, 4s, 6s, 7s, 8s, 9s, 11s, and 12s facts that I do not know automatically if I already know the 1s, 2s, 5s, and 10s.*

Display the strategies above the column and to the left of each row for which the strategy was used. Use the Facts Strategies  display master as needed. Have students do the same on their multiplication table.



TEACHER NOTE

Some students may choose to use the distributive property of multiplication over addition, in which 12 is taken apart differently. For example, 12 can be taken apart into $11 + 1$ or $10 + 2$. Some students may choose to multiply the 2s facts by 6, the 3s by 4, or the 4s by 3. These strategies are applicable and may help students find the 12s more quickly; however, they are not explicitly taught in this lesson.

Practice

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice 1 handout. Select a few students to verbalize their reasoning and each step in the process.

Activity 2: Have students work in pairs or small groups to complete the activity on the Practice 2 handout. Have students verbalize their reasoning and each step in the process to their partners.




WATCH FOR

Some students believe that performing a strategy, such as doubling, changes the total number in the array. Teach students that the doubling strategy changes how the arrays look but not the total number. Use visualizations and manipulatives as needed.

Circulate to monitor student progress. Randomly stop, draw attention to a completed cell, and ask: How did you get this answer? If you know this fact, what other fact do you know? Why?

Example: Draw attention to the cell containing 6×12 on the Practice 1 handout. Listen for a student to explain that doubling 3×12 was used to solve 6×12 because the 6s facts can be found by doubling the 3s facts. Listen for students to explain that if 6×12 is known, 12×6 is also known because of the commutative property of multiplication, and watch for students to complete the row and column.

Display a completed multiplication table. Draw attention to the facts that were completed in this lesson as the multiplication table filled up from facts that we already know.

Display the doubles above the column and to the left of each row for which the strategy was used. Use the Facts Strategies  display master as needed. Have students do the same on their multiplication table.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.
2. Go over the answers (students self-check and correct using a colored pencil).
3. Have students record the number correct in the box and complete their How Am I Doing? graph.
4. Collect the papers to review and monitor student progress.

Closure

Review the key idea. Have students provide examples from the lesson.

Have students discuss their answers to the following questions.

- What is a strategy you can use to find the 6s and 12s facts that you do not automatically know?
- For what other facts can this strategy be used?

Clear up any misconceptions. Students who believe performing a strategy changes the total number in the array need additional instruction.