MSTAR INTERVENTION

L10

Lesson 10: Determine Proportionality by Using Cross Products

Lesson Objective

- Students will determine whether 2 ratios are proportional by using cross products.

Instructional Materials

Material	Quantity	Description
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Index cards with 2 ratios on each	1 per student	
Whiteboard with dry-erase marker	1 per student	
Display Masters	1 each	• Preview: Key Ideas: Determine Proportionality by Using Cross Products • Demonstrate: Salads A–M • Demonstrate: Jellybeans A–E • Demonstrate: Cross Products A–D
Handouts	1 per student	• Cumulative Review • Practice • Independent Practice
Answer Keys	1 each	• Cumulative Review • Practice • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of using common denominators to prove proportionality.

Display and introduce through a brief explanation the key ideas for this lesson:

- Patterns found when using common denominators to prove proportionality lead to the idea of cross products.
- Cross products can be used to prove proportionality. If the 2 numerators are equal, the ratios are proportional. If the 2 numerators are not equal, the ratios are not proportional.

Use the Key Ideas: Determine Proportionality by Using Cross Products DM display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, activate students' background knowledge and preskills by leading activities such as the following.

Before class, create a set of index cards with 2 ratios on each. Have each student choose 1 index card from the stack, decide whether the ratios are proportional, and show his or her reasoning on a whiteboard. Then have students discuss their answers in pairs. Choose 2–3 students to share with the whole group. Ensure that students use the correct mathematical language in their answers and explanations. The following are examples of ratio pairs for the index cards:

- $\frac{2}{6}$ and $\frac{3}{9}$
- $\frac{2}{8}$ and $\frac{4}{12}$

If students cannot complete this activity, stop and explicitly teach the material.

Demonstrate

1. Connect the strategy of using common denominators to the strategy of using cross products to determine proportionality. Ensure that students use the correct mathematical language in their answers and explanations.

Say: *In the previous lesson, we found patterns when we used common denominators to find missing values in a proportion. By identifying these patterns, we developed the idea of cross products. Today, we will use cross products to determine whether 2 ratios are proportional.*

Say: *But first, we need to review our process of using common denominators to prove proportionality.*

Say: *Consider the scenario: The cafeteria serves 2 sizes of salads, large and small. The large salad has 3 pieces of cucumber for every 9 pieces of broccoli. The small salad has 2 pieces of cucumber for every 6 pieces of broccoli. Are the 2 ratios of cucumbers to broccoli proportional for the 2 salads?*

Say: *The ratios* $\frac{3 \text{ cucumbers}}{9 \text{ broccoli}}$ *and* $\frac{2 \text{ cucumbers}}{6 \text{ broccoli}}$ *represent the scenario. To determine whether these 2 ratios are proportional, we must find whether they are equivalent.*

Say: *First, we must find a common multiple of 9 and 6, which are the denominators of the 2 ratios. An obvious common multiple is the product of 9 and 6, which is 54. We can use 54 as our common denominator.*

Use the Salads A DM display master as needed.

Say: *Now that we have a common denominator, we must create a ratio equivalent to* $\frac{3}{9}$ *with a denominator of 54 and a ratio equivalent to* $\frac{2}{6}$ *with a denominator of 54.*

Use the Salads B DM display master as needed.

Say: *Let's begin with $\frac{3}{9}$. I need to create a proportion with a missing value. I can then use the strategy of multiplying by a scale factor to find the numerator of the fraction representing the second ratio.*

Say: *Because I know the 2 ratios must be proportional, I look for the complete ratio that is equivalent to $\frac{3}{9}$ and that has a denominator of 54. I am given both denominators: 9 and 54. So, I think to myself, "9 times what number equals 54?"*

Use the Salads C **DM** display master as needed.

Say: *I know that 9 times 6 is equal to 54.*

Use the Salads D **DM** display master as needed.

Say: *To keep the ratios proportional, we must multiply the fraction representing the ratio by a scale factor. Therefore, if I multiply the denominator by the scale factor 6, I must multiply the numerator by 6 as well.*

Use the Salads E **DM** display master as needed.

Say: *When I multiply the numerator 3 by 6, I get 18 as the numerator for the equivalent ratio. Thus, x = 18, giving us a complete proportion of $\frac{3}{9} = \frac{18}{54}$.*

Use the Salads F **DM** display master as needed.

Use the same language to find the equivalent fraction for $\frac{2}{6}$. Use the Salads G–K **DM** display masters as needed.

Say: *When we find a common denominator, we need to compare the new numerators of the ratios to see whether they are equal. For this example, we found that the new numerators, 18, are equal; thus, the ratios are proportional because they are equivalent.*

Use the Salads L **DM** display master as needed.

Say: *The same pattern found in our previous lesson is present when determining proportionality. I can use cross products to determine whether the 2 ratios are proportional. When applied to this example, if I multiply 6 and 3, I get 18. When I multiply 9 and 2, I get 18. Therefore, the 2 ratios are proportional.*

Use the Salads M DM display master as needed.

2. Determine whether 2 ratios are proportional by using cross products. Ensure that students use the correct mathematical language in their answers and explanations.

Say: *Rob's bag has 3 pink jellybeans for every 4 orange jellybeans. Mark's bag has 4 pink jellybeans for every 6 orange jellybeans. Are the ratios of pink to orange jellybeans in each bag proportional?*

Say: *Consider the ratios* $\frac{3 \text{ pink}}{4 \text{ orange}}$ *and* $\frac{4 \text{ pink}}{6 \text{ orange}}$ *. We will determine whether these ratios are proportional by using cross products.*

Use the Jellybeans A DM display master as needed.

Say: *We will begin with the denominator 4. We cross-multiply the 4 with numerator 4 of the other ratio to get 16.*

Use the Jellybeans B DM display master as needed.

Say: *Next, we cross-multiply the denominator 6 with the numerator 3 of the other ratio to get 18.*

Use the Jellybeans C DM display master as needed.

Say: *If I had used the common denominator strategy, I would have 24 as the common denominator. 16 and 18 would*

TEACHER NOTE

Clarify with students that the use of cross products is derived from using common denominators. Also, mention that cross products may not always be the most efficient method but that you will explore this idea further in an upcoming lesson.

be the numerators, and the ratios would not be equivalent.

Use the Jellybeans D DM display master as needed.

Say: *Using the cross products strategy, because the new numerators are not equal, the 2 ratios are not proportional.*

Use the Jellybeans E DM display master as needed.

Say: *Let's try 1 more example without a scenario. Suppose I was given the ratios* $\frac{2}{6}$ *and* $\frac{5}{15}$ *. Let's use cross products to determine whether these ratios are proportional.*

Use the Cross Products A DM display master as needed.

Say: *We will begin with the denominator 6. We cross-multiply the 6 with the numerator 5 of the other ratio to get 30.*

Use the Cross Products B DM display master as needed.

Say: *Next, we cross-multiply the denominator 15 with the numerator 2 of the other ratio to get 30.*

Use the Cross Products C DM display master as needed.

Say: *If I were to use 90, the product of the 2 denominators, as my common denominator, I would get 30 as the numerator for both ratios. Because they are equal, the ratios are proportional*

Use the Cross Products D DM display master as needed.

Practice

For the practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to

correct their errors. Collect student work to review and monitor student progress.

Activity: Have students complete the activity on the Practice handout. After every 2 problems, have students check their answers with a partner and discuss reasoning. Select a few students to verbalize their reasoning. Ensure that students use the correct mathematical language in their explanations.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.

2. Go over the answers (students self-check and correct, using a colored pencil).

3. Have students record the number correct in the box and complete their How Am I Doing? graph.

4. Collect the papers to review and monitor student progress.

Closure

Review the key ideas. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- When would you use cross products to determine whether ratios are proportional? When would you use a different strategy? Give an example of each.
- When using cross products, how do you know whether 2 ratios are proportional?
- Why does the cross product strategy work?

Clear up any misconceptions. Students who struggle with determining proportionality by using cross products need additional instruction.