

Lesson 12: Determine the Most Efficient Strategy for Finding a Missing Value

Lesson Objective

- Students will determine the most efficient strategy for finding the missing value in a proportion.

Instructional Materials

Material	Quantity	Description
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Popsicle sticks with 1 student name on each	1 per student for teacher use	
Display Masters	1 each	<ul style="list-style-type: none"> • Preview: Key Idea: Determine the Most Efficient Strategy • Demonstrate: Cars and Trucks A-E • Demonstrate: Pizzas A-F • Demonstrate: Magazines A-E
Handouts	1 per student	<ul style="list-style-type: none"> • Cumulative Review • Practice • Independent Practice
Answer Keys	1 each	<ul style="list-style-type: none"> • Cumulative Review • Practice • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of finding a missing value in a proportion by multiplying the fraction representing a ratio by a scale factor.

Display and introduce through a brief explanation the key idea for this lesson:

- 1 method may be more efficient than the others when finding the missing value in a proportion.

Use the Key Idea: Determine the Most Efficient Strategy  display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, activate students' background knowledge and preskills by leading activities such as the following:

- Have students work in pairs to create a list to answer the following question: What methods have we learned to find a missing value in a proportion? Solicit answers from the pairs, asking each pair to answer at least once. Ensure that students use the correct mathematical language in their answers and explanations. Create a running list of student responses.
- Have students work in pairs to answer the following questions. Choose 2–3 students to share their answers and reasoning with the whole group. Have the other students give a thumbs-up or a thumbs-down to show whether they agree with each shared answer. Ensure that students use the correct mathematical language in their answers and explanations.
 - ◇ What is the missing value in the proportion $\frac{1}{3} = \frac{x}{27}$?
 - ◇ What is the missing value in the proportion $\frac{2}{8} = \frac{x}{28}$?

If students cannot complete these activities, stop and explicitly teach the material.

Demonstrate

1. Determine what strategy for finding a missing value in a proportion is most efficient.

Say: *In the previous lessons, we have explored various ways to prove proportionality and to find missing values in proportions. Today, we will look for clues that will help us choose the most efficient strategy for finding missing values in proportions. Although other methods are sometimes valuable, we will mainly focus on 3 strategies: multiplying by a scale factor, using a unit rate, and using cross products.*

Say: *Suppose I am given the following scenario: Jacob has 4 toy cars for every 8 toy trucks. If he continues at this rate, how many toy cars will he have when he has 32 toy trucks?*

Say: *The proportion $\frac{4 \text{ cars}}{8 \text{ trucks}} = \frac{x \text{ cars}}{32 \text{ trucks}}$ represents the scenario.*

Use the Cars and Trucks A  display master as needed.

Say: *I need to find the missing value in the proportion. You might think that we should always use cross products, but sometimes, using cross products is not the most efficient method. To figure out the most efficient method, I should ask myself 2 questions before resorting to cross products:*

- ◇ *Can I easily multiply the fraction representing the complete ratio by a scale factor to find the missing value?*
- ◇ *If not, can I simplify the fraction representing the complete ratio to the unit rate and then easily multiply by a scale factor to find the missing value?*

Say: *If the answer to both questions is “No,” I can always use cross products.*

Say: *When solving a proportion for a missing value, I consider the given*

parts of the proportion. So, I ask myself, "Can I easily multiply the fraction representing the complete ratio by a scale factor to find the missing value?" In this case, I know both denominators in the ratios, 8 and 32. I know that 8 is a factor of 32. So, I can easily multiply 8 by a scale factor to get 32.

Use the Cars and Trucks B  display master as needed.

Say: *What can I multiply 8 by to get 32? (4)*

Use the Cars and Trucks C  display master as needed.

Say: *When multiplying by a scale factor, if I multiply the denominator by 4, I also need to multiply the numerator by 4, so the relationship between the ratios remains proportional.*

Use the Cars and Trucks D  display master as needed.

Say: *When I multiply the numerator 4 by 4, I get a new numerator of 16. Therefore, $x = 16$, meaning that when Jacob has 32 toy trucks, he will have 16 toy cars.*

Use the Cars and Trucks E  display master as needed.

Say: *In this case, I checked first to see whether I could multiply the fraction representing the complete ratio by a scale factor to solve the proportion. I could multiply 8 by 4 to get 32, so I decided to use the scale factor method.*

Say: *Now, listen to this scenario. For a meeting that 27 people attended, Cindy bought 9 pizzas to serve everyone. For the next meeting, Cindy counted the people who would attend, but she lost the list. However, she remembers that she needs to buy only 6 pizzas. Using the same rate, how many people will attend the meeting if Cindy needs to buy only 6 pizzas?*

Use the Pizzas A  display master as needed.

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: I need to find the missing value in the proportion. I know both denominators in the ratios, 9 and 6. First, I ask myself, "Can I easily multiply the fraction representing the complete ratio by a scale factor to find the missing value?" Is there an easy scale factor that I can multiply 9 by to get 6? (no)

Say: Because there is no easy scale factor, I ask myself, "Is 1 of these numbers in the fraction representing the complete ratio the greatest common factor of both numbers?" (yes) This means I can simplify the fraction representing the complete ratio to the unit rate and then easily multiply by a scale factor to find the missing value.

Say: In this situation, we need to simplify the first ratio before finding the scale factor. Consider the ratio, $\frac{27 \text{ people}}{9 \text{ pizzas}}$. We need to find the unit rate.

Say: To simplify a ratio, we need to find the greatest common factor of the numerator and denominator of the fraction representing the ratio. What is the greatest common factor of 27 and 9? (9)

Use the Pizzas B  display master as needed.

Say: To find the unit rate of the ratio, we need to divide the numerator and the denominator of the fraction representing the ratio by the greatest common factor, 9. What do we get when we divide 27 by 9? (3) What do we get when we divide 9 by 9? (1) The new ratio is $\frac{3 \text{ people}}{1 \text{ pizza}}$. Because this ratio represents a number of units for 1 quantity per 1 unit of the other quantity, it is a unit rate.

Use the Pizzas C  display master as needed.

Say: Now that we have the unit rate, we can use it in place of the first ratio to find the missing value in the second ratio. So, we set up the proportion $\frac{3 \text{ people}}{1 \text{ pizza}} = \frac{x \text{ people}}{6 \text{ pizzas}}$. Because we know both of the

denominators of the fractions representing the ratios, we can determine a scale factor to find the missing value. What would we multiply 1 by to get 6? (6)

Say: Therefore, 6 is the scale factor.

Use the Pizzas D  display master as needed.

Say: Now that we have found the scale factor, we need to multiply the numerator of the fraction representing the first ratio, the unit rate, by the scale factor to get the missing value. So, when we multiply 3 by 6, we get what? (18)

Use the Pizzas E  display master as needed.

Say: The missing value of the proportion is 18 people. Therefore, the proportion $\frac{3 \text{ people}}{1 \text{ pizza}} = \frac{18 \text{ people}}{6 \text{ pizzas}}$ is true, as well as the proportion $\frac{27 \text{ people}}{9 \text{ pizzas}} = \frac{18 \text{ people}}{6 \text{ pizzas}}$. We know that 18 people will attend the meeting.

Use the Pizzas F  display master as needed.

3. Find the missing value in a proportion.

Say: Suppose I was given the following scenario: Laura bought 3 magazines last week for a total of \$5.25. This week, she spent \$12.25 on magazines. How many magazines did she buy this week?

Say: The proportion $\frac{3 \text{ magazines}}{\$5.25} = \frac{x \text{ magazines}}{\$12.25}$ represents the scenario.

Use the Magazines A  display master as needed.

Say: I need to find the missing value in the proportion. First, I ask myself, "Can I easily multiply the fraction representing the complete ratio by a scale factor to find the missing value?" Is there a scale factor that I can easily multiply 5.25 by to get 12.25? (no)

Say: *Because there is no easy scale factor, I ask myself, "Is 1 of the numbers in the fraction representing the complete ratio the greatest common factor of both numbers?" (no) So, it would be difficult to simplify the fraction representing the complete ratio to the unit rate and then easily multiply by a scale factor to find the missing value.*

Say: *Because neither strategy would be easy, the best strategy for this scenario is using cross products.* 

Say: *Let's begin with the denominator 5.25. We multiply the denominator 5.25 by the numerator x of the other ratio to get $5.25x$.*

**TEACHER NOTE**

To complete the next step, provide students with a calculator if necessary. It is more important for students to understand the process than to do the computation without a calculator.

Use the Magazines B  display master as needed.

Say: *Next, we multiply the denominator 12.25 by the numerator 3 of the other ratio to get 36.75.*

Use the Magazines C  display master as needed.

Say: *If we were to use the common denominator method, we would get the numerators 36.75 and $5.25x$. The numerators have to be equal; therefore, we set the numerators 36.75 and $5.25x$ equal to each other and solve for x . When I divide both sides by 5.25, I get $x = 7$.*

Use the Magazines D  display master as needed.

Say: *Therefore, $x = 7$ magazines, which creates the proportion $\frac{3 \text{ magazines}}{\$5.25} = \frac{7 \text{ magazines}}{\$12.25}$. Laura bought 7 magazines this week.*

Use the Magazines E  display master as needed.

Say: *In summary, it is a good rule of thumb to check for an easy scale factor first. If the scale factor is difficult, check whether a unit rate for the complete ratio can be found easily. If neither method is easy, use cross products.*

Say: *We can also use the other strategies, such as a table or common denominators, but often, those methods take more time. We want to be efficient; therefore, we want to pick the method that is accurate and requires the least amount of time.*

Practice

For the practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity: Help students complete the activity on the Practice handout. After every 2 problems, have students check their answers with a partner and discuss reasoning. Select a few students to verbalize their reasoning and each step in the process. Ensure that students use the correct mathematical language in their explanations.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.
2. Go over the answers (students self-check and correct, using a colored pencil).
3. Have students record the number correct in the box and complete their How Am I Doing? graph.
4. Collect the papers to review and monitor student progress.

Closure

Review the key idea. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- How can you determine which strategy to use when solving a proportion?
- Could you solve a proportion by using multiple strategies? Why or why not?
Give an example to support your reasoning.

Clear up any misconceptions. Students who struggle to identify an efficient strategy for solving a proportion need additional instruction.