MSTAR INTERVENTION

L13

Lesson 13: Writing Proportions From Word Problems in Multiple Formats

Lesson Objective

- Students will write proportions in multiple formats from word problems.

Instructional Materials

Material	Quantity	Description
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Popsicle sticks with 1 student name on each	1 per student for teacher use	
Display Masters	1 each	• Preview: Key Idea: Writing Proportions From Word Problems • Demonstrate: Basketball A–J • Demonstrate: Mowing Lawns A–K
Handouts	1 per student	• Cumulative Review • Practice • Independent Practice
Answer Keys	1 each	• Cumulative Review • Practice • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of working with proportions to solve and determine proportionality.

Display and introduce through a brief explanation the key idea for this lesson:

- A proportion from a word problem can be written in multiple formats and still accurately describe the scenario.

Use the Key Idea: Writing Proportions From Word Problems DM display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, activate students' background knowledge and preskills by leading activities such as the following.

Have students discuss the following, or similar, questions with a partner. Ask 2–3 pairs to share their answers and reasoning with the whole group. Have the other students give a thumbs-up or a thumbs-down to show whether they agree with each shared answer. Ensure that students use the correct mathematical language in their answers and explanations.

- What quantities are being compared in the ratio $\frac{\text{2 apples}}{\text{4 bananas}}$?
- How can you determine whether ratios are proportional?

If students cannot answer these questions, stop and explicitly teach the material.

Demonstrate

1. Prove proportionality in different formats.

Say: *Today, we will learn how to write proportions from word problems in multiple formats.*

Say: *Consider the following problem. Robert makes 12 basketball shots for every 50 he tries. He has made 24 shots; therefore, he has attempted 100 shots. Set up a proportion that represents Robert's basketball statistics.*

Use the Basketball A DM display master as needed.

Say: *Let's highlight the key information.*

Use the Basketball B DM display master as needed.

Say: *From this situation, we can write a proportion comparing 2 ratios. It is important that when writing the proportion, we record the units, or labels, of the quantities that are compared to ensure that like ratios are compared.*

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *Let's use the Understand, Plan, Solve, Check graphic organizer to guide our thinking through this problem. Let's begin with understanding the problem. What is this problem asking? (to set up a proportion that represents Robert's basketball statistics)*

Say: *Now, let's plan how to solve this problem. What quantities am I comparing? (shots made and shots attempted)*

TEACHER NOTE
Model highlighting the key information. Have students use colored pencils to highlight and underline key information. Use the Basketball B display master as needed.

Say: *What do I know? (Robert made 12 shots out of 50 attempted and 24 shots out of 100 attempted)*

Say: *What quantities go together? (12 shots made and 50 shots attempted; 24 shots made and 100 shots attempted)*

Say: *How could I set this up? ($\frac{\text{shots made}}{\text{shots attempted}} = \frac{\text{shots made}}{\text{shots attempted}}$)*

Use the Basketball C **DM** display master as needed.

Say: *When comparing these 2 ratios, we can create the proportion $\frac{12 \text{ shots made}}{50 \text{ shots attempted}} = \frac{24 \text{ shots made}}{100 \text{ shots attempted}}$. Notice that the numerators of both ratios represent the number of shots Robert made, and the denominators of both ratios represent the total number of shots Robert attempted.*

Use the Basketball D **DM** display master as needed. 2

Say: *Now, I want to prove that we have created a proportion. First, let's look at the relationship between the 2 ratios. If Robert made 12 of 50 shots, then he made 24 of 100 shots. Is there a scale factor that we can multiply $\frac{12}{50}$ by to get $\frac{24}{100}$ (yes)*

Say: *What would we multiply the numerator and denominator by? (2) Because there is a relationship between the 2 ratios and we can multiply 1 fraction representing a ratio by a scale factor to get the fraction representing the other ratio, the ratios are proportional.*

Say: *We can also use cross products to prove proportionality. When we cross-multiply 50 by 24, we get 1,200. When we cross-multiply 100 by 12, we get 1,200. Because the products are equal, we have confirmed that we have a proportion.*

2 **TEACHER NOTE**

Allow students to use a calculator for computation as needed. It is more important for students to understand the concept than to do the calculations without a calculator.

Use the Basketball E **DM** display master as needed.

Say: *Another way to set up the proportion is to write it so that the number of shots attempted is in the numerator, and shots made are in the denominator. So, we could say* $\frac{50 \text{ shots attempted}}{12 \text{ shots made}} = \frac{100 \text{ shots attempted}}{24 \text{ shots made}}$.

Use the Basketball F **DM** display master as needed.

Say: *We could also set up the proportion with shots made in 1 ratio and compare them to shots attempted in the other ratio. So, we could say* $\frac{12 \text{ shots made}}{24 \text{ shots made}} = \frac{50 \text{ shots attempted}}{100 \text{ shots attempted}}$ *. Notice that the numerators of both ratios reflect the 12 shots made out of 50 shots attempted, and the denominators of both ratios reflect the 24 shots made out of 100 shots attempted.*

Use the Basketball G **DM** display master as needed.

Say: *The last way we could write the proportion builds off the last example. We could again compare shots made in 1 ratio and shots attempted in the other ratio. In this proportion, we could say* $\frac{24 \text{ shots made}}{12 \text{ shots made}} = \frac{100 \text{ shots attempted}}{50 \text{ shots attempted}}$ *. Notice that the numerators of both ratios reflect the 24 shots made out of 100 shots attempted, and the denominators of both ratios reflect the 12 shots made out of 50 shots attempted.*

Use the Basketball H **DM** display master as needed.

Say: *We can confirm that all the ratios are, in fact, proportional by using cross products. Each time we cross-multiply a numerator of 1 ratio by a denominator of the other ratio, we get the product 1,200.*

Say: *Now, we check the reasonableness of our proportions. Is my answer reasonable? (yes) How do I know? (because we proved a relationship exists and all cross products are the same, proving proportionality)*

Use the Basketball I **DM** display master as needed.

Say: *To model the different ways that we could write this proportion, we can begin with the original proportion. Then, we can turn our paper a quarter-turn to see the different proportion formats that are possible.* 3

Use the Basketball J DM display master as needed.

2. Write a proportion in different formats from a word problem.

Say: *Consider the following problem situation. Lucy receives $25 for mowing 2 lawns. At this rate, she would make $175 for mowing 14 lawns. Set up a proportion that represents Lucy's rate for mowing lawns.*

Use the Mowing Lawns A DM display master as needed.

Say: *Let's highlight the key information.* 4

Use the Mowing Lawns B DM display master as needed.

Say: *From this situation, we can write a proportion comparing 2 ratios.*

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *Let's begin with the Understand section of the graphic organizer. What is this problem asking? (to set up a proportion that represents Lucy's rate for mowing lawns)*

Say: *Now, let's plan how to solve this problem. What quantities am I comparing? (dollars made and lawns mowed)*

Say: *What do I know? (Lucy made $25 for mowing 2 lawns; Lucy made $175 for mowing 14 lawns)*

3 **TEACHER NOTE**
Model the quarter turns to show the different formats for the proportion. Stress that this is a way to model and see the different formats. Each turn will represent 1 of the 4 ratios created in the Solve quadrant of the graphic organizer.

4 **TEACHER NOTE**
Model highlighting the key information. Have students use colored pencils to highlight and underline key information. Use the Mowing Lawns B display master as needed.

Say: *What quantities go together? ($25 made and 2 lawns; $175 made and 14 lawns)*

Say: *How would I set this up? (* $\frac{\text{dollars made}}{\text{lawns mowed}} = \frac{\text{dollars made}}{\text{lawns mowed}}$ *)*

Use the Mowing Lawns C **DM** display master as needed.

Say: *When comparing these ratios, we can create the proportion* $\frac{\$25 \text{ made}}{2 \text{ lawns mowed}} = \frac{\$175 \text{ made}}{14 \text{ lawns mowed}}$. *Notice that the numerators of both ratios represent the amount of money Lucy made, and the denominators of both ratios represent the number of lawns Lucy mowed.*

Use Mowing Lawns D **DM** display master as needed. 5

Say: *Now, let's solve the problem by proving that we have created a proportion that represents the relationship described in the problem.*

Say: *First, let's look at the relationship between the 2 ratios. If Lucy made $25 for 2 lawns, then she made $175 for 14 lawns. Is there a scale factor that we can multiply* $\frac{25}{2}$ *by to get* $\frac{175}{14}$ *? (yes)*

Say: *What would we multiply the numerator and denominator by? (7) Because there is a relationship between the 2 ratios and we can multiply 1 fraction representing a ratio by a scale factor to get the fraction representing the other ratio, the ratios are proportional.*

Say: *We can also use cross products to prove proportionality. When we cross-multiply 2 by 175, we get 350. When we cross-multiply 14 by 25, we get 350. Because the products are equal, we have confirmed that we have a proportion.*

5 **TEACHER NOTE**

Allow students to use a calculator for computation as needed. It is more important for the students to understand the concept than to do the calculations without a calculator.

Use the Mowing Lawns E **DM** display master as needed.

Say: *Another way to set up the proportion is to write it so that the number of lawns mowed is in the numerator and the dollars made are in the denominator. So, we could say* $\frac{2 \text{ lawns mowed}}{\$25 \text{ made}} = \frac{14 \text{ lawns mowed}}{\$175 \text{ made}}$.

Use the Mowing Lawns F **DM** display master as needed.

Say: *We could also set up the proportion with dollars made in 1 ratio and compare it to lawns mowed in the other ratio. So, we could say* $\frac{\$25 \text{ made}}{\$175 \text{ made}} = \frac{2 \text{ lawns mowed}}{14 \text{ lawns mowed}}$ *. Notice that the numerators of both ratios reflect the $25 made from 2 lawns mowed, and the denominators of both ratios reflect the $175 made from 14 lawns mowed.*

Use the Mowing Lawns G **DM** display master as needed.

Say: *The last way we could write the proportion builds off the last example. We could again compare dollars made in 1 ratio and lawns mowed in the other ratio. In this proportion, we could say* $\frac{\$175 \text{ made}}{\$25 \text{ made}} = \frac{14 \text{ lawns mowed}}{2 \text{ lawns mowed}}$ *. Notice that the numerators of both ratios reflect the $175 made from 14 lawns mowed, and the denominators of both ratios reflect the $25 made from 2 lawns mowed.*

Use the Mowing Lawns H **DM** display master as needed.

Say: *We can confirm that all the ratios are, in fact, proportional by using cross products. Each time we cross-multiply a numerator of 1 ratio by a denominator of the other ratio, we get the product 350.*

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *Now, we check the reasonableness of our proportions. Is my answer reasonable? (yes) How do I know? (we proved a relationship exists and all cross products are the same, proving proportionality)*

Use the Mowing Lawns I **DM** display master as needed.

Say: *To model the different ways that we could write this proportion, we can begin with the original proportion. Then, we can turn our paper a quarter-turn to see the different proportion formats that are possible.*

Use the Mowing Lawns J **DM** display master as needed.

Say: *From the proportions displayed on the screen, we will choose which represent our problem. We will put a check mark by the proportions that represent our problem and an "x" on the proportions that do not.*

Use the Mowing Lawns K **DM** display master as needed.

Guide students through choosing the proportions that represent the problem.

Say: *Name the letters of the proportions that represent the problem. (A, D, E, H) Which proportions do not represent this situation? (B, C, F, G)*

Say: *Let's look at the proportion in the B row. This proportion is comparing dollars made in the numerator to lawns mowed in the denominator. The first ratio says Lucy made $25 for 2 lawns, which is a true statement. However, the second ratio says Lucy made $14 for 175 lawns. This is not a true statement. We can use cross products to prove this is not a proportion. If we multiply the denominator 2 by the numerator 14, what will we get? (28) If we multiply the denominator 175 by the numerator 25, what will we get? (4,375) Because the cross products are not equal, a proportion does not exist.*

Discuss answer choices C, F, and G, using the language from above. Students should understand why these answer choices do not represent the relationship described in the problem and are not proportional.

Practice

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice handout. Have students check their answers with a partner and discuss reasoning. Select a few students to verbalize their reasoning. Ensure that students use the correct mathematical language in their explanations.

Activity 2: Have students work in pairs to come up with multiple proportions for a given situation. Then, provide students with a set of cards representing possible proportion formats. Students should divide the cards into proportions that do and do not represent the relationship described in the word problem. Have students explain how they determined which proportions are correct.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.
2. Go over the answers (students self-check and correct, using a colored pencil).
3. Have students record the number correct in the box and complete their How Am I Doing? graph.
4. Collect the papers to review and monitor student progress.

Closure

Review the key idea. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- How do you know when a proportion represents the relationship described in a word problem?
- Why is it important to know that several different proportions can accurately represent the same relationship?

Clear up any misconceptions. Students who struggle to write proportions in multiple formats or to determine which formats are correct need additional instruction.