MSTAR INTERVENTION

L16

Lesson 16: Applying Proportionality to Similar Figures

Lesson Objectives

- Students will set up and solve proportions in similar figure application problems involving missing lengths.

Instructional Materials

Material	Quantity	Description
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Display Masters	1 each	• Preview: Key Ideas: Similar Figures • Demonstrate: Door A–E • Demonstrate: Triangle A–E
Handouts	1 per student	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice
Answer Keys	1 each	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of using proportions to find missing values in proportional relationships.

Display and introduce through a brief explanation the key ideas for this lesson:

- For 2 figures to be similar, all corresponding angles must be congruent and all pairs of corresponding lengths must be proportional, or form the same ratio.
- To find the missing length in a pair of similar figures, set up a proportion comparing corresponding side lengths.

Use the Key Ideas: Similar Figures DM display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, present problems to activate students' background knowledge and preskills, such as the following:

Set up and solve the proportions from the following problem situations:

- In a bag of jelly beans, there are 5 red for every 6 blue. If there are 30 blue jelly beans, how many red jelly beans are in the bag?
- On the highway, there are 12 cars for every 3 trucks. If there are 18 trucks, how many cars are there?

Demonstrate

1. Show that corresponding side lengths of similar figures are proportional and that the ratio comparing any 2 lengths of 1 figure is equal to the ratio comparing corresponding lengths in a similar figure.

Say: *In the previous lesson, we used proportions to find missing values in problems involving percents. Today we will use proportions to find missing lengths in similar figures.*

Say: *First, we need to recall the definition of "similar figures." For 2 figures to be similar, all corresponding angles must be congruent and all pairs of corresponding lengths must be proportional, or form the same ratio.* 1

Say: *Can you think of an example of 2 figures that have the same shape but are different sizes?*

Allow students to share their ideas before discussing them with the group. Provide examples of figures that are similar, such as squares, and examples of figures that are nonsimilar.

Say: *In this example, we will explore 2 additional characteristics of similar figures:*

- *Corresponding side lengths of similar figures are proportional.*
- *The ratio comparing any 2 lengths in 1 figure is equal to the ratio comparing corresponding lengths in a similar figure.*

1 **TEACHER NOTE**
Students should understand the meaning of "congruent," but review the concept if necessary.

TEACHER NOTE
For this and subsequent examples, use colored markers to connect the side of each figure to its corresponding place in the proportion.

Say: *Consider the following scenario: A scale drawing of a door measures 9 centimeters high and 3 centimeters wide. The actual door measures 180 centimeters high and 60 centimeters wide. The scale drawing and the actual door are similar figures.*

Use the Door A **DM** display master as needed. Point to each side of the figure as you identify it by the appropriate label.

Say: *Let's explore the first characteristic: Corresponding side lengths of similar figures are proportional.*

Say: *In this drawing, side AB corresponds to side EF, side CD corresponds to side GH, side AC corresponds to side EG, and side BD corresponds to side FH. If the characteristic is true, then* $\frac{\overline{CD}}{\overline{GH}} = \frac{\overline{BD}}{\overline{FH}}$*. Let's see whether this characteristic holds true by substituting the values for each side length. The ratio is* $\frac{3\text{ cm}}{60\text{ cm}}$ *and* $\frac{9\text{ cm}}{180\text{ cm}}$.

Use the Door B **DM** display master as needed.

Say: *We need to see whether the 2 ratios,* $\frac{3}{60}$ *and* $\frac{9}{180}$*, are proportional. We know that if 2 ratios are proportional, they simplify to the same ratio.*

Say: *Consider* $\frac{3}{60}$ *. What is the greatest common factor of 3 and 60? (3) To simplify the ratio, we must divide the numerator and denominator of the fraction representing the ratio by 3. What is the simplified ratio? (*$\frac{1}{20}$*)*

Say: *Now, let's consider* $\frac{9}{180}$ *. What is the greatest common factor of 9 and 180? (9) We must divide the numerator and denominator of the fraction representing the ratio by 9. What is the simplified ratio? (*$\frac{1}{20}$*) Because the ratios simplify to the same ratio, they are proportional.*

Use the Door C DM display master as needed.

Say: *Next, we will explore the second characteristic: The ratio comparing 2 lengths in 1 figure is equal to the ratio comparing corresponding lengths in a similar figure.*

Say: *We can set up the ratios* $\frac{\overline{BD}}{\overline{CD}}$ *and* $\frac{\overline{FH}}{\overline{GH}}$*. Corresponding sides are in the same positions in the fraction representing each ratio. When using the measurements of the figures, we are considering the ratios* $\frac{9 \text{ cm}}{3 \text{ cm}}$ *and* $\frac{180 \text{ cm}}{60 \text{ cm}}$*.*

Use the Door D DM display master as needed.

Say: *To determine whether the ratios comparing the lengths in each figure are equal, we must simplify the ratios.*

Say: *Let's begin with the ratio* $\frac{9}{3}$*. What is the greatest common factor of 9 and 3? (3) We divide the numerator and denominator of the fraction representing the ratio by 3. What is the simplified ratio? (*$\frac{3}{1}$*)*

Say: *Now, let's consider the ratio* $\frac{180}{60}$*. What is the greatest common factor of 180 and 60? (60) Divide the numerator and denominator of the fraction representing the ratio by 60. What is the simplified ratio? (*$\frac{3}{1}$*) The ratios are equivalent, proving that the ratios comparing corresponding sides of 2 similar figures are equal.*

Use the Door E DM display master as needed.

2. Find the missing length in a set of similar figures, using a proportion.

Say: *Now that we have shown both characteristics to be true, we can use them to set up a proportion to find missing side lengths. In this example, we will use both characteristics.*

Say: *Consider the following situation: The 2 right triangles shown are similar. If the shortest length of the smaller triangle is 9 centimeters, what is the length of the longest side, or hypotenuse, of the smaller triangle?*

Use the Triangle A **DM** display master as needed.

Say: *First, we will set up a proportion that uses corresponding side lengths of the similar figures to find the missing length.*

Say: *In this problem, the shortest sides of both triangles are given and the longest side, or hypotenuse, of the larger triangle is given. We are looking for the length of the hypotenuse of the smaller triangle.*

Say: *We set up a proportion that compares the shortest sides of the triangles and the hypotenuses of the triangles:* $\frac{\text{shortest of smaller}}{\text{shortest of larger}} = \frac{\text{hypotenuse of smaller}}{\text{hypotenuse of larger}}$*. By substituting the side lengths, we get the proportion* $\frac{9\text{ cm}}{12\text{ cm}} = \frac{x}{20\text{ cm}}$ *. The x represents the unknown side length.*

Use the Triangle B **DM** display master as needed.

Say: *Now that we have a proportion, we can use a strategy for solving proportions to find the missing length. Because there is not a friendly scale factor that we can multiply by, we will use cross products to find the missing length.*

Say: *What are the cross products that we can set equal to each other? (12x = 180) What do you get when you solve for x? (15) So, x = 15 centimeters. The length of the hypotenuse of the smaller triangle is 15 centimeters.*

Use the Triangle C **DM** display master as needed.

Say: *Another way to solve this problem is to set up a proportion that uses side lengths from the smaller triangle in the first ratio and side lengths from the larger triangle in the second ratio.*

Say: *We set up the proportion comparing* $\frac{\text{shortest of larger}}{\text{hypotenuse of larger}} = \frac{\text{shortest of smaller}}{\text{hypotenuse of smaller}}$ *.*

Substituting the side lengths, we get the proportion $\frac{12\text{ cm}}{20\text{ cm}} = \frac{9\text{ cm}}{x}$*. The x represents the unknown side length.*

Use the Triangle D **DM** display master as needed. Allow students to attempt solving the proportion before walking them through the steps.

Say: *Because there is not a friendly scale factor that we can multiply by, we will use cross products to find the missing length.*

Say: *What are the cross products? (180 and 12x) Set the products equal to each other: 180 = 12x. When you solve for x, what do you get? (15) So, 15 centimeters = x. Using a different set of proportions, we have found the length of the hypotenuse of the smaller triangle to be 15 centimeters.*

Use the Triangle E **DM** display master as needed.

Say: *These 2 examples show how we can find a missing side length by using either of the 2 characteristics discussed in this lesson.*

Practice

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice 1 handout.

Activity 2: Have students work in pairs to complete the activity on the Practice 2 handout.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.
2. Go over the answers (students self-check and correct, using a colored pencil).
3. Have students record the number correct in the box and complete their How Am I Doing? graph.
4. Collect the papers to review and monitor student progress.

Closure

Review the key ideas. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- What do you have to know about 2 figures before you can use proportions to compare their side lengths?
- Can you think of a shape that is always proportional to another figure that has its same shape? (square)

Clear up any misconceptions. Students who are not confident with the process of using proportions to find missing lengths in similar figures need additional instruction.