Lesson 3: Determine Multiplicative and Additive Thinking

Lesson Objective

- Students will determine the difference between additive and multiplicative thinking by using a table.

Instructional Materials

Material	Quantity	Description
Color tiles	1 set of red, 1 set of yellow	
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Popsicle sticks with 1 student name on each	1 per student for teacher use	
Whiteboard and dry-erase marker	1 per student	
Display Masters	1 each	• Preview: Key Ideas: Multiplicative vs. Additive Thinking • Demonstrate: Apple Addition • Demonstrate: Apple Multiplication • Demonstrate: Fruit Punch A–F • Demonstrate: How Old? A–F • Demonstrate: Grapes and Apples A–E
Handouts	1 per student	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice
Answer Keys	1 each	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of using addition or multiplication to determine the solution to a problem situation.

Display and introduce through a brief explanation the key ideas for this lesson:

- Additive thinking is present when a constant number is added to a value to get the resulting value.
- Multiplicative thinking is present when a value is multiplied by a constant to get the resulting value.
- A proportional relationship exists when multiplicative thinking is present.

Use the Key Ideas: Multiplicative vs. Additive Thinking **DM** display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, activate students' background knowledge and preskills by leading 1 of the following activities:

- Have students write their answer to the following questions on a whiteboard. Then, have students compare their answers with a partner's. After discussion, have students hold up their whiteboards. Finally, check the answers and discuss why some responses are different.
  - ◇ If I begin with 2 and end with 6, what process might I have used? (add 4 or multiply by 3)
  - ◇ If I begin with 4 and end with 8, what process might I have used? (add 4 or multiply by 2)
  - ◇ If I begin with 8 and end with 24, what process might I have used? (add 16 or

multiply by 3)

- ◇ If I begin with 5 and end with 20, what process might I have used? (add 15 or multiply by 4)

- Have each student draw this image on a whiteboard:

  _________ ☐ ________ = ____________

  Then, have each student write a number in the first blank and in the answer blank. Finally, have students trade with a partner and fill in the operation and second number blank to make the number sentence true. Repeat this activity 3 times.

If students cannot complete these activities, stop and explicitly teach the material.

Demonstrate

1. Define additive and multiplicative thinking.

Say: *In the previous lesson, we learned how to determine proportionality by using simplification. Today, we will determine whether relationships require additive thinking or multiplicative thinking.*

Say: *When you were in elementary school, you learned that addition is used to combine values. For example, if I have 3 apples and add 6 apples, I have 9 apples total. This is additive thinking.*

Use the Apple Addition DM display master as needed.

Say: *When you learned to multiply, you were probably told that multiplication is the addition of equal groups. Though repeated addition is still the underlying concept of multiplication, we developed a more efficient way to add equal groups. If I have 3 groups of 6 apples, I have 18 apples total. So 3 times 6 is equal to 18. This is multiplicative thinking.*

Use the Apple Multiplication DM display master as needed.

Say: *It is important to understand that 3 more than a number and 3 times a number have a different meaning.*

Say: *In algebraic terms, it is important to understand the difference*

1 TEACHER NOTE
It is important to emphasize throughout this demonstration the relationship between proportionality and multiplicative thinking. Use color tiles to model the relationship in the examples. Ensure that students use the appropriate mathematical language when responding to questions or explaining their thinking.

2 TEACHER NOTE
Set out another red tile and 3 more yellow tiles. Use the Process column on the display master to record students' findings about the possible processes being used. Model the remaining questions with the tiles.

between additive and multiplicative thinking because multiplicative thinking refers to proportional relationships. We will explore this idea further in the examples.

2. Differentiate between multiplicative and additive thinking in a table. 1

Say: *Suppose I am given the following scenario: 3 cups of orange juice are used for every cup of cranberry juice when making fruit punch.*

Use the Fruit Punch A **DM** display master as needed.

Set out 1 red color tile to represent the cup of cranberry juice and 3 yellow color tiles to represent the cups of orange juice.

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *Let's look at this table. If I use 1 cup of cranberry juice, then I use 3 cups of orange juice. What process might I use to get from 1 to 3 in the table? (add 2 or multiply by 3) If I use 2 cups of cranberry juice, then I use 6 cups of orange juice. What process might I use to get from 2 to 6? (add 4 or multiply by 3)*

2 Use the Fruit Punch B **DM** display master as needed.

Say: *Before, we thought about the process to get from 1 to 3. We thought it might be adding 2 or multiplying by 3. Now that we have modeled this situation with color tiles and filled out our table, what is the common process between the rows? (multiplying by 3)*

How many cups of orange juice for 3 cups of cranberry juice? (9) For 4 cups of cranberry juice? (12)

Use the Fruit Punch C **DM** display master as needed.

> **Say:** *If the process I use to get from the x column (cranberry juice) to the y column (orange juice) is to multiply by 3, how many cups of orange juice are used for 10 cups of cranberry juice? (30)*

> **Say:** *To determine how many cups of orange juice are used for any amount of cranberry juice, I can generate a rule that will always hold true. A rule will make our work more efficient. Following our pattern, if I have any amount of cranberry juice, how would I find the amount of orange juice needed? (multiply it by 3)*

> **Say:** *When writing an algebraic expression, I do not need to use parentheses to show that I am multiplying the number and the variable together, so I just write "3x."*

> **Say:** *Therefore, y = 3x is the algebraic rule for finding the number of cups of orange juice for a given number of cups of cranberry juice. Because this process involves multiplying the x value by a constant number or rate to get the y value, it represents multiplicative thinking.*

Use the Fruit Punch D **DM** display master as needed.

> **Say:** *Multiplicative thinking indicates that a proportional relationship exists. To prove this, let's take 2 sets of values from the table to determine whether they are proportional. I need to set up the 2 ratios in a proportion. Let's use* $\frac{2}{6} = \frac{3}{9}$.

Use the Fruit Punch E **DM** display master as needed.

> **Say:** *Remember that to prove whether 2 ratios are proportional, I need to simplify each ratio and compare. If I begin with* $\frac{2}{6}$ *, I ask myself, "What*

is the common factor between 2 and 6?" (2) Because 2 is the common factor, I divide the numerator and denominator by 2. When I divide the numerator, 2, by 2, I get 1. When I divide the denominator, 6, by 2, I get 3. Therefore, $\frac{2}{6}$ *simplifies to* $\frac{1}{3}$.

Say: *Now, let's consider* $\frac{3}{9}$. *What is the common factor between 3 and 9? (3) Because 3 is the common factor, I divide the numerator and denominator by 3. When I divide the numerator, 3, by 3, I get 1. When I divide the denominator, 9, by 3, I get 3. Therefore,* $\frac{3}{9}$ *simplifies to* $\frac{1}{3}$. *Because both* $\frac{2}{6}$ *and* $\frac{3}{9}$ *simplify to* $\frac{1}{3}$, *the ratios are proportional.*

Use the Fruit Punch F **DM** display master as needed.

Say: *Now, suppose I am given the following scenario: James was 4 years old when his sister Lisa was born.*

Use the How Old? A **DM** display master as needed.

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *When Lisa turned 1, James was 5. What processes can I use to get from 1 to 5? (add 4 or multiply by 5) When Lisa was 2, James was 6. What processes could I use to get from 2 to 6? (add 4 or multiply by 3)*

Record the possible processes in the Process column. Use the How Old? B **DM** display master as needed.

Say: *What is the common process for all the rows to get from Lisa's age, x, to James' age, y? (adding 4) How old was James when Lisa turned 3? (7)*

Use the How Old? C **DM** display master as needed.

Say: *If the process that I am applying to the x column is adding 4, how old would James be when Lisa turns 12? (16) If Lisa's age is x, what is the general rule to find James' age, y? (y = x + 4)*

Say: *Therefore, $y = x + 4$ is the algebraic rule for finding James' age, given Lisa's age. Because the process involves adding a constant to the x value to get the y value, it represents additive thinking.*

Use the How Old? D DM display master as needed.

Say: *When additive thinking is present in an algebraic relationship, a proportional relationship does not exist. To prove this, let's consider 2 sets of values from the table. Let's consider the 2 ratios $\frac{1}{5}$ and $\frac{2}{6}$.*

Use the How Old? E DM display master as needed.

Say: *Remember that to determine whether 2 ratios are proportional, I need to simplify each ratio and compare.*

Simplify the 2 ratios to determine whether they are equivalent. Demonstrate the steps as necessary. Use the How Old? F DM display master as needed.

Say: *Because the ratios do not simplify to the same ratio, they are not proportional.*

3. Create a table of values from a given scenario.

Say: *In the previous 2 examples, we learned how to determine whether a situation reflected multiplicative or additive thinking, based on a table of values. We also found the rule for the table. Now, we will create a table of values from a given scenario, determine whether the scenario reflects multiplicative or additive thinking, and find the rule for the table of values.*

Say: *Consider this scenario. Julie is making a fruit salad. The recipe asks for 4 apples for every 1 bunch of grapes.*

Use the Grapes and Apples A DM display master as needed. 3

Say: *Now, we need to develop a table. X will represent the number of grape bunches, and y will represent the number of apples. From the recipe, we*

TEACHER NOTE
Use colored tiles to model the number of grape bunches and the number of apples. As you are asking guiding questions, complete the table on the display master. Record the possible processes in the Process column.

know that for every 1 bunch of grapes, she uses 4 apples.

Use the Grapes and Apples B DM display master as needed.

Select a popsicle stick to choose a student to answer each of the following questions. Ensure that students use the correct mathematical language in their responses.

Say: *What processes might we use to get from 1 to 4? (add 3 or multiply by 4)*

Set out 1 yellow color tile to represent the grape bunch and 4 red color tiles to represent the apples.

Say: *How many apples will Julie use if she uses 2 bunches of grapes? (8) What process might we use to get from 2 to 8? (add 6 or multiply by 4)*

Set out another yellow tile to represent the second grape bunch. Then, model setting out 4 more red color tiles to represent 4 more apples. Now, you should have 2 yellow tiles and 8 red tiles.

Use the Grapes and Apples C DM display master as needed.

Say: *Let's look at the table. What is the common process between all the rows? (multiply by 4) Using this process, how many apples should there be for 3 bunches of grapes? (12) Let's check our model.*

Set out 1 more bunch of grapes and 4 more apples. Confirm that there are 12 apples in all. Repeat with 4 bunches of grapes and 16 apples.

Use the Grapes and Apples D DM display master as needed.

Say: *If the process that I am applying to the x column is multiplying by 4, what is the general rule if Julie uses x* *grape bunches? (y = 4x)*

Select a popsicle stick to choose a student to answer. Ensure that students use the correct mathematical language in their responses.

Use the Grapes and Apples E **DM** display master as needed.

Say: *Given the completed table and rules, what type of thinking does this scenario reflect? (multiplicative)*

Practice

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice 1 handout. Have students share their answers with a partner and discuss their reasoning. Select a few pairs to verbalize their reasoning and each step in the process. Ensure that students use the correct mathematical language in their explanations.

Activity 2: Have students work in pairs to complete the activity on the Practice 2 handout. Have students verbalize their reasoning and each step in the process to their partners.

Circulate to monitor student progress.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.
2. Go over the answers (students self-check and correct, using a colored pencil).
3. Have students record the number correct in the box and complete their How Am I Doing? graph.
4. Collect the papers to review and monitor student progress.

Closure

Review the key ideas. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- A proportional relationship is present when which operation is used to get from the *x* value to the *y* value? How can you show this is true?
- What kind of situation uses additive thinking? Give an example.

Clear up any misconceptions. Students who struggle with identifying the type of thinking used in a table need additional instruction.