MSTAR INTERVENTION

L7

Lesson 7: Determine Proportionality, Using Common Denominators

Lesson Objective

- Students will determine whether 2 ratios are proportional by using common denominators.

Instructional Materials

Material	Quantity	Description
How Am I Doing? graph	1 per student	
Colored pencils	1 per student	
Index cards with questions	1 per group	Each index card should have all 3 questions (see below), labeled A, B, and C.
Display Masters	1 each	• Preview: Key Idea: Determine Proportionality • Demonstrate: Boys to Girls A–I • Demonstrate: Tootsie Rolls to Smarties A–F
Handouts	1 per student	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice
Answer Keys	1 each	• Cumulative Review • Practice 1 • Practice 2 • Independent Practice

Cumulative Review

Have students answer the questions on the Cumulative Review handout. Go over the answers. Correct misconceptions. Have students use a colored pencil to make corrections as needed. Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' conceptual knowledge of determining proportionality and finding common denominators.

Display and introduce through a brief explanation the key idea for this lesson:

- 2 ratios are proportional if the equivalent ratios resulting from finding a common denominator have the same numerator.

Use the Key Idea: Determine Proportionality DM display master as needed.

Engage Prior/Informal Knowledge

To open the lesson, activate students' background knowledge and preskills by leading activities such as the following:

Before class, write the following 3 questions on index cards. Divide students into groups of 3. Give 1 index card to each group and assign each person in the group 1 of the questions (A, B, or C). Have each person in the group explain his or her answer to the other members in the group. Choose 3 individuals to share their answers with the whole group. Ensure that students use the correct mathematical language in their answers and explanations.

A. What is a common denominator for the fractions $\frac{2}{3}$ and $\frac{3}{4}$? (12, 24)

B. How do you know whether 2 ratios are proportional? (they are equivalent, they simplify to the same ratio)

C. Are the ratios $\frac{2}{6}$ and $\frac{4}{12}$ proportional? (yes)

If students cannot answer these questions, stop and explicitly teach the material.

Demonstrate

1. Find a common denominator to determine proportionality.

Say: *Previously, we learned that 2 ratios are proportional if they are equivalent. Today, we will learn how to use common denominators to determine whether 2 ratios are equivalent, thus making them proportional.*

Say: *Listen to this scenario. In Math Club, there are 6 boys and 9 girls. In Book Club, there are 4 boys and 6 girls. Are the ratios of boys to girls in each club proportional?*

Say: *Consider the 2 ratios* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *and* $\frac{4 \text{ boys}}{6 \text{ girls}}$*. To determine whether these 2 ratios are proportional, we must find whether they are equivalent.*

Say: *Because there is not a friendly scale factor to multiply the numerators and denominators of the first ratio by to get the second ratio, I will have to create 2 ratios with a common denominator that are equivalent to* $\frac{6}{9}$ *and* $\frac{4}{6}$ *and then compare them to determine whether they are proportional. If the 2 new ratios are proportional, then* $\frac{6}{9}$ *and* $\frac{4}{6}$ *are proportional as well.*

Say: *First, we must find a common multiple of 9 and 6, which are the denominators of the 2 ratios. An obvious common multiple is the product of 9 and 6, which is 54. We can use 54 as our common denominator. We will be able to find out how many boys there would be for every 54 girls by using each ratio. If the ratios are proportional, we will get the same number of boys for 54 girls.*

Use the Boys to Girls A **DM** display master as needed.

Say: *Now that we have a common denominator, we will create a ratio equivalent to* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *with a denominator of 54 girls and a ratio*

equivalent to $\frac{4 \text{ boys}}{6 \text{ girls}}$ *with a denominator of 54 girls.*

Say: *Let's begin with* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *. I need to create a proportion to find the missing number of boys. I can then use the strategy of multiplying by a scale factor to find the number of boys in the second ratio.*

Use the Boys to Girls B **DM** display master as needed.

Say: *Because I know that the 2 ratios must be proportional, I am looking for the complete ratio that is equivalent to* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *and has a denominator of 54 boys. I am given both denominators, representing the number of girls: 9 and 54. So, I think to myself, "9 times what number equals 54?"*

Use the Boys to Girls C **DM** display master as needed.

Say: *I know that 9 times 6 equals 54. This means there are 6 times as many girls in the second ratio as in the first.*

Use the Boys to Girls D **DM** display master as needed.

Say: *To keep the ratios proportional, we must multiply by a scale factor. Therefore, if I multiply the denominator by 6, I must multiply the numerator by 6 as well. If there are 6 times as many girls, there must be 6 times as many boys for the ratios to be proportional.*

Use the Boys to Girls E **DM** display master as needed.

Say: *When I multiply the numerator, 6, by 6, to find out how many boys are in the second ratio, I get 36 as the numerator for the equivalent ratio. If there are 6 times more boys in the second ratio, the second ratio has 36 boys to 54 girls. This means that if the ratio of boys to girls is 6 to 9, and there are 54 girls total, there would have to be 36 boys in Math Club.*

Use the Boys to Girls F **DM** display master as needed.

Say: *Now, let's consider the ratio* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *. I need to create a proportion with a missing value.*

Use the Boys to Girls G **DM** display master as needed. Give students a chance to find the missing value on their own.

Say: *Because I know that the 2 ratios must be proportional, I am looking for the complete ratio that is equivalent to* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *and has a denominator of 54 girls. I can use the same strategy for finding a missing value as in the first example. What is the equivalent ratio? (* $\frac{36 \text{ boys}}{54 \text{ girls}}$ *)*

Use the Boys to Girls H **DM** display master as needed.

Say: *Because we found that* $\frac{6 \text{ boys}}{9 \text{ girls}}$ *and* $\frac{4 \text{ boys}}{6 \text{ girls}}$ *are both equivalent to* $\frac{36 \text{ boys}}{54 \text{ girls}}$ *by using a common denominator, we know they are equivalent. Therefore, the ratios of boys to girls in each club are proportional.*

Use the Boys to Girls I **DM** display master as needed.

2. Find a common denominator to determine proportionality.

Say: *Let's try another scenario: Juan and Olivia both had birthday parties this weekend. Each person gave away candy bags to all of their guests. In each of Juan's candy bags, he put 2 Tootsie Rolls for every 3 Smarties. In each of Olivia's bags, she put 5 Tootsie Rolls for every 8 Smarties. Are the ratios of Tootsie Rolls to Smarties in Juan and Olivia's bags proportional?*

Say: *Consider the ratios* $\frac{2 \text{ Tootsie Rolls}}{3 \text{ Smarties}}$ *and* $\frac{5 \text{ Tootsie Rolls}}{8 \text{ Smarties}}$ *. To determine whether these 2 ratios are proportional, we must find whether they are equivalent.*

Say: *First, we must find a common multiple of 3 and 8, which are the*

denominators of the 2 ratios. An obvious common multiple is the product of 3 and 8, which is 24. We can use 24 as our common denominator. To decide whether the 2 ratios are equivalent, we will figure out how many Tootsie Rolls there would be if 24 Smarties were given out for each ratio.

Say: *Now that we have a common denominator, we must create a ratio equivalent to* $\frac{2 \text{ Tootsie Rolls}}{3 \text{ Smarties}}$ *with a denominator of 24 Smarties and a ratio equivalent to* $\frac{5 \text{ Tootsie Rolls}}{8 \text{ Smarties}}$ *with a denominator of 24 Smarties.*

Use the Tootsie Rolls to Smarties A **DM** display master as needed.

Say: *Let's begin with* $\frac{2 \text{ Tootsie Rolls}}{3 \text{ Smarties}}$ *. I need to create a proportion with a missing value. I can then use the strategy of multiplying by a scale factor to find the numerator of the second ratio. The scale factor would tell me how many times more Smarties are in the second ratio.*

Use the Tootsie Rolls to Smarties B **DM** display master as needed. Give students a chance to find the missing value on their own and share answers with a partner. Ensure that students use the correct mathematical language in their answers and explanations.

Say: *Because I know that the 2 ratios must be proportional, I am looking for the complete ratio that is equivalent to* $\frac{2}{3}$ *and has a denominator of 24. I can use the same strategy for finding a missing value as in the first example. Work on your own to answer the following questions and then share your answers with a partner:*

- *How many times more Smarties are in the second ratio than in the first? (8 times)*
- *What is the equivalent ratio? (* $\frac{16 \text{ Tootsie Rolls}}{24 \text{ Smarties}}$ *)*

Choose 2–3 students to share their answers with the group.

Use the Tootsie Rolls to Smarties C **DM** display master as needed.

Say: *Now, let's consider* $\frac{5 \text{ Tootsie Rolls}}{8 \text{ Smarties}}$ *. I need to create a proportion with a missing value.*

Use the Tootsie Rolls to Smarties D DM display master as needed. Give students a chance to find the missing value on their own and share answers with a partner. Ensure that students use the correct mathematical language in their answers and explanations.

Say: *Because I know the 2 ratios should be proportional, I am looking for the complete ratio that is equivalent to* $\frac{5}{8}$ *and has a denominator of 24. I can use the same strategy for finding a missing value as in the first example. Work on your own to answer the following questions and then share your answers with a partner:*

- *How many times more Smarties are in the second ratio than in the first? (3 times)*
- *What is the equivalent ratio? (* $\frac{15}{24}$ *)*

Choose 2–3 students to share their answers with the group.

Use the Tootsie Rolls to Smarties E DM display master as needed.

Say: *By comparing the number of Tootsie Rolls with a common denominator of 24 Smarties for each ratio, I can find out whether the 2 ratios are proportional. Because the equivalent number of Tootsie Rolls to 24 Smarties is not the same, I know that* $\frac{2}{3}$ *and* $\frac{5}{8}$ *are not equivalent. Therefore, the ratios of Tootsie Rolls to Smarties in Juan and Olivia's bags are not proportional.*

Use the Tootsie Rolls to Smarties F DM display master as needed.

Practice

For each practice activity, provide detailed feedback to students, highlighting what was done correctly and what needs improvement. Provide opportunities for students to correct their errors. Collect student work to review and monitor student progress.

Activity 1: Help students complete the activity on the Practice 1 handout. At each of the indicated places, have students check their answers with a partner and discuss reasoning. Select a few students to verbalize their reasoning and each step in the process. Ensure that students use the correct mathematical language in their explanations.

Activity 2: Have students work in pairs to complete the activity on the Practice 2 handout. Have students verbalize their reasoning and each step in the process to their partner.

Independent Practice

1. Have students work independently to complete the activity on the Independent Practice handout.

2. Go over the answers (students self-check and correct, using a colored pencil).

3. Have students record the number correct in the box and complete their How Am I Doing? graph.

4. Collect the papers to review and monitor student progress.

Closure

Review the key idea. Have students provide examples from the lesson.

Have students discuss their answer to the following questions:

- In this lesson, what steps did we use to determine whether 2 ratios are proportional?
- How does finding a common denominator help you determine whether 2 ratios are proportional?

Clear up any misconceptions. Students who struggle with determining proportionality by using common denominators need additional instruction.